

## STUDY OF THE CNC LATHE CONTROL PARAMETER INFLUENCE UPON THE CONTOURING ACCURACY AT THE LINEAR INTERPOLATION

Dumitru DUMITRU<sup>1</sup>, Eugen STRĂJESCU<sup>2</sup>

*În această lucrare se studiază influențele parametrilor servo și dinamicii axelor asupra preciziei de conturare la interpolarea liniară într-un sistem de două axe numerice, specific strungurilor cu CNC.*

*Pentru aceasta se propune un model simplificat al sistemului de avans al strungului care include structura mecanică și parametrii principali servo, pe baza căruia se obține prin simulare în mediul Matlab/Simulink estimarea erorii la interpolarea liniară. Pentru generarea parametrilor cinematici de referință utilizați la simularea interpolării se propune un model Matlab simplu, validat de rezultatele simulărilor realizate pe prototipul mecatronic al strungului cu CNC.*

*The influences of the axes servo-parameters and dynamics on the contouring accuracy at the linear interpolation in a system with two numerical axes, typical to CNC lathes, are studied in this paper.*

*For this, a simplified model of the lathe feed system is proposed, where the mechanical structure and the main servo-parameters are included. On its basis, the estimation of the linear interpolation error is obtained by simulation in the Matlab/Simulink software. A simple Matlab model is proposed for the generation of the reference kinematic parameters used in the interpolation simulation. This model is validated by the results of the simulations performed on the mechatronic prototype of the CNC lathe.*

**Key words:** interpolation, contouring accuracy, simulation, CNC lathe.

### 1. Introduction

The main task of the CNC lathe numerical axis servo-system resides in the machine axis control in order to ensure that the geometry described by the part program is manufactured as quickly as possible, with the required contouring accuracy. This accuracy is limited by phenomena connected to: (a) the cutting process effects, (b) the inaccuracy of the mechanical components and (c) the servo-control and the dynamics of the axes, [1]. The last category of error sources is very important, especially for the high speed CNC machine-tools.

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<sup>1</sup> Lecturer, Department of Mechanical Engineering, University "Valahia" of Târgoviște, Romania

<sup>2</sup> Professor, Chair of Machine and Manufacturing Systems, University POLITEHNICA of Bucharest, Romania, e-mail: eugen\_strajescu@yahoo.com

A mechatronic approach integrated on a virtual lathe model, where both the typical mechanical structure and its numerical control system are included, can be used as the influence of the servo-control parameters and the axis dynamics upon the CNC lathe interpolation accuracy to be investigated.

This approach is more and more used in the preliminary design of the CNC machine-tools, [2], [3], [4], having as purpose the fast and significant analysis of more possible solutions in the stage of preliminary development of this product, leading to the best mechatronic structure for the final assumed solution.

## **2. Modeling of the CNC lathe for the interpolation simulation**

The problem of the numerical axis motion control includes two aspects:

- the case of fast motions for the tool positioning between two points. During the motion, cutting does not occur (positioning point by point). It is important to reach the final point as precisely as possible in the shortest time. In this case, the performed path does not have a significant importance;
- the case of the tool motions performed with material removal, on one or on more axes (contouring). In this case, the performed path, respectively the fidelity in the imposed path reproduction, is essential.

In a point by point positioning cycle, *the tracking error*, (defined as the distance between the instantaneous values of the performed position and the imposed position) does not affect the final value of the motion; a time delay in the reach of the final motion value is induced. The tracking error removal involves the reach of the final point in the stated time, without delay.

The in-time correlation of the motions on the numerically controlled machine axes in the contouring stage involves the successive control of a very great number of points that define the part profile. This big quantity of data can be introduced through a program in the numerical control equipment by means of some proper program carriers, but the solution is practically a non advisable one. Another variant resides in the getting of the points, which define the path, following some calculations made by an own unit within the numerical control equipment. This method, generally applied today, will constitute the single solution further used in the model of the CNC lathe numerical control simulation.

