

CALCULATION OF THE PROBABILITY FOR THE FLASHOVER OF THE INSULATOR STRING DUE TO THE LIGHTNING STROKES ON THE PHASE CONDUCTOR. PART II. MATHEMATICAL SOLUTION AND NUMERICAL APPROACH

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În prezent, dezbaterile cu privire la raportul preț-calitate se concentrează pe ideea de performanță, o performanță înțeleasă sub dimensiunile economiei, eficienței și economicității. Iată de ce devine tot mai important alegerea zonei de amplasare a conductoarelor de protecție a LEA pe baza unor calcule riguroase privind probabilitatea de conturare a lanțurilor de izolatoare datorită loviturilor directe de trăsnet pe conductoarele active.

The discussions related to the price-quality report are concentrating now-a-days on the idea of performance, a performance under the concept of efficiency, effectiveness, economy. That's why, choosing the location for the ground wire of the Over Head Lines becomes more and more important. This selection must be done after some rigorous calculations related to the probability for the flashover of the insulator string due to the direct lightning on the phase conductors.

Keywords: OHL, flashover, leader, lightning protection, lightning model, shielding failure

1. Introduction

The author proposed in [1], a formula to calculate the probability for the flashover of the insulators string due to direct lightning on the phase conductors.

$$P_{flashover} = 2 \cdot D_{ground} \cdot L \cdot (PC_{point_A} \cdot k_{egalizare}) \quad (1)$$

were:

D_{ground} - ground flash density;

L – the length of the Over Head Line;

$k_{egalizare}$ – the constant which takes into consideration the coefficient of equalization between ground wire and phase conductor.

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- I_{cont} – the limit value of the lightning's current for which one gets the flashover of the insulator string;
- I_{max} – the maximal amplitude of the lightning's current;
- $D_{or_cp_I_{min}}$, $D_{or_cp_I_{cont}}$, $D_{or_cp_I_{max}}$ – the striking distances for the case when the lightning strikes the ground wire, distances corresponding to I_{min} , I_{cont} , and I_{max} , respectively;
- $D_{or_ca_I_{min}}$, $D_{or_ca_I_{cont}}$, $D_{or_ca_I_{max}}$ – the striking distances for the case when the lightning strikes the phase conductor, distances corresponding to I_{min} , I_{cont} , and I_{max} , respectively;
- D_{or} - striking distance;
- LD – the lateral distance defined in [2], [3] and [4];
- LPM – Leader Progression Model;
- OHL - Over Head Lines.

2. Mathematical solution

For symmetry and easier calculations, we have picked the y-axis to pass through the center of the tower (a guyed tower type is presented in figures 1 and 2). If there exists only one ground wire (this is the case of most double circuit OHL), then the y-axis will pass through the center of this conductor. If there are two ground wires (case met on most single circuit OHL), then the y-axis will pass through the middle point of the distance between these wires.

The phase conductor will be represented by the point $A(x_A, y_A)$ and the ground wire one by the point $B(x_B, y_B)$.

To calculate the probability of a lightning current appearance on the phase conductor in A with an amplitude that will produce the flashover of the insulators string, probability found in (30), we must complete the following steps:

- A) write the expressions for all the functions met in the figure 1;
- B) calculate the coordinates for all the intersection points;
- C) determine the areas affected by the lightning strokes and evaluate the integrals met in the formula.

A) Writing the expressions for all the functions met in the figure 1

A1) The equations for the circles with centers in the print of the ground wire and of radii $D_{or_cp_I_{min}}$ and $D_{or_cp_I_{max}}$.

$$\begin{cases} y = y_B + \sqrt{D_{or_cp_I_{min}}^2 - (x - x_B)^2} \\ y = y_B + \sqrt{D_{or_cp_I_{max}}^2 - (x - x_B)^2} \end{cases} \quad (3)$$

An arc of radius $D_{or_cp_I}$ represents the geometrical space of the points from where a lightning with amplitude I of the current strikes the ground wire.

