

## AN EXPERIMENTAL STUDY OF SHAFT SPILLWAY WITH A POLYGONAL SECTION

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*The present paper reports the results of an experimental study of a novel design of shaft spillway geometry proposed in order to improve hydrodynamic flow condition during floods. The proposed conception consists of replacing the classical circular cross section of the entrance with a 12-section polygonal configuration which allows the entry of the water jet without detachment in the connecting elbow and a direct continuity to the well. Experiments on physical model showed that the maximum water level (MWL) during the maximum flood has decreased by 0.68 m, allowing a more reliability and avoiding cavitation risk. In addition, the new design increases discharge coefficient up to 0.52 which in turn can allow a gain in terms of dam volume.*

**Keywords:** shaft spillways; polygonal section; flow; well; connecting elbow; design.

### 1. Introduction

A flood discharge is one of the major problems occurring in the process of construction of large dams. It is necessary to ensure the passage of exceptionally high discharges, even higher than what the most pessimistic forecasts can predict [1, 2].

In addition, it is necessary to ensure that these masses of water fall from a height equal to the height of the dam, often more than one hundred meters. Thus, the difficulty is twofold: first of all, it is necessary to ensure the passage of water, and then dissipate its kinetic energy acquired in this way.

It is known that the energy dissipation usually takes place either by means of a hydraulic jump in a stilling basin or, if the geological conditions permit, by the throwing of the flow in the riverbed. The problem of the passage of water is to provide a smooth flow structure at the inlet to the water conveying structure, aeration of the tunnel. Fewer difficulties arise when the flow is diverted by the open channel [3, 4, 5].

Numerous spillway structures can be divided into two main types, depending on whether they take water from the surface of the reservoir or have a

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deep-water intake. Among the surface water intake structures, in turn, we can distinguish two alternatives: straight weir, usually along the dam crest, or circular weir (followed by a water fall shaft and a horizontal diversion gallery), located near the dam [6, 7].

One of the main structures of reservoir hydrosystems is a weir designed to discharge excess water. The costs for construction of spillways should meet the requirements of reliability and safety of the spillway system operation, in connection with which they are calculated for the discharges of rare frequency, forcing to increase the size of the spillway structures and increase the cost of the entire complex. To reduce the cost of construction of spillway structures, the spillways used that require a minimum amount of expensive construction materials [8].

These conditions are fully satisfied by shaft spillways, which are widely used in hydraulic engineering practice because of the advantages that they have, such as: compact design, high discharge capacity and automatic operation. As a rule, they are used in rocky areas and relatively narrow areas, where other types of weirs become expensive [9, 10, 11].

To provide favorable hydraulic conditions for operation of the water intake funnel, it is necessary to install devices to prevent rotation of the flow at the inlet and eliminate the danger of cavitation phenomena [12, 13].

To reduce the cost of spillways of new dams and overcome the problem of cavitation as well as to facilitate construction procedure, a special design was developed [14, 15]. It consists on replacing the circular cross section of the shaft with a polygonal cross section that ensures the establishment of a continuous flow without bending points along the vertical conveying shaft and simplifies the formwork, which is confirmed by the following circumstances:

First, the polygonal shape replaces the inner surface of the shaft with two-dimensional curvature with a system of cylindrical surfaces with one-dimensional curvature, the formwork for which is performed without bends in the direction of water movement. The working surface of the formwork can be formed with flat sheets, for example, plywood, steel sheets, etc. [16, 17].

Secondly, in the cross sections of the water conveying system, the hooped reinforcement is replaced with a system of straight elements, which greatly simplifies the performance of reinforcement work.

Thirdly, the appropriate selection of the outline of the shaft discharge surface allows for a continuous flow from the shaft to the connecting bend [18].

Fourth, depending on the design discharge, the choice of a rational number of sides of a regular polyhedron makes it possible to perform the connecting elbow as a direct continuation of the shaft without bends of the surface across the longitudinal axis of the flow.

In order to increase the discharge capacity and simplify the design of the intake funnel head, the easiest is to perform circular cylindrical cross section. The adoption of a regular dodecagon facilitates the design of a water intake funnel [19, 20].

In the practice of model hydraulic studies, the scale of modelling and the assignment of the model operation modes are determined by the technical capabilities of the hydraulic laboratory, the limiting parameters of which are either the size of the laboratory or its discharge.

