

## NUMERICAL STABILITY OF ADAPTIVE CONTROL ALGORITHMS

Ciprian LUPU<sup>1</sup>, Andreea UDREA<sup>2</sup>, Dumitru POPESCU<sup>3</sup>, Cătălin PETRESCU<sup>4</sup>,

*Această lucrare analizează impactul modificării parametrilor unei structuri adaptive de reglare. Elementul central, este metoda propusă pentru a atenua efectele negative ale modificărilor importante ale ieșirii algoritmului de reglare. Metoda este inclusă în cadrul unui algoritm de conducere în timp real bazat pe un model de referință și pe un mecanism de adaptare pentru o clasă de sisteme neliniare.*

*This paper analyzes the impact of modifying the control algorithm parameters by an adaptation law on the immediate variation of the command calculated for an adaptive control structure. A method to limit negative effect of the large command variations due to the adaptation algorithm is proposed. The procedure is included in a real-time control algorithm with a model reference adaptive control mechanism designed for a class of nonlinear systems*

**Keywords:** adaptive control, real-time, application, nonlinear

### 1. Introduction

In the last 20 years, due to computational advancement, adaptive control solutions became very appealing and were largely investigated [1], [2], [3].

Adaptive control encountered challenges in the field of real-time systems (preserving the closed-loop performances in case of non-linearity, structural disturbances or process uncertainties), which do not have precise classical models admissible to existing control designs [4].

A simple and clarifying example on the numeric stability of adaptive algorithms is the following: suppose given a numerical control system that contains at least a proportional component (P, PI, PID, RST etc.). In conformity with the control algorithm:  $\varepsilon(k) = r(k) - y(k)$  and  $du(k) = K * \varepsilon(k)$ , where  $\varepsilon$  is the error – calculated as the difference between the set point  $r(k)$  and the output

<sup>1</sup> Assoc. Prof., Computer Science and Engineering Department, University POLITEHNICA of Bucharest, Romania, e-mail: cip@indinf.pub.ro

<sup>2</sup> Assistant, Computer Science and Engineering Department, University POLITEHNICA of Bucharest, Romania, e-mail: andreea@indinf.pub.ro

<sup>3</sup> Prof., Computer Science and Engineering Department, University POLITEHNICA of Bucharest, Romania, e-mail: dpopescu@indinf.pub.ro

<sup>4</sup> Assoc. Prof., Computer Science and Engineering Department, University POLITEHNICA of Bucharest, Romania, e-mail: catalin@indinf.pub.ro

$y(k)$  and  $du(k)$  and  $K$  are the calculated variation of the command corresponding to the proportional component, respectively the algorithm amplification. If, initially,  $K=2$ , for a change of set point of 10% it results  $\varepsilon(k)=10$  and  $du(k)=20$  – an important, but manageable command variation for the closed loop system. Suppose that, after modifying the set point, the system will transit to another functioning regime where an amplification of  $K = 3.5$  is needed. This change of the  $K$  parameter in the context of  $\varepsilon(k) = 10$  makes  $du(k) = 35$  with cannot be tolerate by the control system, this situation may lead to instability or other negative aspects. In this context, this paper proposes a stable real-time control algorithm based on a reference model adaptive control mechanism for a class of nonlinear systems.

The reference model is computed off-line, while the controller's parameters are determined on-line, via a static gain adaptation criterion. For this study there is used an RST control algorithm, presented in the next figure:

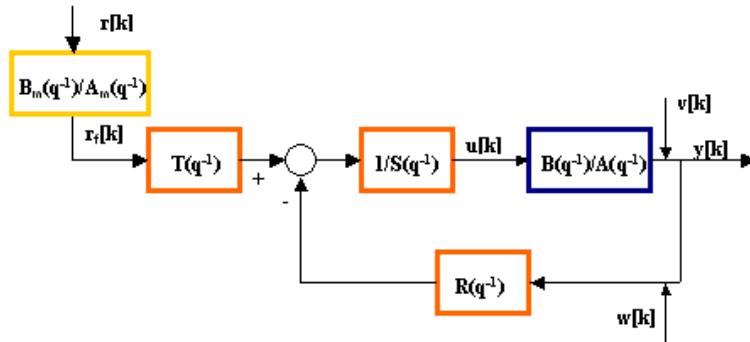


Fig. 1. Classic RST control scheme

Where the R, S and T polynomials are:

$$\begin{aligned} R(q^{-1}) &= r_0 + r_1 q^{-1} + \dots + r_{nr} q^{-nr} \\ S(q^{-1}) &= s_0 + s_1 q^{-1} + \dots + s_{ns} q^{-ns} \\ T(q^{-1}) &= t_0 + t_1 q^{-1} + \dots + t_{nt} q^{-nt} \end{aligned} \quad (1)$$

where  $n_r$ ,  $n_s$  and  $n_t$  are their degrees.

