

## OPTIMAL OPERATION OF LARGE HYDROPOWER RESERVOIRS WITH UNREGULATED INFLOWS

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*Articolul prezintă un model de optimizare stocastică de tip explicit, bazat pe programarea dinamică și destinat operării pe termen lung a unui lac energetic de mari dimensiuni. Ca și caz de studiu s-a folosit lacul Dokan din Irak. Tabloul deciziilor optime găsit de model permite să se traseze o serie de traiectorii optime pentru variația nivelului în lac pe parcursul anului, care sunt utile pentru a asista operatorul în efectuarea exploatării curente.*

*This paper presents an explicit stochastic optimization model based on dynamic programming, for long-term operation of a large hydropower reservoir. The Dokan reservoir from Iraq was selected as case study. The table of optimal decisions allows to tracing several optimal storage guide curves, which are useful to assist decision maker in current operation.*

**Keywords:** explicit stochastic optimization, stochastic dynamic programming, optimal hydropower reservoir operation

### 1. Introduction

Although the reservoir system optimization is placed among the few areas of application of optimizations models with many opportunities, actual implementations of these models in real-world are still limited. Often, in public water management agencies, avoidance of difficulties is the major goal, rather than improving efficiency or reducing costs. A reason for this situation may be that the reservoir operators have lacked confidence in models, which purport to replace their judgment and prescribe solution strategies under risk and uncertainty.

Determining operational policies for the efficient management of available water in large reservoirs is a complex problem because it involves random hydrological events. For such a reservoir, a long-term (annual, with monthly time step) strategy must be derived, taking into account the hydrologic uncertainties. However, if long unregulated inflow time series are available, there are some optimization models for reservoir operation in stochastic conditions. Authors as

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Yeh [1], Wurbs et al. [2], Simonovic [3], Wurbs [4] have summarized and reviewed the use of optimization models and their applications to reservoir systems. Two basic approaches may be defined.

In *implicit stochastic optimization* methods, some deterministic models are used together with long historical or synthetically generated inflow time series, or several shorter equally-likely sequences, to derive optimal operational policies. Unfortunately, these policies are unique to the assumed hydrologic time series and a multiple regression analysis can be then applied to the optimization results for developing monthly operating rules. The operating rules provide the optimal decision variables as functions of observable information (such as current storage levels, previous period inflows) and forecasted inflows for current month. Among the most used optimization methods is the simplex of linear programming and its variants, favored by [5] for a large reservoirs system. Paper [6] derived general operating rules using regression from dynamic programming results, while in [7] a neural network model is proposed to treat the same results. Some other nonlinear programming models have been developed in [8], [9], [10].

*Explicit stochastic optimization* procedures attempt to operate directly on probabilistic descriptions of random inflows or other random variables, rather than long recorded (or generated) hydrologic time series. In this way, optimization is performed without the presumption of perfect foreknowledge of future inflows, and optimal policies are obtained without the need for regression analysis or other manipulation on the optimization results. Some chance-constrained models have been developed using linear decision rule and several extensions to this one, as in [11], [12], [13] etc. In [14] the optimal storage guide curves are derived by a stochastic dynamic programming (SDP) formulation. Other authors have applied SDP to single reservoir problems as in [15] and [16], but there are few extensions of SDP to multireservoir systems where the curse of dimensionality is more aggravated than in the deterministic case.

In this paper, an explicit stochastic optimization is performed by an SDP-type model for the large hydropower Dokan reservoir from Iraq. Its size and much unregulated monthly inflows justify an optimization analysis to improve the efficiency of hydroelectric energy generation and tradeoff with other conflicting project uses and purposes.

Section 1 describes the mathematical formulation of a SDP model for optimal long-term reservoir operation.

In Section 2, some relevant input data to this project and case study are presented and prepared.

Section 3 includes the main results of optimization analysis, under stochastic unregulated inflows conditions with priority on power generation.

A next paper will be devoted to develop a simulation model for system operation, based on stochastic optimization results, but corrected to allow the current operation with medium-term forecasting on inflow discharges. Some different objective functions will be tested, and simulation model performances will be compared with recorded data from real-world operation.

## 2. SDP model formulation

Discrete dynamic programming (DP) is an ideal optimization procedure for solving sequential decision problems such as reservoir system optimization. Other advantages of DP are as follow: 1) Objective function and constraints can be nonlinear, nonconvex, and even discontinuous functional relationships; 2) A large number of constraints can be imposed, so that the number of feasible combinations of discrete decisions is highly reduced; 3) Produces closed-loop decision policies that allow flexibility in operation by conditioning optimal decision on current state of the reservoir; 4) Is more readily extensible to explicit stochastic optimization problems than the other procedures.

As originally developed by Bellman [17], DP decomposes the original problem into subproblems, which are solved sequentially over each stage (time periods in operational context, but can generally represent locations, activities etc.). Fig. 1 presents a stage from a typical multistage problem.