A2) The equations for the circles with centers in the print of the phase conductor and of radii $D_{or_ca_I_{min}}$ and $D_{or_ca_I_{cont}}$:

$$\begin{cases} y = y_A + \sqrt{D_{or_ca_I_{min}}^2 - (x - x_A)^2} \\ y = y_A + \sqrt{D_{or_ca_I_{cont}}^2 - (x - x_A)^2} \end{cases} \quad (4)$$

An arc of radius $D_{or_ca_I}$ represents the geometrical space of the points from where a lightning with amplitude I of the current strikes the phase conductor.

A3) The expressions of the two functions, see [1], coming from the intersections of the circles (with radii $D_{or_cp_I}$ and $D_{or_ca_I}$) with the cylinders (with radius LD):

$$\begin{cases} f_{cp}(x) = y_B + \sqrt{9,4^2 \cdot \left[\frac{(x - x_B) - (\gamma_1 \cdot y_B + \tau_1)}{\alpha_1 \cdot y_B + \beta_1} \right]^{4/3} - (x - x_B)^2} \\ f_{ca}(x) = y_A + \sqrt{9,4^2 \cdot \left[\frac{(x - x_A) - (\gamma_1 \cdot y_A + \tau_1)}{\alpha_1 \cdot y_A + \beta_1} \right]^{4/3} - (x - x_A)^2} \end{cases} \quad (5)$$

According to LPM, the lightning striking a structure are found inside of a cylinder of radius LD . The author [1] has determined equations (5) (see [1], [5]) using the intersections of these cylinders with the afferent arcs to the distance D_{or} from where the lightning become oriented. In other words, the lightning sitting inside the curves $f_{cp}(x)$ and $f_{ca}(x)$ will strike the elements of the Over Head Lines, and lightning outside of these curves will strike elements on the ground level.

B) Calculating the coordinates for all the intersection points

B1) The coordinates of the point $C(x_C, y_C)$

The point $C(x_C, y_C)$ is the intersection of the circle with center in the print of the ground wire and of radius $D_{or_cp_I_{min}}$ with the function given in (5).

$$\begin{cases} y = y_B + \sqrt{D_{or_cp_I_{\min}}^2 - (x - x_B)^2} \\ y = y_B + \sqrt{9,4^2 \cdot \left[\frac{(x - x_B) - (\gamma_1 \cdot y_B + \tau_1)}{\alpha_1 \cdot y_B + \beta_1} \right]^{4/3} - (x - x_B)^2} \end{cases} \quad (6)$$

The values for the coordinates x_C, y_C will therefore be:

$$\begin{cases} x_C = x = x_B + (\alpha_1 \cdot y_B + \beta_1) \cdot I_{\min}^{2/3} + (\gamma_1 \cdot y_B + \tau_1) \\ y_C = y = y_B + \sqrt{D_{or_cp_I_{\min}}^2 - (x - x_B)^2} \end{cases} \quad (7)$$

B2) The coordinates of the point $D(x_D, y_D)$

The point $D(x_D, y_D)$ is the intersection of the circle with center in the print of the phase conductor and of radius $D_{or_ca_I_{cont}}$ with the function given in (5).

$$\begin{cases} y = y_A + \sqrt{D_{or_ca_I_{cont}}^2 - (x - x_A)^2} \\ y = y_B + \sqrt{9,4^2 \cdot \left[\frac{(x - x_B) - (\gamma_1 \cdot y_B + \tau_1)}{\alpha_1 \cdot y_B + \beta_1} \right]^{4/3} - (x - x_B)^2} \end{cases} \quad (8)$$