The purpose of this paper is to present results of an experimental study conducted on the spillway geometry with a novel design. It consists in replacing the circular cross section of the well with a 12-section polygonal configuration. The model hydraulic studies showed an increase in discharge of the mine by 15% compared with the mine spillway of traditional shape.

In addition, the head of the water intake funnel is cylindrical, which allows to increase the flow coefficient of 0.46 a 0.52, without subjecting the head of the shaft to the danger of cavitation effects. The conducted experiments provided new design for dam shaft spillways which have been proposed for the construction of the Djedra dam in Algeria.

## 2. Materials and Methods

### 2.1 Description of the proposed model

Shaft spillway with a polygonal cross section is performed in accordance with the patent of the Russian Federation No.2341516 (Figure 1). Water intake funnel is performed in plan in the form of dodecagon. The lower part of the spillway shaft is made in the form of a transition section with a radial bend, adjacent to the tailrace conduit. The inlet section of the transition section is made in the form of a square, two opposite sides  $b_1$  (Fig. 2) of which are parallel to the longitudinal axis of the tailrace conduit. The inner surface of the shaft is made of four faces 1-1 and 2-2 of trapezoidal shape and eight faces 3-4 of triangular shape, which converge by two at the corner points of the inlet section of the transition section. The lower edges of the trapezoidal faces form an inlet square cross section of the transition section (Fig. 1), and the upper edges 9-10 form two pairs of mutually orthogonal edges of the water intake funnel, one of which is parallel to the longitudinal axis of the tailrace conduit. All faces of the inner surface of the shaft tangentially conjugate with the corresponding faces of the water intake funnel head (Fig.1 and Fig.3). Such an internal surface of the shaft spillway provides the formation of a discharge surface without bends across the flows, which in the traditional design of shaft spillways cause a flow separation from the surface of the shaft and create the danger of cavitation erosion of concrete. The square inlet section of the transition section of the shaft spillway at Djedra Dam

has sides  $b_1 = b_2 = 6.0$  m and the outlet section of rectangular shape has dimensions of the outlet section and has sides  $b_1 \times b_2 = 5.0 \times 6.0$  m.

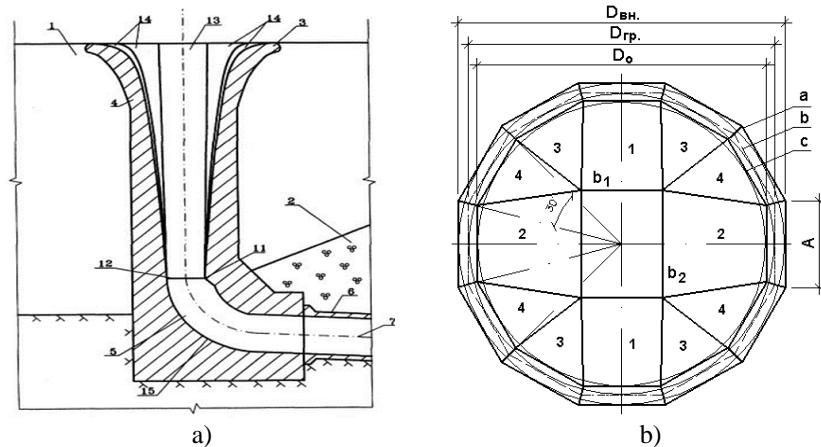


Fig. 1.a) shows the vertical sectional view of a weir in wells along the longitudinal axis of the discharge pipe; b) Plan of the shaft of the polygonal cross-section with a 12-hole water inlet funnel, (1, 2) - Trapezoidal faces;(3, 4) - corner faces.

The following elements are indicated in Figure 1:

A – length of the outer edge of the face of the intake funnel head;

$D_{out}$  – diameter of the inscribed circle of the outer boundary of the funnel;

$D_{crest}$  – diameter of the inscribed circle of the funnel crest;

$D_o$  – diameter of the inscribed circle of the conjugation line of the funnel crest and spillway faces of the shaft;

$b_1$  and  $b_2$  – sides of the rectangular outlet section of the shaft.

a - outer edge of the intake funnel

b - axis of the intake funnel;

c – conjugation line of the inner surface of the funnel faces and corresponding faces of the intake funnel crest.

Fig. 2 shows the location of the elliptical guides of the profile of the shaft inner surface.