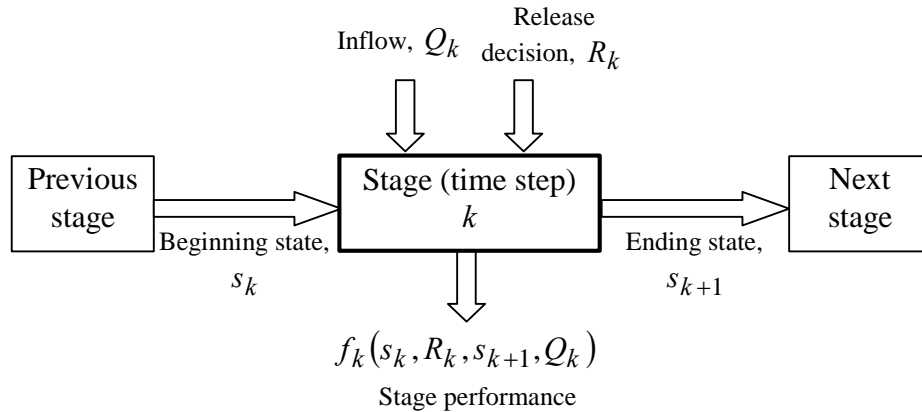


Fig. 1. Illustration of reservoir operation as sequential decision process.

Any DP formulation involves specification of two variables: state variable and decision (or control) variable. In reservoir optimization problem, the *state variable* is selected as storage level at beginning of period  $k$ ,  $s_k$  and the system evolution is completely defined by sequence of states during the all  $k = 1, 2, \dots, K$  stages of interest. *Decision variable* must be among the controllable parameters, at system operator disposal (i.e. volume of release during time step  $k$ ,  $R_k$ ).

The *state transformation equation* describes relation between the two state and decision variable from stage  $k$ , to generate ending period state,  $s_{k+1}$ .

A return / cost is produced at each stage  $k$  by a *performance function*  $f_k(s_k, R_k, s_{k+1}, Q_k)$ , and is assumed independent of decisions made in other stages. This separability property implies that the system state must embody all information to be communicated from stage to stage. The objective may be to maximize total returns or minimize accumulated costs *over all  $K$  stages*. By “return” may be defined a monetary or technical objective (i.e., annual total hydro-energy, annual firm energy, annual water yield etc.), and also for cost case.

Finally, system operation must be accomplished assuring mass balance on the reservoir content, and under *constraints* associated with both state and decision variables, for each stage.

Solution of DP optimization problem is obtained by calculation of an *optimal performance* (return or cost) *function*  $C_k(s_k)$  representing the optimum performance accumulated from the current stage  $k$  through the final stage  $K$ , conditioned on a given initial state  $s_k$ . This function is evaluated using a *backward-looking DP recursion relation* based on Bellman’s principle of optimality, which states that: no matter what the initial stage and state of a Markovian decision process, there exists an optimal policy from that stage and state to the end.

For Dokan reservoir case study the above-mentioned DP elements are as follow:

*state variable:*  $V_k^i$  – stored volume in reservoir at the beginning of month  $k$ ;

*decision variable:*  $V_k^f$  – stored volume in reservoir at the end of month  $k$ .

By this selection the *state transformation equation* becomes very simple:

$$V_{k+1}^i = V_k^f, \text{ for } k = 1, 2, \dots, K-1. \quad (1)$$

*Stage performance function:*

$$f_k(V_k^i, V_k^f, Q_k) = E_k - S_k, \quad (2)$$

where  $E_k$  is hydro-energy produced during month  $k$ , and  $S_k$  – hydro-energy spilled during month  $k$ , so that  $f_k$  represents a sort of “net” energy production in

stage  $k$ . This performance function is conditioned on the state and decision variables, and also on inflow discharge during month  $k$ ,  $Q_k$  (expressed in storage units per time interval), accepted as random variable with known probabilities.

*Optimization objective* is then:

$$\max E \left[ F = \sum_{k=1}^{K=12} f_k(V_k^i, V_k^f, Q_k) \right], \quad (3)$$

where  $E$  represents the expectation operator applied to annual net hydro-energy production, which must be maxim in stochastic conditions.

*Reservoir mass balance* is computed by:

$$V_k^f = V_k^i + Q_k - R_k, \quad (4)$$

where  $R_k$  is monthly released discharge in storage units, and any losses due to evaporation or seepage have been neglected (input data on inflows were obtained by operation balance, and thus the losses are implicitly included).

*Operational constraints* are concerned with bounds on storage and releases.