Mathematically solving the system of equations (8), one will obtain a polynomial equation of degree 8 of the following form:

$$\left[M^3 (x - N)^4 - f(x) \right]^2 = g^2(x) \cdot (\sqrt{h(x)})^2 \quad (9)$$

where:

$$\begin{cases} M = \frac{9,4^2}{(\alpha_1 \cdot y_B + \beta_1)^{4/3}} \\ N = x_B + \gamma_1 \cdot y_B + \tau_1 \\ \gamma = 2 \cdot (y_A - y_B) \\ \beta = 2 \cdot (x_A - x_B) \\ \tau = (x_A^2 - x_B^2) - D_{or_ca_I_{cont}}^2 - (y_A - y_B)^2 \end{cases} \quad (10)$$

and respectively,

$$\begin{cases} f(x) = 3\gamma^2 \cdot (\sqrt{h(x)})^2 (\beta x - \tau) + \beta^3 x^3 - 3\beta^2 x^2 \tau + 3\beta x \tau^2 - \tau^3 \\ g(x) = \gamma^3 \cdot (\sqrt{h(x)})^2 + 3\lambda\beta^2 x^2 - 6\lambda\beta x \tau + 3\gamma\tau^2 \\ h(x) = D_{or_ca_I_{cont}}^2 - (x - x_A)^2 \end{cases} \quad (11)$$

Therefore, the coordinates of the point K will be:

$$\begin{cases} x_D = \text{ecuation.solution}(9) \\ y_D = y_A + \sqrt{D_{or_ca_I_{cont}}^2 - (x_K - x_A)^2} \end{cases} \quad (12)$$

B3) The coordinates of the point $E(x_E, y_E)$

The point $E(x_E, y_E)$ is the intersection of the two functions given in (5).

$$\begin{cases} y = y_B + \sqrt{9,4^2 \cdot \left[\frac{(x - x_B) - (\gamma_1 \cdot y_B + \tau_1)}{\alpha_1 \cdot y_B + \beta_1} \right]^{4/3} - (x - x_B)^2} \\ y = y_A + \sqrt{9,4^2 \cdot \left[\frac{(x - x_A) - (\gamma_1 \cdot y_A + \tau_1)}{\alpha_1 \cdot y_A + \beta_1} \right]^{4/3} - (x - x_A)^2} \end{cases} \quad (13)$$

(13) will give a nonlinear equation in X which can be solved numerically using for example bisection method, Newton's method or a homotopy method. The solution of this equation will be the x -coordinate of the point E .

Therefore, the coordinates of the point E will be:

$$\begin{cases} x_E = \text{ecuation.solution}(13) \\ y_E = y_B + \sqrt{9,4^2 \cdot \left[\frac{(x - x_B) - (\gamma_1 \cdot y_B + \tau_1)}{\alpha_1 \cdot y_B + \beta_1} \right]^{4/3} - (x_E - x_B)^2} \end{cases} \quad (14)$$

B4) The coordinates of the point $F(x_F, y_F)$

The point $F(x_F, y_F)$ is the intersection of the circle with center in the print of the ground wire and of radius $D_{or_cp_I_{max}}$ with the function given in (5).

$$\begin{cases} y = y_B + \sqrt{D_{or_cp_I_{max}}^2 - (x - x_B)^2} \\ y = y_B + \sqrt{9,4^2 \cdot \left[\frac{(x - x_B) - (\gamma_1 \cdot y_B + \tau_1)}{\alpha_1 \cdot y_B + \beta_1} \right]^{4/3} - (x - x_B)^2} \end{cases} \quad (15)$$

Similarly with B1), will obtain:

$$\begin{cases} x_F = x_B + (\alpha_1 \cdot y_B + \beta_1) \cdot I_{\max}^{2/3} + (\gamma_1 \cdot y_B + \tau_1) \\ y_F = y_B + \sqrt{D_{or_cp_I_{\max}}^2 - (x - x_B)^2} \end{cases} \quad (16)$$

B5) The coordinates of the point $G(x_G, y_G)$

The point $G(x_G, y_G)$ is the intersection of the circle with center in the print of the phase conductor and of radius $D_{or_ca_I_{\min}}$ with the function given in (5).