In practice, it is mainly necessary to make interpolation for the straight line and the circle, the most usual curves. Most of the parts have profiles that can be divided in these two types of curves. Some necessities for a parabola or ellipse interpolation seldom appear, only in very special cases for curves that cannot be defined by an explicit mathematical equation but are determined by a number of points. In this last case, the role of the interpolator is to provide the coordinates of the intermediate curve points on the basis of some formula for their approximation.

In order to simulate the machine motion, it is necessary to describe the model, besides the mechanical structure and the components of the drive and the control system. So, it is necessary to describe motors, sensors, CNC system. In this way, it is possible to obtain a complex mechatronic model, a model able to reproduce the dynamical interactions between the mechanical structure and the numerical control system.

Structurally, a CNC lathe consists of two feed numerical axes and their dynamic behavior mostly determines the accuracy of the tool positioning related to the part.

In this paper, the model of each numerical axis is a standard one, where a recirculating ballscrew converts the rotational movements of the d. c. servomotor in the linear movement of the slide. A reducing gear mechanism is placed between the servomotor axis and the ballscrew (Fig. 1).

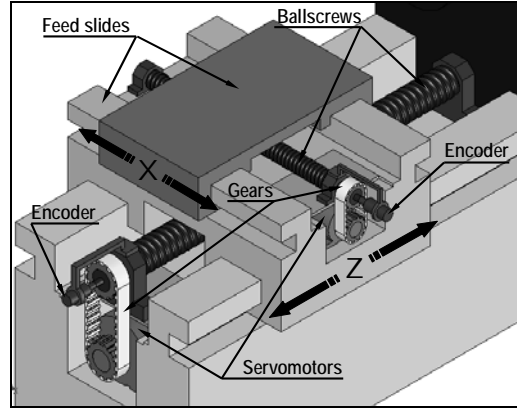


Fig. 1 – Typical structure of the CNC lathe

The base of the numerical axis model made for simulation is represented by the simple arrangement of a load with the moment of inertia,  $J_{eff}$ , that must be driven by a motor with the moment of inertia  $J_M$ . The connection between the motor and the load is made by using of an element with the stiffness,  $k_c$ , and the damping,  $c$ , [9].

The complete model for simulation shown in the Figure 2 is obtained by addition of the driving and the control systems typical for the numerically driven axes [9] to the mechanical model. This model is characterized by the following parameters:  $T_M$  – total motor torque;  $\theta_M$  – servomotor rotation angle;  $J_M$  – motor moment of inertia;  $B_M$  – motor viscous friction coefficient;  $N_1$  – teeth number of the drive gear from the reducing gear mechanism;  $N_2$  – teeth number of the driven gear;  $N = (N_1/N_2) \cdot (p/2\pi)$  – mechanical drive coefficient;  $B_L$  –

load viscous friction coefficient;  $\theta_L$  – ballscrew rotation angle;  $x$  – linear shifting of the numerical axis slide;  $e_a$  – supply voltage of the rotor circuit, as input value in system;  $R_a$  – rotor circuit resistance;  $L_a$  – total self-induction of the rotor circuit;  $e_b$  – counter electromotive voltage induced in the rotor winding;  $i_a$  – intensity of the current from the rotor circuit.

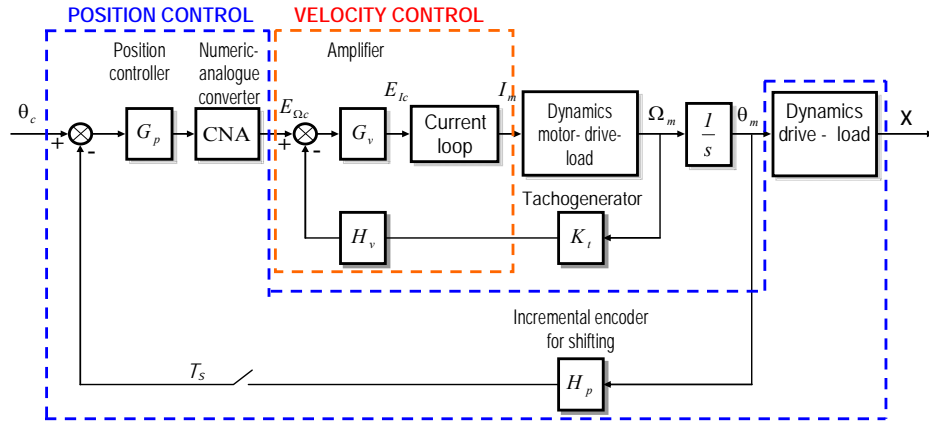


Fig. 2 – Block diagram and the signal flux in the mechatronic model of the numerical axis

The proposed model for the digital position controller is shown in the Fig. 3.