The live storage in reservoir during stage  $k$  should be less than or equal to the maximum active storage capacity,  $V_k^{Max}$ , and also greater than or equal to the minimum storage capacity,  $V_k^{\min}$ , accepted for this stage:

$$V_k^{\min} \leq V_k^f \leq V_k^{Max}, \text{ for } k = 1, 2, \dots, K. \quad (5)$$

By eq. (1), this constraint operates on state variable as well.

Irrigation demand constraint is imposed by lower bound of release during each month:

$$R_k \geq R_k^{\min}, \text{ for } k = 1, 2, \dots, K, \quad (6)$$

where  $R_k^{\min}$  is the minimum irrigation demand to sustain the crops.

If  $R_k^{Max}$  denotes the monthly release corresponding to maximum capacity of turbines, then the two terms in eq. (2) are computed by:

$$\begin{aligned} E_k &= e(\bar{V}_k) R_k, & \text{if } R_k \leq R_k^{Max}, \\ D_k &= 0, & \text{if } R_k \leq R_k^{Max}, \\ D_k &= e(\bar{V}_k) (R_k - R_k^{Max}), & \text{if } R_k > R_k^{Max}, \end{aligned} \quad (7)$$

where  $e(\bar{V}_k)$  is specific hydro-energy production in (GWh/  $10^6 \text{ m}^3$ ), conditioned on average storage level over period  $k$ ,  $\bar{V}_k$ .

The *backward DP recursion relation* for optimal return function computation in stochastic problem is:

$$C_k(V_k^i) = \max \left\{ \sum_{j=1}^{J_k} p_{kj}(Q_{kj}) [f_{kj}(V_k^i, V_k^f, Q_{kj}) + C_{k+1}(V_{k+1}^i)] \right\}, \quad (8)$$

where  $Q_{kj}$  denotes the  $j$ -th discrete realization of random variable  $Q_k$ , with associated independent discrete probability of occurrence  $p_{kj}(Q_{kj})$ , and  $J_k$  is number of classes used in frequency analysis for stage  $k$ . Obviously, it must that

$$\sum_{j=1}^{J_k} p_{kj}(Q_{kj}) = 1, \text{ for } k = 1, 2, \dots, K, \quad (9)$$

and backwards recursive process begins with terminal condition

$$C_{K+1}(V_{K+1}^i) = 0. \quad (10)$$

Each successive  $C_k(V_k^i)$  is evaluated as a function of all discrete values of  $V_k^i$  used in analysis for stage  $k$ , and stored together with the corresponding optimal decision  $V_k^*(V_k^i)$ . The two matrices obtained for all  $K$  stages represent the results of SDP analysis.

### 3. Input data for SDP analysis in Dokan reservoir case study

Dokan dam and reservoir is a multipurpose project built on Little Zab river in Kurdistan, Iraq. This project was conceived primarily for flood control, with other purposes being irrigation, hydropower, pisciculture and recreation. The catchment area of reservoir up to the dam site is about  $12,000 \text{ km}^2$ , and the multiannual mean inflow attains about  $200 \text{ m}^3/\text{s}$ .

The active storage capacity is placed between the reservoir levels of 480 m (minimum hydropower operating level), and 511 m respectively (level of an uncontrolled bell mouth spillway), involving a volume of about  $5,400 \cdot 10^6 \text{ m}^3$ . The gross storage capacity is about  $7,000 \cdot 10^6 \text{ m}^3$ , and reservoir water surface at level 511 m exceeds  $270 \text{ km}^2$ .

There are 5 Francis type turbines at the Dokan power plant with 80 MW power capacity each, operating at net heads between 60 and 95 m, with discharges of  $50 - 111 \text{ m}^3/\text{s}$ .

In present-day conditions, the irrigation demand is reduced to only about  $20 \text{ m}^3/\text{s}$ , and this value was used to compute the lower bound  $R_k^{\min}$ , which is imposed for all  $K$  months. The maximum capacity of turbines,  $R_k^{\max}$ , was limited to  $470 \text{ m}^3/\text{s}$  (94  $\text{m}^3/\text{s}$  per turbine being a discharge possible on the whole domain of heads).

Specific hydro-energy production is estimated by relation:

$$e = 0.0023 \cdot (z - 415), (\text{GWh}/10^6 \cdot \text{m}^3), \quad (11)$$

where  $z$  represents storage level (m), connected by volume in storage as in Table 1.

The lower bound for volume in storage,  $V_k^{\min}$ , is imposed to  $1,400 \cdot 10^6 \cdot \text{m}^3$  over months  $k=1,2,\dots,11$ , and to  $2,400 \cdot 10^6 \cdot \text{m}^3$  at the end of year, while upper bound,  $V_k^{\max}$ , is accepted as  $6,800 \cdot 10^6 \cdot \text{m}^3$  during the all stages. Using a discretization step  $\Delta V = 50 \cdot 10^6 \cdot \text{m}^3$ , a total of 109 discrete values results for state / decision variables.