$$\begin{cases} y = y_A + \sqrt{D_{or_ca_I_{\min}}^2 - (x - x_A)^2} \\ y = y_A + \sqrt{9,4^2 \cdot \left[\frac{(x - x_A) - (\gamma_1 \cdot y_A + \tau_1)}{\alpha_1 \cdot y_A + \beta_1} \right]^{4/3} - (x - x_A)^2} \end{cases} \quad (17)$$

The coordinates of the point G are:

$$\begin{cases} x_G = x_A + (\alpha_1 \cdot y_A + \beta_1) \cdot I_{\min}^{2/3} + (\gamma_1 \cdot y_A + \tau_1) \\ y_G = y_A + \sqrt{D_{or_ca_I_{\min}}^2 - (x - x_A)^2} \end{cases} \quad (18)$$

B6) The coordinates of the point $H(x_H, y_H)$

The point $H(x_H, y_H)$ is the intersection of the circle with center in the print of the phase conductor and of radius $D_{or_ca_I_{cont}}$ with the function given in (5).

The coordinates of the point H are:

$$\begin{cases} x_H = x_A + (\alpha_1 \cdot y_A + \beta_1) \cdot I_{cont}^{2/3} + (\gamma_1 \cdot y_A + \tau_1) \\ y_H = y_A + \sqrt{D_{or_ca_I_{cont}}^2 - (x - x_A)^2} \end{cases} \quad (19)$$

B7) The coordinates of the point $I(x_I, y_I)$

The point $I(x_I, y_I)$ is the intersection of the circle with center in the print of the ground wire and of radius $D_{or_cp_I_{\max}}$ with the y -axis.

The coordinates of I are:

$$\begin{cases} x_I = 0 \\ y_I = y_B + \sqrt{D_{or_cp_I_{\max}}^2 - (x - x_B)^2} \end{cases} \quad (20)$$

B8) The coordinates of the point $J(x_J, y_J)$

The point $J(x_J, y_J)$ is the intersection of the circle with center in the print of the ground wire and of radius $D_{or_cp_I_{\min}}$ with the y -axis.

The coordinates of the point J are:

$$\begin{cases} x_J = 0 \\ y_J = y_B + \sqrt{D_{or_cp_I_{\min}}^2 - (x - x_B)^2} \end{cases} \quad (21)$$

B9) The coordinates of the point $K(x_K, y_K)$

The point $K(x_K, y_K)$ is the intersection of the two circles with centers in the prints of the ground wire and phase conductor and of radii $D_{or_cp_I_{\min}}$ and $D_{or_ca_I_{\min}}$, respectively.

$$\begin{cases} y = y_A + \sqrt{D_{or_ca_I_{\min}}^2 - (x - x_A)^2} \\ y = y_B + \sqrt{D_{or_cp_I_{\min}}^2 - (x - x_B)^2} \end{cases} \quad (22)$$

The coordinates of the point K are:

$$\begin{cases} x_K = \frac{(pq + r^2 x_A) \pm \sqrt{(pq + r^2 x_A)^2 - (q^2 + r^2)(p^2 - r^2(D_{or_ca_I_{\min}}^2 - x_A^2))}}{q^2 + r^2} \\ y_K = y_A + \sqrt{D_{or_ca_I_{\min}}^2 - (x - x_A)^2} \end{cases} \quad (23)$$

where:

$$\begin{cases} p = (x_A^2 - x_B^2) - (D_{or_ca_I_{\min}}^2 - D_{or_cp_I_{\min}}^2) - (y_A - y_B)^2 \\ q = 2 \cdot x \cdot (x_A - x_B) \\ r = 2 \cdot (y_A - y_B) \end{cases} \quad (24)$$

C) Determining the areas affected by the lightning strokes and evaluating the integrals met in the formula

After describing the expressions of the functions (see step A) and calculating the coordinates of the intersection points (see step B), one can calculate the probability of lightning current appearance on the phase conductor in A with an amplitude that will produce the flashover of the insulators string.