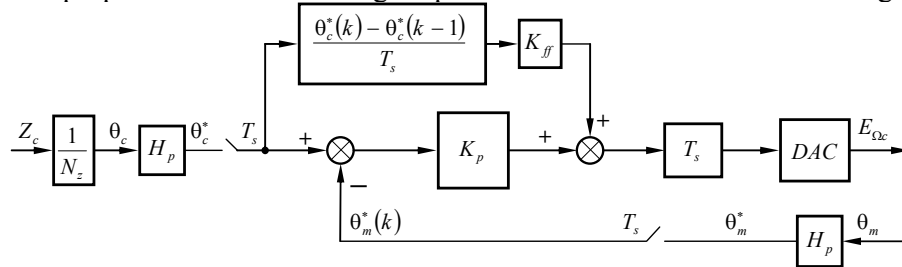


Fig. 3: Digital position controller

The control of the motor shaft position  $\theta_c$  is updated at each  $T_s = 12$  [ms] and compared with the motor position  $\theta_m$ , which is sampled at each 12 ms from the output of the incremental encoder that generates  $H_p 2048/2\pi$  pulses per radian of the motor rotation. A proportional control strategy is implemented with the total value of the position loop, which is adjusted by the determination of the position error gain value  $K_p$ . The prevision is made for the control velocity estimation, its feedforward being obtained by the gain  $K_{ff}$ .

The position loop is a system of type 1. In the absence of the velocity feedforward, when an input of constant velocity (ramp) follows, the steady state position error will be determined by the velocity error coefficient  $K_v$ .

In the above described controller, the proportional gain is firstly achieved at  $K_p = 1.310$  for Z axis, and 1.335 for X axis, by adjustment, with known methods.

The numerical control model must be able to construe an ISO program section, with G code standard instructions, which define displacements on segments of linear motion – G01, circular motion – G02 or G03, or with spline functions – G05, and to generate the reference positions for each machine axis.

The main characteristic of such systems is the discrete running given by the digital character of the elements from their structure. The values that are used in the system are estimated and adapted at equal discrete time intervals, (sample periods).

The programmed motion is reached by the control of a certain profile for the velocity of each numerical axis servomotor. The graph of the servomotor velocity variation during the programmed motion execution, assumes a  $T_c$  long cycle consisted of the acceleration time,  $T_1$ , the constant velocity time,  $T_2$ , and the deceleration time  $[T_c - (T_1 + T_2)]$ . Two velocity profiles are frequently used by the control equipments of the CNC machine-tools: the trapezoidal velocity profile and the parabolic velocity profile.

Although it is easy to be practically implemented, the trapezoidal velocity profile has the disadvantage of the sudden acceleration variation that is very important, especially for the contouring manufacturing. The sudden variation of the acceleration generates the phenomenon named jerk, which has a negative influence on the feed system behavior, especially for the contouring manufacturing. The jerk determines not only the influence on the accuracy of the performed profile, but also the oscillation introduction in system. For this reason, the jerk limitation is an important method for the improvement of the dynamic numerically controlled machine-tool behavior, respectively the programmed profile performance accuracy.

However, according to the simulations further presented, the phenomenon of sudden acceleration increase (jerk) is actually much decreased. The delay effects introduced by the different elements of the feed system, make the real shape of the velocity variation close to a parabolic one.

In the model proposed for simulation, the trapezoidal velocity profile (Figure 3) is a symmetric one, the acceleration and deceleration phases having the same length in time and the same value of the acceleration,  $a$ , a positive value for the acceleration phase and a negative value for the deceleration phase.