An imposed trajectory is sometimes useful. It can be produced by a trajectory model generator that can has the following form:

$$y^*(k+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})} r(k) \quad (2)$$

where the  $A_m$  and  $B_m$  polynomials have these forms:

$$\begin{aligned} A_m(q^{-1}) &= 1 + a_{m1}q^{-1} + \dots + a_{mn}q^{-n} \\ B_m(q^{-1}) &= b_{m0} + b_{m1}q^{-1} + \dots + b_{mn}q^{-n} \end{aligned} \quad (3)$$

The algorithm using pole placement design procedure is based on the identified process's model.

The plant model is obtained considering an ARX form:

$$y(k) = \frac{q^{-d} B(q^{-1})}{A(q^{-1})} u(k) \quad (4)$$

where:

$$\begin{aligned} B(q^{-1}) &= b_1 q^{-1} + b_2 q^{-2} + \dots + b_{nb} q^{-nb} \\ A(q^{-1}) &= 1 + a_1 q^{-1} + \dots + a_{na} q^{-na} \end{aligned} \quad (5)$$

For this structure, in situation where  $R(q^{-1}) = T(q^{-1})$  (PID in RST form [4]), the command value calculated by the real-time adaptive algorithm is:

$$u(k) = \frac{1}{S_0} \left( - \sum_{i=1}^{n_S} s_i u(k-i) + \sum_{i=0}^{n_R} r_i \varepsilon(k-i) \right) \quad (6)$$

where  $\varepsilon(k) = r_j(k) - y(k)$  is the error,  $R$  and  $S$  polynomials describing the controller,  $y(k)$  and  $u(k)$  represent the process output, respectively computed command value, as in Fig. 1. Between  $R$  and  $S$  polynomials coefficients and  $\varepsilon(k)$ ,  $u(k)$  measures vectors, relation (1) establishes a “numeric connection” given by  $u(k)$  value.

After algorithm's parameter adaptation,  $R$ 's coefficients are replaced with a new set of values:

$$R^a(q^{-1}) = r_0^a + r_1^a q^{-1} + \dots + r_{nR}^a q^{-nR} \quad (7)$$

the adapted value of command is calculated based on the new adapted coefficients and the precedent measurements  $u(k-i)$ ,  $r(k-i)$ ,  $y(k-i)$  vectors.

$$u_a(k) = \frac{1}{s_0} \left( - \sum_{i=1}^{n_s} s_i u(k-i) + \sum_{i=0}^{n_r} r_i^a \varepsilon(k-i) \right) \quad (8)$$

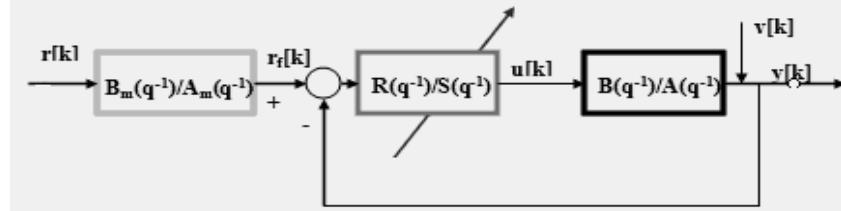


Fig. 2. Classic adaptive control scheme

The difference between  $u(k)$  and  $u_a(k)$  must be less than an imposed value - namely  $-K_a$  - maximal adaptation impact factor:

$$|u_a(k) - u(k)| < K_a \quad (9)$$

The value of this factor is given by the process particularities (actuators, dynamics, stability, numerical implementation of the numeric control algorithm etc.).

## 2. Real Time adaptive algorithm

The proposed structure (Fig. 3.) implies a series of steps that are made before the real-time running of the application [5].

- identification and storage of the static characteristic of the process - determined by meaning a series of measurements;
- selecting the inflexion points of the static characteristic - the points where the process changes visible its dynamics;
- identification of the process model (dynamical) in a randomly chosen functioning point;
- design of the regulation algorithm (PID, RST, other) - based on the dynamic model.