From a geometrical point of view, this probability can be expressed as:

- the quotient between the probability of lightning appearance on the phase conductor (with an amplitude of lightning current equal or more than I_{cont}) and the probability of lightning appearance on the Over Head Line, or
- the quotient of the following areas

$$PC_{po\ int_A} = \frac{S_{DEH}}{S_{JCDEFIJ} + S_{CKGHEDC}} = \frac{S_{DEH}}{S_{total}} = \quad (25)$$

were:

$$S_{DEH} = \int_D^E f_{cp} \cdot dx - \int_D^H D_{or_ca_Icont} \cdot dx - \int_H^E f_{ca} \cdot dx \quad (26)$$

$$S_{JCDEFIJ} = \int_I^F D_{or_cp_Imax} \cdot dx - \int_J^C D_{or_cp_Imin} \cdot dx - \int_C^F f_{cp} \cdot dx \quad (27)$$

$$S_{CKGHEDC} = \int_C^E f_{cp} \cdot dx - \int_C^K D_{or_cp_Imin} \cdot dx - \int_K^G D_{or_ca_Imin} \cdot dx - \int_G^E f_{ca} \cdot dx \quad (28)$$

Having in mind that the amplitudes of the lightning currents are log-normal distributed:

$$p(\ln I) = \frac{1}{\sigma I \sqrt{2\pi}} \cdot e^{-\frac{(\ln I - \ln \bar{I})^2}{2\sigma^2}} \quad (29)$$

we will rewrite the probability of a lightning current appearance on the phase conductor in A with an amplitude that will produce the flashover of the insulators string as:

$$PC_{po\ int_A} = \frac{\iint_{DEH} p(\ln I) \cdot dx \cdot dy}{\iint_{JCDEFIJ+CKGHEDC} p(\ln I) \cdot dx \cdot dy} \quad (30)$$

3. Numerical approach

The calculations seem to be complicated; it is sufficient to see that we would have to analytically solve a polynomial equation of degree 8. From a

mathematical point of view, analytically solving such an equation is very difficult, if not impossible. From this reason, we should look for a numerical approach to calculate the probability for the flashover of the insulators string due to lightning strokes on the phase conductors.

The actual presented model is sketched also in figure 2, but in polar coordinates (R, Θ) . It might be easier to use polar coordinates and arrive at (46) to calculate the probability for the flashover of the insulators string. For this, one needs to complete the following two steps:

- A) rewrite the expressions for all the functions in the figure.
- B) determine the areas affected by the lightning strokes and evaluate the integrals.

A) Rewriting the functions in polar coordinates (R, Θ)

One will start from the known fact that:

$$\begin{cases} x - x_A = R \cdot \cos \theta \\ y - y_A = R \cdot \sin \theta \end{cases} \quad (31)$$

A1) Rewriting the functions $y = f_{ca}(x)$ and $y = f_{cp}(x)$ in polar coordinates:

$$y = f_{ca}(x) = \sqrt{9,4^2 \cdot \left[\frac{(x - x_A) - (\gamma_1 \cdot y_A + \tau_1)}{\alpha_1 \cdot y_A + \beta_1} \right]^{4/3} - (x - x_A)^2} \quad (32)$$

becomes

$$\Theta = \tilde{f}_{ca}(R) \quad (33)$$

where

$$\tilde{f}_{ca}(R) = \arccos \left[\frac{\left(\frac{R}{9,4} \right)^{3/2} \cdot (\alpha_1 \cdot y_A + \beta_1) + (\gamma_1 \cdot y_A + \tau_1)}{R} \right] \quad (34)$$

and,

$$y = f_{cp}(x) = \sqrt{9,4^2 \cdot \left[\frac{(x - x_B) - (\gamma_1 \cdot y_B + \tau_1)}{\alpha_1 \cdot y_B + \beta_1} \right]^{4/3} - (x - x_B)^2} \quad (35)$$

becomes

$$\Theta = \tilde{f}_{cp}(R) \quad (36)$$

where

$$\tilde{f}_{cp}(R) = \arccos \left[\frac{\left(\frac{R}{9,4} \right)^{3/2} \cdot (\alpha_1 \cdot y_B + \beta_1) + (\gamma_1 \cdot y_B + \tau_1)}{R} \right] \quad (37)$$