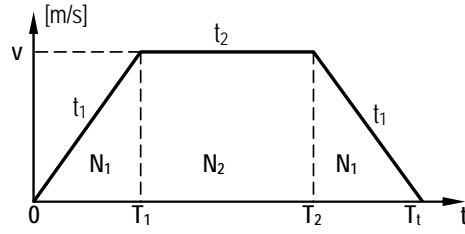


Fig. 3: Velocity profile with zero initial feed

The meaning of the used parameters are as follows:

$v$  – programmed feed velocity, [m/s];

$a$  – programmed acceleration for the acceleration phases, [m/s<sup>2</sup>];

$T_1$  – duration of acceleration and deceleration segments, [s];

$T_2$  – duration of constant velocity segment, [s];

$N_1$  – number of interpolation steps (elementary increments) for the acceleration and deceleration phases;

$N_2$  – number of interpolation steps (elementary increments) for the constant velocity phase;

$t_1$  – set of the interpolation points on the acceleration segments, [s];

$t_2$  – set of the interpolation points on the constant velocity segment, [s];

$T_a$  – dwell time for the complete performance of the controlled motions, [s];  $T_a = 0,5$  [s];

$N_a$  – number of interpolation steps (elementary increments) for the dwell period,  $T_a$ ;

$t_a$  – set of the interpolation points from the dwell period,  $T_a$ ;

$T_s$  – sample period, [s];  $T_s = 0,012$  [s];

$s_1$  – distance traveled on path in the acceleration and the deceleration phases, [m];

$s_2$  – distance travelled on path in the constant velocity phase, in [m].

If the initial velocity is zero, the total length traveled during the acceleration period ( $0 < t < T_1$ ), has the value:

$$s_1 = \int_0^{T_1} a \cdot t \cdot dt = \frac{a \cdot T_1^2}{2} \quad (1)$$

and the duration of the acceleration and deceleration phases is:

$$T_1 = \frac{v}{a} \quad (2)$$

If the total length of the trapezoidal velocity profile is  $s$ , and the path

distances are equal for the acceleration and deceleration segments, the path distance for the constant velocity segment will be:

$$s_2 = s - 2s_1 \quad (3)$$

and the duration of the constant velocity segment is:

$$T_2 = \frac{s_2}{v} \quad (4)$$

The number of interpolation steps in the constant velocity phase is:

$$N_2 = \text{round} \frac{T_2}{T_s} \quad (5)$$

and this is always rounded at the further high level as integer for the calculation efficiency.

With this value thus determined for the number  $N_2$ , the duration of the constant velocity segment is again calculated as:

$$T_2 = N_2 \cdot T_s \quad (6)$$

and the set of points representing the limits of the elementary interpolation intervals for the constant velocity segment is generated:

$$t_2 = [0 : T_s : T_2]^T \quad (7)$$

The adjusted path distance for the constant velocity segment is also recalculated with the formula:

$$s_2 = v \cdot N_2 \cdot T_s \quad (8)$$

For the generation of the interpolation points from the acceleration and deceleration phases (which are the symmetric ones, shown above) the length of the acceleration segment was calculated:

$$s_1 = \frac{s - s_2}{2} \quad (9)$$

The duration of the acceleration segment is:

$$T_1 = \sqrt{\frac{2 \cdot s_1}{a}} \quad (10)$$

Further on, the same calculations for the length and duration adjustment of the acceleration and deceleration intervals are made, where the above calculations are repeated, respectively:

$$N_1 = \text{round} \frac{T_1}{T_s}; \quad T_1 = N_1 \cdot T_s; \quad t_1 = [0 : T_s : T_1]^T \quad (11)$$

Then, the acceleration values are again calculated:

$$a = \frac{v}{T_1} \quad (12)$$

and the values of displacements on each interpolation elementary interval for the

acceleration phase:

$$s_1 = \frac{a \cdot t_1^2}{2} \quad (13)$$

and for the constant velocity phase:

$$s_2 = v \cdot t_2 \quad (14)$$

For the generation of the graphs related to the axis controls and the path controls respectively, the facilities typical to Matlab software are used.