During run time, the adaptive algorithm follows the next steps every sampling period:

- establish the plant's gain; this is made considering the process position relatively to the inflexion points of the static characteristic. The gain is determined based on the inflexion point that is immediately inferior;
- command limitation procedure;
- controller adaptation; adaptation of the  $R$  polynomial coefficients in order to

maintain the product between the plants gain and the controllers gain constant, so that the initial (off-line) closed-loop performances remain unchanged.

### 2.1. Real time plant's gain determination

The plant's gain is determined based on the inflexion point that is immediately inferior  $-y_{i-1}$  considering the process position at moment  $k$   $- y_p$ .

$$K_{\text{plant}} = (y_p - y_{i-1}) / (u_p - u_{i-1}) \quad (10)$$

Using the inflexion points of a predetermined characteristic (off-line) can successfully replace a continuous estimation (on-line) of the process parameters (amplification) which is tributary to real-time implementation, validation and unwanted effects produced by the disturbances and noise.

### 2.2. Adaptive law

The adaptive law compares the output of the system (process) to the output values of the precalculated model and modifies the controllers' parameters in order to maintain the product between the controllers' gain and the plants' gain constant:

$$K_{\text{controller}} \cdot K_{\text{plant}} = ct. \quad (11)$$

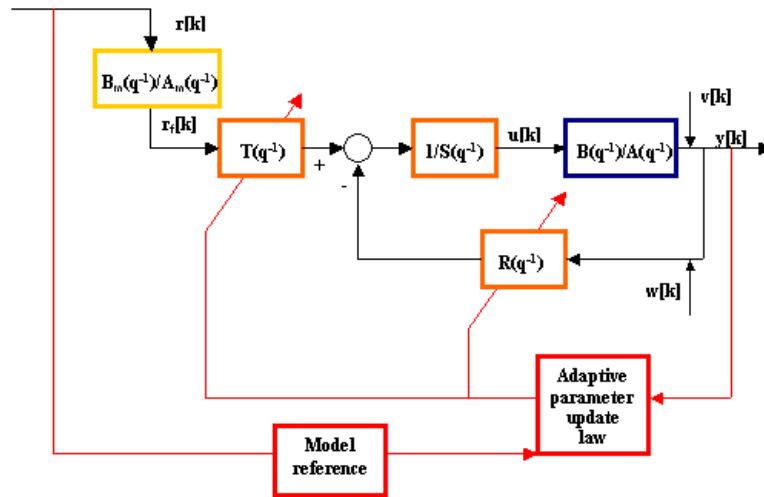


Fig. 3. Implemented real-time adaptive scheme

That implies that the controller's parameters are to be recalculated in order to satisfy relation (11).

The justification for the parameters adaptation procedure is the following: the transfer function for the closed loop system has the form in (12) and the model's gain is given by (13):

$$H(q^{-1}) = \frac{R(q^{-1})B(q^{-1})}{S(q^{-1})A(q^{-1})} / [1 + \frac{R(q^{-1})B(q^{-1})}{S(q^{-1})A(q^{-1})}] \quad (12)$$

$$K_{\text{plant}} = \sum_{i=0}^{n_B} b_i / [1 + \sum_{j=1}^{n_A} a_j] \quad (13)$$

If the static characteristic varies creating an inflexion point, we have a new  $K_{\text{plant}}'$  and we need to recalculate the value for the controller's gain in order to satisfy the adaptive law.

The controller's gain would be:

$$K_{\text{controller}}' = K_{\text{controller}} \frac{K_{\text{plant}}'}{K_{\text{plant}}} = K_{\text{controller}} F \quad (14)$$

where  $F$  is called correction factor.

From this and in order to maintain unchanged relation (11), the rapport:

$$rap = \sum_{i=0}^{n_R} r_i / [1 + \sum_{j=1}^{n_S} s_j] \quad (15)$$

must be multiplied by  $F$ .

If we multiply the parameters  $r_i$  of the controller's polynomial  $R$  by  $F$ , the correction factor, we satisfy the adaptive law.

Using this solution, these parameters modification do not influence the stability of the system as it can be seen from the experimental results.