A2) Rewriting the equation $x = 0$ in polar coordinates

$$x = 0 \quad (38)$$

becomes

$$\Theta = \tilde{f}_{IJ}(R) \quad (39)$$

where

$$\tilde{f}_{IJ}(R) = \arccos \left(-\frac{x_B}{R} \right) \quad (40)$$

A3) Rewriting R

Knowing the log-normal repartition for the I

$$\begin{cases} CDF : F_I(i) = \Phi \left(\frac{\ln I - \mu}{\sigma} \right) = P(I \leq i) \\ PDF : f_I(i) = \frac{1}{i\sigma\sqrt{2\pi}} \cdot e^{-\frac{(\ln i - \mu)^2}{2\sigma^2}} \end{cases} \quad (41)$$

we would like to calculate the CDF (cumulative density function) and the PDF (probability density function) for the variable R , where:

$$R = \alpha \cdot I^\beta \quad (42)$$

$$CDF : F_R(r) = F_R(r, \mu, \sigma) = P(R \leq r) = P(\alpha \cdot I^\beta \leq r) = P\left(I \leq \left(\frac{r}{\alpha}\right)^{1/\beta}\right) =$$

$$= \Phi \left(\frac{\ln \left[\left(\frac{r}{\alpha} \right)^{1/\beta} \right] - \mu}{\sigma} \right) = \Phi \left(\frac{\frac{1}{\beta} \cdot \ln \frac{r}{\alpha} - \mu}{\sigma} \right) \quad (43)$$

$$\begin{aligned}
PDF : f_R(r) &= f_R(r, \mu, \sigma) = \frac{d}{dr}[F_R(r)] = \frac{d}{dr} \left[\Phi \left(\frac{\frac{1}{\beta} \cdot \ln \frac{r}{\alpha} - \mu}{\sigma} \right) \right] = \\
&= \Phi^I \left(\frac{\frac{1}{\beta} \cdot \ln \frac{r}{\alpha} - \mu}{\sigma} \right) \cdot \frac{d}{dr} \left(\frac{\frac{1}{\beta} \cdot \ln \frac{r}{\alpha} - \mu}{\sigma} \right) = \Phi^I \left(\frac{\frac{1}{\beta} \cdot \ln \frac{r}{\alpha} - \mu}{\sigma} \right) \cdot \frac{1}{\sigma} \cdot \frac{1}{r} = \\
&= \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{\left(\frac{1}{\beta} \cdot \ln \frac{r}{\alpha} - \mu \right)^2}{2\sigma^2}} \cdot \frac{1}{r\beta\sigma} = \frac{1}{r\beta\sigma\sqrt{2\pi}} \cdot e^{-\frac{\left(\frac{1}{\beta} \cdot \ln \frac{r}{\alpha} - \mu \right)^2}{2\sigma^2}} \quad (44)
\end{aligned}$$

Since R and Θ are independent variables, we have:

$$f_{R,\Theta}(r, \theta) = f_R(r) \cdot f_{\Theta}(\theta) = \frac{1}{r\beta\sigma\sqrt{2\pi}} \cdot e^{-\frac{\left(\frac{1}{\beta} \cdot \ln \frac{r}{\alpha} - \mu \right)^2}{2\sigma^2}} \cdot 1_{\left[0, \frac{\pi}{2}\right]}(\theta) \quad (45)$$