The appraisal of the contouring accuracy for the linear interpolation can be made through the generation of a straight line, conducted at a certain angle related to the Cartesian coordinate system of the CNC lathe. Indeed, to follow a straight line in a plane, the motion of two axes must be coordinated so that the contouring error is minimum. The contour error,  $\varepsilon_c$ , is defined as the minimum distance between the actual position and the desired path, [1], Fig. 4.

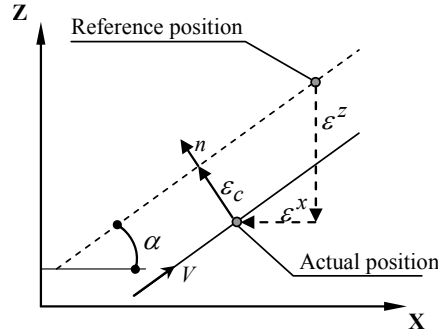


Fig. 4. Contouring error in straight line interpolation

According to [1], the anticipated contouring error, for a path travelled with a constant velocity  $v$  at an angle  $\alpha$  related to the X axis is:

$$\varepsilon_c = \frac{v \sin 2\alpha}{2} \left( \frac{1}{k_{vx}} - \frac{1}{k_{vz}} \right) \quad (15)$$

where  $k_{vx}$  and  $k_{vz}$  are the obtained axis velocity coefficients.

### 3. Results obtained by simulation

The simulation model above described allows investigating the influence of the structure dynamics and the control system performances on the lathe interpolation accuracy. Further on, examples of results are presented. These results were obtained at the linear interpolation simulation of a straight line segment with 100 mm length and having a  $15^\circ$  angle related to the X axis direction. The possibilities to decrease the error were looked for by the control system parameter adjustment.



The mechanical parameters of the lathe numerical axis models are detailed in the Table 1.

Table 1

Parameter denomination	Value - X axis	Value – Z axis
Gear ratio, $N$ , [m/rad]	1/124	1/277
Motor inertia, $J_m$ , [Kg·m <sup>2</sup> ]	0,00482	0,00926
Motor viscous friction, $B_m$ , [N·s/m]	0,00224	0,0166
Motor Coulomb friction, $\tau_c$ , [N·m]	0,23	0,60
Effective motor viscous friction $B'_m$ , [N·s/m]	0,0532	0,136
Load moment of inertia, $J_b$ , [Kg·m <sup>2</sup> ]	0,00146	0,00517
Torsional stiffness, $k$ , [N·m/rad]	10,8	9,2
Torsional damping, $c_{ts}$ , [N·m/rad]	0,01	0,00947

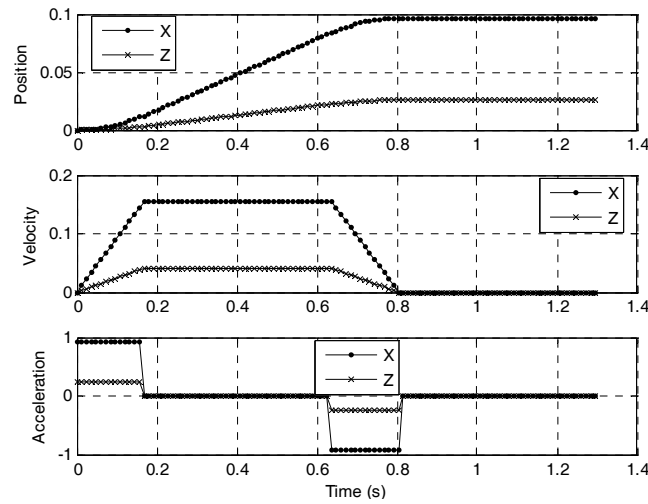


Fig. 5. Reference kinematic parameters generated by the interpolator;  $v=10$  [m/min];  $a=1$  [m/s<sup>2</sup>]

The generation of the reference kinematic parameters for each of those two lathe axes by the above described interpolator is shown in Figure 5 and the velocity response, respectively the position response are shown in Figures 6 and 7.

In this case, the interpolation contouring error is too high, being unacceptable. The simulations with changed dynamic and control parameters will be resumed. So, obtained velocity coefficients equal for those two axes will be assumed and the feed velocity will be decreased at  $v=3$  [m/min]. For this case, the velocity response of the axes is shown in the Fig. 8 and the interpolation performance in the Fig. 9.