### 2.3. Limitation procedure

There are some considerations to be made when using a correction factor. If the controller's parameters are modified with a correction factor that is too large, the adapted command value  $u_a(k)$ :

$$u_a(k) = \frac{1}{s_0} \left( -\sum_{i=1}^{n_s} s_i u(k-i) + \sum_{j=0}^{n_R} Fr_j \varepsilon(k-j) \right) \quad (16)$$

will be different in comparison to the command value that would result from the unmodified controller –  $u(k)$ . This difference between the expected and applied command can send shocks to the process leading to instability or, depending on the process, a variation of this sort is not allowed (for example a much too rapid heating leading to deterioration of tanks walls).

From this point of view, a limit to the difference between  $u(k)$  and  $u_a(k)$  must be imposed. From (6), (16) and (9) we obtain the limits for the correction factor  $F$ :

$$|1-F| < \frac{K_a s_0}{\sum_{j=0}^{n_R} r_j \varepsilon(k-j)} \quad (17)$$

If relation (17) is respected, the correction with  $F$  is applied, otherwise, the maximal value is calculated by solving (17) and selecting the largest admitted value.

The obtained effect is that the variation of parameters is slowed down in favor of the limitation of the command variation. However, the controller's parameters adaptation takes places continuously and after a series of limitation operations the limitation would not be necessary.

### 3. Experimental results

In order to demonstrate the applicability of this solution, a nonlinear process consisting in a tank with a filling point and multiple evacuation points was chosen as an example. The purpose of the control system is to maintain constant the liquid level in the tank. A real time software process simulator for this system has been created using LabWindows/CVI:

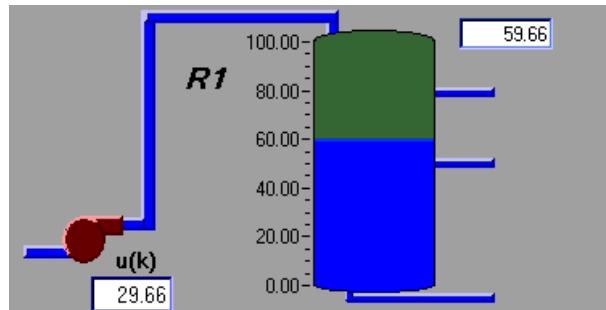


Fig. 4. Tank with single filling-multiple evacuation points simulator

The process simulator permits adding disturbances, in order to test the controller dynamics in real functioning.

Also, the evolution of the liquid in the tank is graphically represented.

The simulator communicates with the control platform using a dedicated communication file.

Using this process simulator, one can generate a nonlinear characteristic.

The input-output characteristic for this system can be approximated by a set of medium input and output values and has the form showed in Fig. 5.

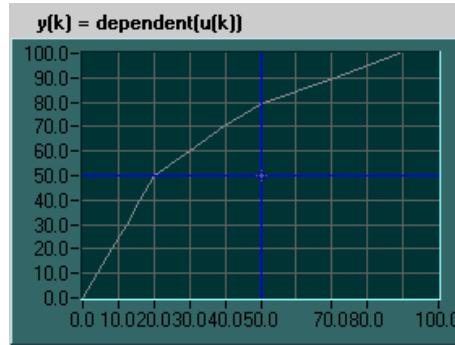


Fig. 5. Input-output characteristic for this system

In order to test the adaptive controller (Fig. 3), one must load this characteristic – model reference in the adaptive controller real-time software application and select a number of inflection points, where the process dynamic changes.

The set of inflection points can be easily observed and learned, so that the new process's gain and controller's parameters will be calculated by using the adaptive law.

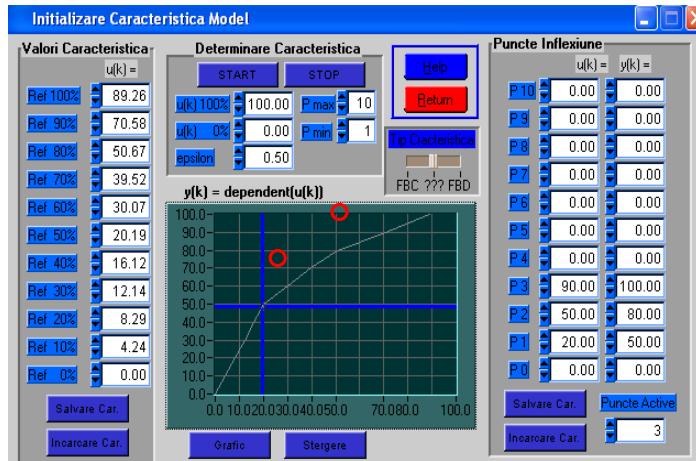


Fig. 6. Choosing the inflection points on the static characteristic

Accordingly to the figure above, we've identified the following three functioning intervals (0-50%), (50-80%), respectively (80-100%).