Table 1

**Water level (m) – volume in storage ( $10^6 \text{ m}^3$ ) relation for Dokan reservoir**

$z$	470	474	478.11	480.5	484	486	488	490.03	492.01
$V$	741.5	969.1	1261.5	1462.6	1801.5	2021.8	2262.4	2528.6	2810.2

$z$	494.06	496.01	498.04	500.04	502.1	504.1	506.4	508.04	510.75	512.36
$V$	3126.1	3450.2	3813.2	4196.8	4619.2	5055.4	5589.8	5991.2	6694.5	7135.3

From the system operator, a lot of recorded data time series has been obtained, including monthly inflow discharges between October 1958 and January 2001. These values vary from a minimum of  $11 \text{ m}^3/\text{s}$  in August to about  $1510 \text{ m}^3/\text{s}$  in March, while mean multiannual data are placed between  $64 \text{ m}^3/\text{s}$  for September and  $483 \text{ m}^3/\text{s}$  for April.

Because unregulated monthly inflows in Dokan reservoir are poorly correlated from month to month, these have been accepted as independent random variables for each stage. A number of 10 classes is selected for frequency analysis, and the classical synthetic generation Thomas–Fiering model was used to give more consistency to frequency distributions. For example, Table 2 includes some results of frequency analysis over a 200 years time series, where inflows are expressed in storage units ( $10^6 \text{ m}^3/\text{month}$ ), and computed as average value of all data in any class.

#### 4. Main results of SDP analysis for Dokan reservoir

Input data as shown in Section 2 were used in SDP optimization model described in Section 1.

From table with optimal return function as function of storage level for first stage (October), one ascertains that the expected maximum annual net hydro-energy is placed between 913 GWh (489.05 m storage level at beginning of year) and 1903 GWh (511.14 m as initial storage level). One note that recorded data during 1980 – 1999 interval reveals an average annual production of about 963 GWh/year.

However, more useful are results from optimal decisions table because these allow to tracing several optimal storage guidecurves. A fragment with optimal decisions (expressed as ending storage level for each month) is included in Table 3.

The data from this table must be understood as follows (for example):

- If storage level at beginning of March is 490.53 m (first column), then the optimal ending storage level for March is 492.27 m (column of month 3);
- To the same initial storage level but for September corresponds an optimal ending storage level in September of 489.05 m.

Positions marked by “0” values in Table 3 correspond to unacceptable states for the used input data, and in the context of SDP formulation.

Fig. 2 presents a lot of optimal storage guidecurves traced with data from Table 3 for various storage levels at the beginning of hydrological year.

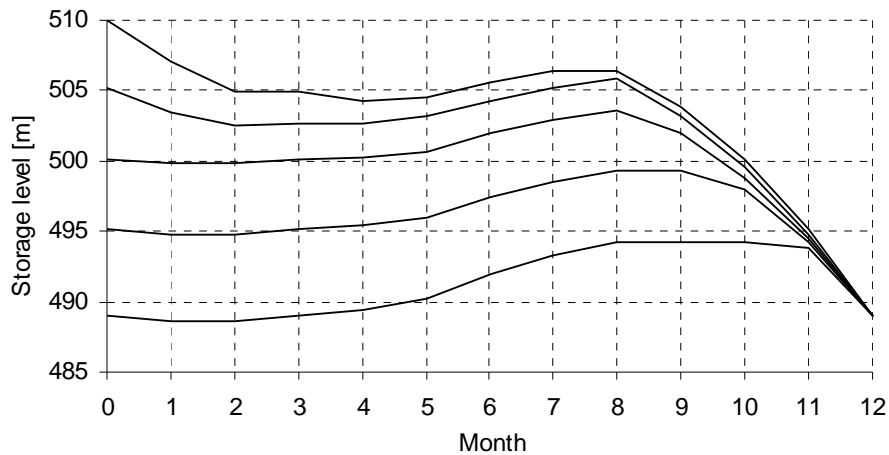


Fig. 2. Several optimal storage guidecurves for Dokan reservoir in stochastic conditions.



Such a guidecurve may be or may not be followed during a year of real operation, depending on the effective monthly inflows. However, these guidecurves are very useful to assist the decision maker for reservoir operation.

Because of the objective function (expected maximum annual net hydro-energy), all guidecurves attains minimum storage level accepted for ending of the year.

Using only the inflow volumes (lower guidecurve), an expected annual production of 913 GWh is obtained by this stochastic optimization model, very closed to the recorded value of 963 GWh/year during a 19 years time interval.

## 5. Conclusion

SDP optimization model for Dokan reservoir produced a table with optimal decisions (ending storage level) as functions of the beginning storage level for each month, under unregulated inflows conditions.

A simulation model, runned with the recorded monthly inflows, will verify performances of these optimal decisions.

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