B) Determining the areas affected by the lightning strokes and evaluating the integrals

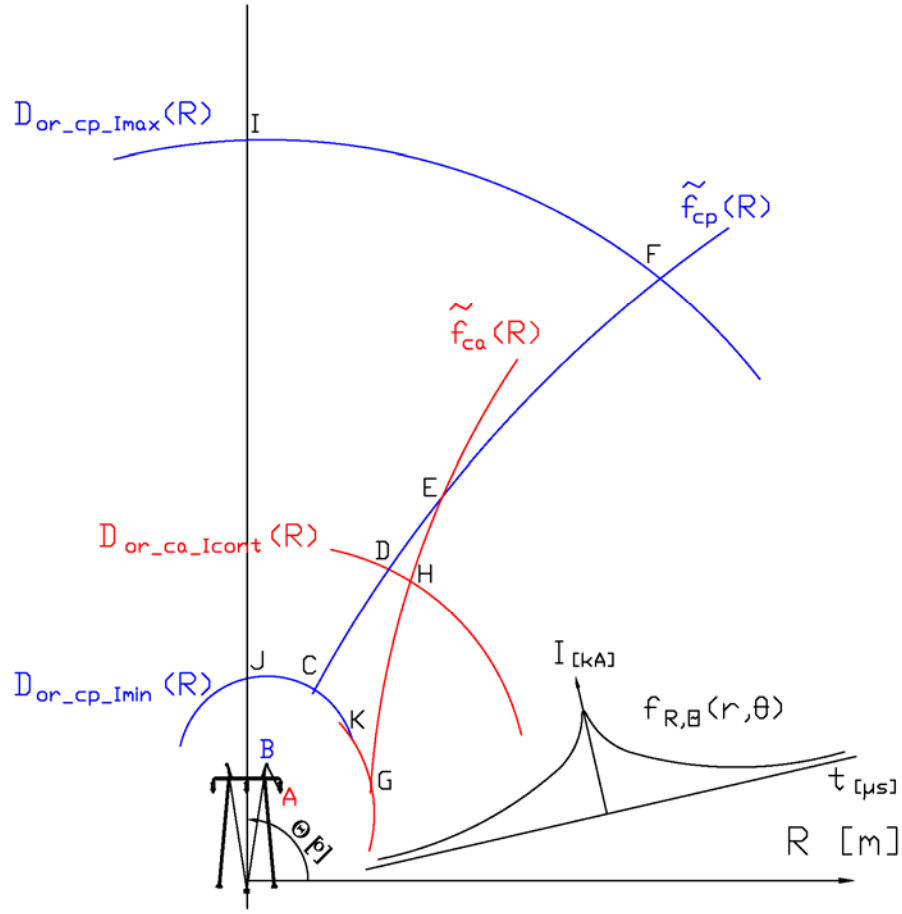


Fig. 2 Calculation of the probability for the flashover of the insulator string due to the direct lightning on the phase conductor – Polar coordinate system.

Relation (30) becomes:

$$PC_{po\ int_A} = \frac{\iint_{[DEH]} f_{R,\Theta}(r,\theta) \cdot dr \cdot d\theta}{\iint_{[JCDEFHIJ]} f_{R,\Theta}(r,\theta) \cdot dr \cdot d\theta + \iint_{[CKGHEDC]} f_{R,\Theta}(r,\theta) \cdot dr \cdot d\theta} \quad (46)$$

where:

$$\iint_{[DEH]} f_{R,\Theta}(r,\theta) \cdot dr \cdot d\theta = \int_{D_{or_ca_Icont}}^{D_{or_ca_IE}} \int_{\tilde{f}_{ca}(R)}^{\tilde{f}_{cp}(R)} f_{R,\Theta}(r,\theta) \cdot d\theta \cdot dr =$$