In order to identify the model for the first interval, a sampling period  $T_e=0.6$  sec was used and least-squares identification method from Adaptech/WinPIM platform [10] was employed:

$$M_1(q^{-1}) = \frac{1.13044}{1-0.54787q^{-1}}$$

For this model, we have computed the corresponding RST control algorithm using a pole placement procedure and the Adaptech/WinREG platform.

For the poles placement procedure we've used a second order system, defined by the dynamics natural frequency  $\omega_0 = 0.5$  and the damping factor  $\xi=0.95$  for tracking performances and  $\omega_0 = 1.25$ ,  $\xi = 0.8$  for disturbance rejection performances, keeping the same sampling period as for identification  $T_e=0.6$  sec.

The obtained parameters for the first functioning interval are as it follows:

$$\begin{aligned} R_1(q^{-1}) &= 0.494956 - 0.218212 q^{-1} \\ S_1(q^{-1}) &= 1.000000 - 1.000000 q^{-1} \\ T_1(q^{-1}) &= 0.884611 - 0.874307 q^{-1} + 0.266440 q^{-2} \end{aligned}$$

These initial values for the RST controller, are loaded into the simulator, see Fig. 7.

The pair model – controller can be identified, also, on another region and used in the same way.

Using this application, a few tests were made in order to demonstrate the fact that the adaptive control mechanism that was implemented can guarantee the closed loop stability under changes of reference and disturbances influence.

We first impose a disturbance of 1% and we test the performances for changes of reference:

- from 10% (where the first pre calculated RST algorithm is active) to 40% (where, normally, because the static gain value maintains the RST parameters) – Fig. 7.
- from 40% to 60% - Fig. 8;
- from 60% to 90% - Fig. 9;
- from 90% to 30% - Fig. 10.

As it can be observed, the tracking and rejection performances are quite good and the system remains stable.

The effective modification of parameters is done when the filtered process output becomes greater than 50% and 80%.

For the first 3 changes of reference, the difference between the filtered set point and the process output is quite insignificant, for the last example the difference is larger, but not compromising the quality of the control procedure.

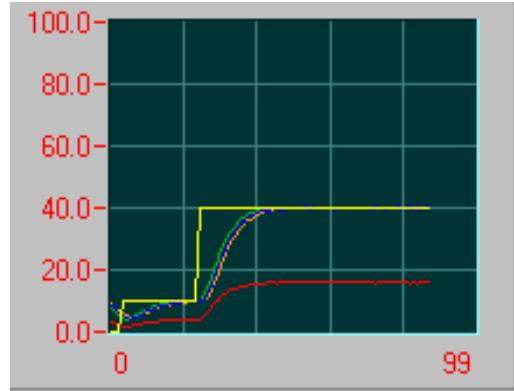


Fig. 7. Performances when changing the reference from 10% to 40% (reference - yellow, filtered reference – green, system output – blue, command – red, model - orange)