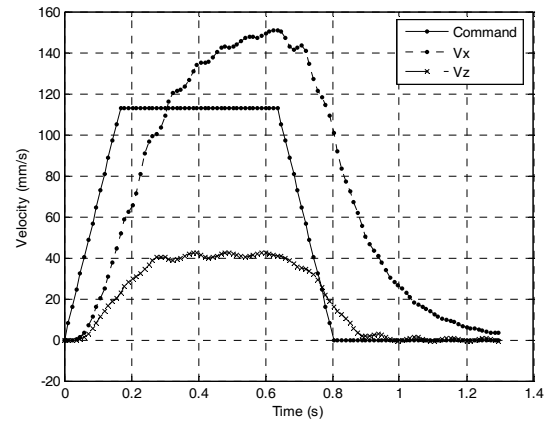


Fig. 6. Velocity response of axes; obtained velocity coefficients:  $K_{vx}=0.3097$ ;  $K_{vz}=0.689$

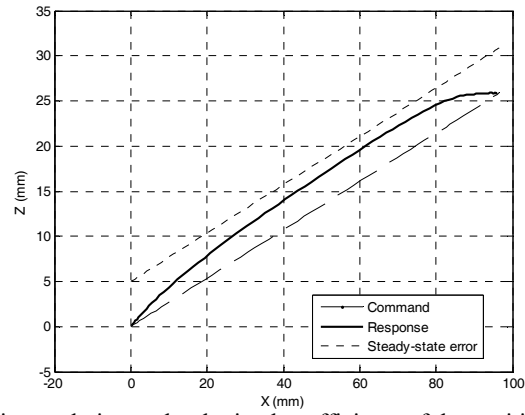


Fig. 7. Performed interpolation path; obtained coefficients of the position loop:  $K_{px}=1.335$ ;  $K_{pz}=1.31$

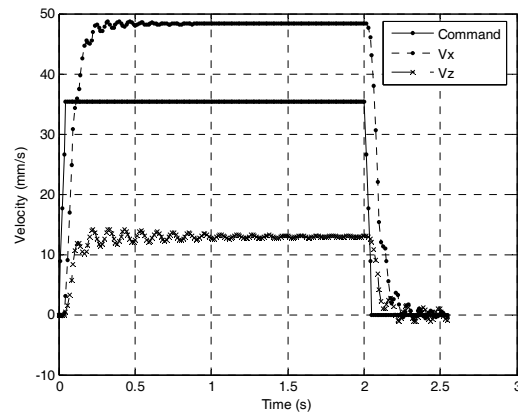


Fig. 8. Velocity response of the axes; obtained velocity coefficients:  $K_{vx}=0.689$ ;  $K_{vz}=0.689$

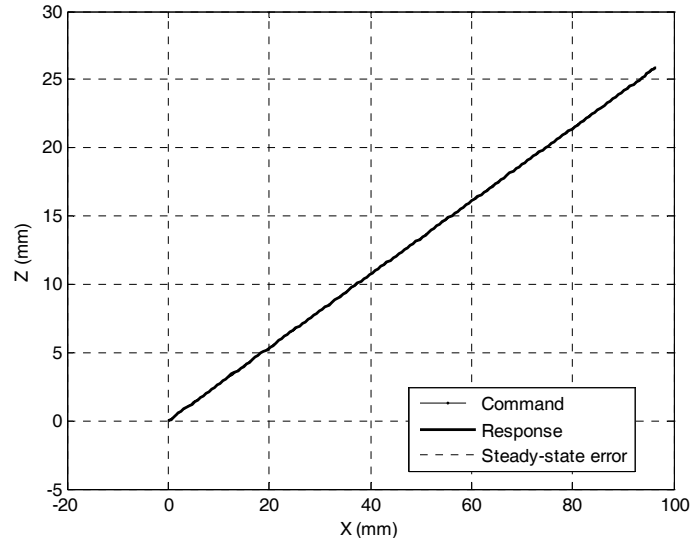


Fig. 9. Performed interpolation path; obtained coefficients of the position loop:  $K_{px}=1.335$ ;  $K_{pz}=1.335$

One can remark that the contouring error was considerably decreased, so the controlled path and the performed path are nearly overlapped. In order to point out the contouring error value, a detail of the interpolation path from Figure 9 is shown in Figure 10.