$$= \int_{D_{or_ca_lcont}}^{D_{or_ca_lmax}} f_{R,\Theta}(r, \theta) \cdot [\tilde{f}_{cp}(R) - \tilde{f}_{ca}(R)] \cdot dr \quad (47)$$

$$\begin{aligned} \iint_{[JCDEFHIJ]} f_{R,\Theta}(r, \theta) \cdot dr \cdot d\theta &= \int_{D_{or_cp_lmin}}^{D_{or_cp_lmax}} \tilde{f}_{IJ}(R) \int f_{R,\Theta}(r, \theta) \cdot d\theta \cdot dr = \\ &= \int_{D_{or_cp_lmin}}^{D_{or_ca_lmax}} f_{R,\Theta}(r, \theta) \cdot [\tilde{f}_{IJ}(R) - \tilde{f}_{cp}(R)] \cdot dr \quad (48) \end{aligned}$$

$$\begin{aligned} \iint_{[CKGHEDC]} f_{R,\Theta}(r, \theta) \cdot dr \cdot d\theta &= \int_{D_{or_cp_lmin}}^{D_{or_cp_IE}} \tilde{f}_{cp}(R) \int f_{R,\Theta}(r, \theta) \cdot d\theta \cdot dr - \\ &- \int_{D_{or_ca_lmin}}^{D_{or_cp_lmin}} \tilde{f}_{ca}(R) \int f_{R,\Theta}(r, \theta) \cdot d\theta \cdot dr = \\ &= \int_{D_{or_cp_lmin}}^{D_{or_cp_IE}} [\tilde{f}_{cp}(R) - \tilde{f}_{ca}(R)] \cdot \\ &- \int_{D_{or_ca_lmin}}^{D_{or_cp_lmin}} [\Theta_K - \tilde{f}_{ca}(R)] \quad (49) \end{aligned}$$

For an OHL having two ground wires, length L and being in an area with ground flash density D_{ground} , the probability for the flashover of the insulators string due to direct lightning hits on the phase conductors can be written as:

$$P_{flashover} = 2 \cdot D_{ground} \cdot L \cdot (PC_{point_A} \cdot k_{equalization}) \quad (50)$$

4. Conclusions

The probability for the flashover of the insulators string due to direct lightning strokes on the phase conductors can be calculated using (46). The accuracy of the results depends on a series of factors as:

- methods of calculations: the impedances and earth-resistance for the towers and the method of calculation other elements that are needed to evaluate: I_{cont} and $D_{or_ca_I_{cont}}$ for example;
- determining the amplitudes of the lightning currents I_{min} and I_{max} , etc.

For a better approximation of this probability, the authors propose to take into account the following:

- for a spam without dislevelment, the average height of the conductors above the ground is given by the relation:

$$h_{conductor} = h_{clamps} - \frac{2}{3} \cdot f_{conductor} \quad (51)$$

were:

$h_{conductor}$ - the average height of the conductors above the ground;

h_{clamps} - the height of the clamps above the ground;

$f_{conductor}$ - the arrow of the ground wire.

- the quotient between arrow of the ground wire and the one of the phase conductor is sub-unitary. To correlate the two types of conductors to the arrow, one usually impose the relation:

$$f_{ground_wire} = k_{co} \cdot f_{phase_conductor} \quad (52)$$

were:

f_{ground_wire} - the arrow of the ground wire;

$f_{phase_conductor}$ - the arrow of the phase conductor;

$k_{co} = 0.95$ - the arrow equalization coefficient at the -5 °C in the white frost existence case on the conductor.

The role of the relations (51) and (52) in calculating the probability for the flashover of the insulators strings is presented in PhD thesis [5].

In the next article, “Calculation of the probability for the flashover of the insulators string due to direct lightning strokes on the phase conductors. Part III. Applications.”, the authors will present some results obtained using relation (50).

We believe, it is absolutely necessarily to confront these data with the values offered by the Lightning Location System as well as the staff working inside the power plants.

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