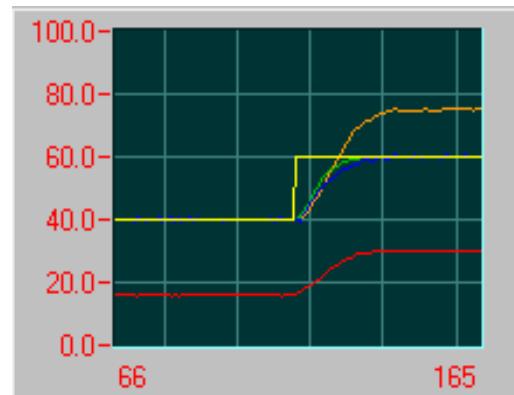


Fig. 8. Performances when changing the reference from 40% to 60%

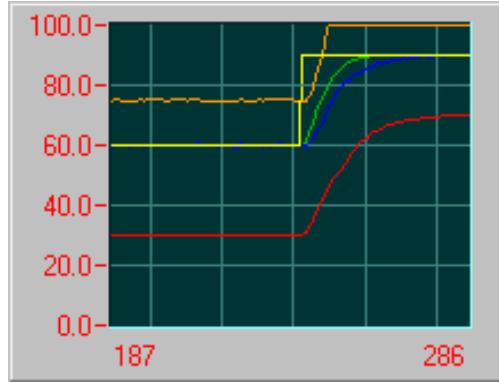


Fig. 9. Performances when changing the reference from 60% to 90%

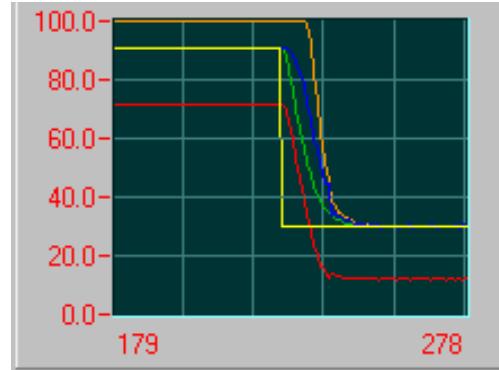


Fig. 10. Performances when changing the reference from 90% to 40%

In all tests, one can see that there are no shocks or oscillations in the control evolution by applying this approach neighed in the process output, nor in the command.

Increasing the number of selected inflection points improves the performances if the characteristic substantially changes its form in those points.

#### 4. Conclusions

The solution based on using the inflexion points of the off-line determined static characteristic together with limiting the control algorithm's parameters variation offers to the closed loop control system the qualities of a numerical stable adaptive system that is real-time easy to implement. This solution avoids the closed loop identification procedure which implies solving a series of specific problems.

The limitation procedure was successfully tested on several nonlinear processes from the same class, using a controller simulator software application implementing the proposed reference model identification and adaptive law. One of the process simulators results has been presented.

The solution that has been proposed is simple and easy to implement. It doesn't imply any kind of problems concerning the sample period value (satisfying tests were performed for a sample period of 10 ms).

The tracking performances are good; the disturbances rejection for this adaptive solution is well performed.

The command and process output values do not present brusque changes while crossing an inflexion point and for this reasons, the closed loop can conserve the imposed performances.

With regards to the results obtained in the paper, this adaptive method can be successfully recommended for real-time control structures for this class of nonlinear processes.

### Acknowledgement

This work was partially supported by CNCSIS - IDEI Program of Romanian Research, Development and Integration National Plan II, Grant no. 1044/2007.

### R E F E R E N C E S

- [1] *Z. Bubnicki*, Modern Control Theory, Springer-Verlag, 2005
- [2] *I. Dumitache*, Ingineria Reglării Automate (Process Control Techniques), Politehnica Press, Bucharest, 2005
- [3] *G. Tao*, Adaptive control design and analysis, Wiley-Interscience, John Wiley&Sons Inc., 2003
- [4] *I.D. Landau, R. Lozano, M. M'Saad*, Adaptive Control, Springer Verlag, London, 1997
- [5] *C. Lupu, D. Popescu, A. Udrea*, Real-time Control Applications for Nonlinear Processes Based on Adaptive Control and the Static Characteristic, In: WSEAS Transactions on Systems and Control, **Vol. 3**, No. 6, June 2008, 607-616, ISSN 1991-8763
- [6] *I. D. Landau, A. Karimi*, Recursive algorithm for identification in closed loop: a unified approach and evaluation, Automatica, **vol. 33**, no. 8, 1997, pp. 1499-1523
- [7] *C. Lupu, D. Popescu, C. Petrescu, A. Ticlea, B. Irimia, C. Dimon, A. Udrea*, Multiple – model Design and Switching Solutions for Nonlinear Processes Control, EUROSIS Publication, ISBN: 978-90-77381-4-03, Industrial Simulation Conference Proceedings, pag. 71-76, Lyon, France, 2008
- [8] *Kuan-Yu Chen, Mong-Tao Tsai, Pi-Cheng Tung*, An Experimental Analysis of an Active Magnetic Bearing System Using PID-Type Fuzzy Controllers with Parameter Adaptive Methods, 6th WSEAS International Conference on CIRCUITS, SYSTEMS, ELECTRONICS, CONTROL & SIGNAL PROCESSING, Cairo, Egypt, Dec 29-31, 2007
- [9] *J. Richalet*, Practique de la Commande Predictiv, Editura Herms, Paris, 1993
- [10] *I.D. Landau*, Identification et commande des systemes, Ed. Hermes, Paris, 1993