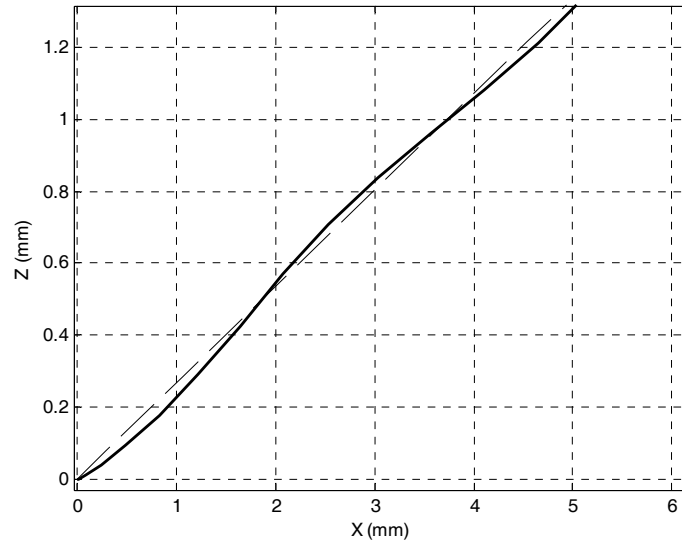


Fig. 10. Detail of the interpolation path simulated in Figure 9.

#### 4. Conclusions

Using mechatronic syntheses of analysis and optimization, the paper investigates the influence of the CNC lathe control parameters on the interpolation accuracy. For this, a lathe model, built in Matlab/Simulink software, is proposed, where are included both the mechanical structure and the control numerical part, by means of which an imposed path is simulated in order to show the possibilities to decrease the contouring error.

The proposed method can be used in the preliminary stage of a CNC lathe design, the purpose being the selection of the optimum variant for the product development, without the automatic generation of the detailed lathe project. The creators of some more laborious models of this type, same as that performed within the European project MECOMAT, [2], underline that the designer knowledge is essential at the detailed design for the appraisal of many other constraints of the final product.

#### REFERENCES

- [1] *Y. Koren*, Computer Control of Manufacturing Systems, Mc-Graw Hill, 1983.
- [2] *G. Bianchi, F. Paolucci, P. Van den Braembussche, H. Van Brussel*, Towards Virtual Enginee-loop in Machine Tool Design, Annals of CIRP, Vol. 45/1/1996
- [3] *R. Maj, G. Bianchi*, Mechatronic analysis of machine tools, 9th SAMTECH Users Conference, 2005.
- [4] *E. Schäfers, J. Hamann, H.-P. Tröndle*, Mechatronic Optimization, Analysis and Simulation of Machines, Reprint of a presentation given at the SPS/IPC/Drives 2001 exhibition, Siemens AG, Germany, 2001.
- [5] *G. Spinnler*, Conception des Machines – Principes et applications, Tome 3, Dimensionnement, Presses Polytechniques et Universitaires Romandes, Lousanne, 1997.
- [6] *P.-J. Barre, R. Bearee, P. Borne, E. Dumetz*, Influence of a Jerk Controlled Movement Law on the Vibratory Behaviour of High-Dynamics Systems, Journal of Intelligent and Robotic Systems (2005) 42, Springer 2005, pp. 275-293.
- [7] *Y. Altinas*, Manufacturing Automation, Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design, Cambridge University Press, 2006.
- [8] *H. Groß, J. Harmann, and G. Wiegartner*, Electrical Feed Drives in Automation, MCD Corporate Publishing, Siemens, 2001.
- [9] *D. Dumitru, E. Străjescu*, Study of the dynamic CNC lathe behavior using the modeling and the simulation, Bulletin of the Polytechnic Institute of Iași, Volume LIV (LVIII), Fascicle 4, Machine Construction, 2008, pp. 329-338.