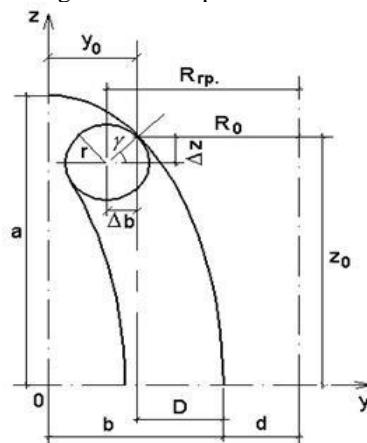


Fig. 2. Diagram of construction the faces of the elliptic profile edges.

The guides of the elliptical faces of the discharge surface are described by the equation of the ellipse in parametric form:

$$F = \frac{z^2}{a^2} + \frac{y^2}{b^2} - 1 \quad (1)$$

After the corresponding mathematical transformations, we obtain the current coordinates of the normal line for the construction of the outer surface of the shaft:

$$\left. \begin{aligned} Y &= \frac{b}{a} \cdot \sqrt{a^2 - z^2} \cdot \left[ 1 - \frac{t \cdot a^2}{b \cdot \sqrt{a^4 - z^2 \cdot (a^2 - b^2)}} \right] \\ Z &= z - \frac{t \cdot b \cdot z}{\sqrt{a^4 - z^2 \cdot (a^2 - b^2)}} \end{aligned} \right\} \quad (2)$$

At  $z = 0$ , we obtain for the outer surface of the shaft wall:

$$Y = b, \quad Z = 0.$$

$$\text{At } z = a, \quad Y = 0, \quad Z = a - t.$$

The shaft spillway design described above was adopted in the final design of Djedra Dam

The initial data for calculating the parameters of the outline of the internal faces of the shaft spillway have the following values:

Elevation of the weir crest :  $\nabla_{crest} = 555$  m;

Elevation of the end section :  $\nabla_{end\ sect.} = 520$  m;

Diameter of the intake funnel crest  $D_{crest} = 22.5$  m;

Radius of the shaft circular head:  $r = 0.75$  m;

Angle of tangent slope with vertical line:  $\gamma = 33.29^\circ$ ,

Width of the tailrace tunnel  $b = 6.0$  m;

The shaft section at the outlet is square,  $b_1 \times b_2 = 6.0 \times 6.0$  m in size

The design spillway described above is adopted in the final design of Djedra Dam in Algeria. This dam is located in Souk Ahras Province, occupying an area of  $150 \text{ km}^2$ , and is expected to annually provide  $12 \text{ million m}^3$  for drinking water supply to the main city and its suburb and  $2 \text{ million m}^3$  for the needs of the industrial zone.

The spillway is designed as a single structure with a water intake tower. The main features of Djedra Dam are as follows:

Reservoir yield is  $15 \text{ million m}^3/\text{year}$ ;

Full supply level in the upstream is  $555.00$  m;

Maximum water level is  $558.28$  m;

Total reservoir capacity is  $35 \text{ million m}^3$ ;

Elevation of the dam crest is 555.00 m;

Dam height is 60 m;

Crest length is 425 m;

Crest width is 22.50 m

The tailrace conduit of the shaft spillway is designed as a free-flow tunnel with U-shaped cross section with width  $b=6.0$  m and height  $a=7.0$  m and with a crown of a semi-circular outline with radius  $R=3.0$  m. The tunnel is connected to the shaft using an elbow with rectangular cross section  $6 \times 7$  m with radius  $R=13.0$  m. After the elbow outlet there is a transition section, in which the rectangular cross section of the elbow passes into U-shaped section of the tailrace tunnel.

The length of the tailrace tunnel is 173.40 m; the slope of the tunnel bottom is 0.0225.

To dissipate the flow energy coming out of the tunnel, a stilling basin expanding in plan was designed and studied with a blind end wall. Water from the stilling basin was released into the river through a cut-out in the side wall facing the river flow. Since the stilling basin axis is at an angle of  $55^\circ$  to the river axis, such a design of the stilling basin provided the most favorable connection with the river flow with the minimum specific discharges. This decision was embodied in the construction of Babana Dam in Syria [21].

The operation of the tailrace tunnel and the expanding stilling basin with a cut-out in the side wall was checked in all ranges of possible operation modes of the shaft spillway on a hydraulic model on a scale of 1:60 made in laboratory of hydroelectric power stations of Moscow State University of Environmental Engineering.

Fig. 3 shows the vertical longitudinal section of the shaft spillway with a polygonal cross section at Djedra Dam.

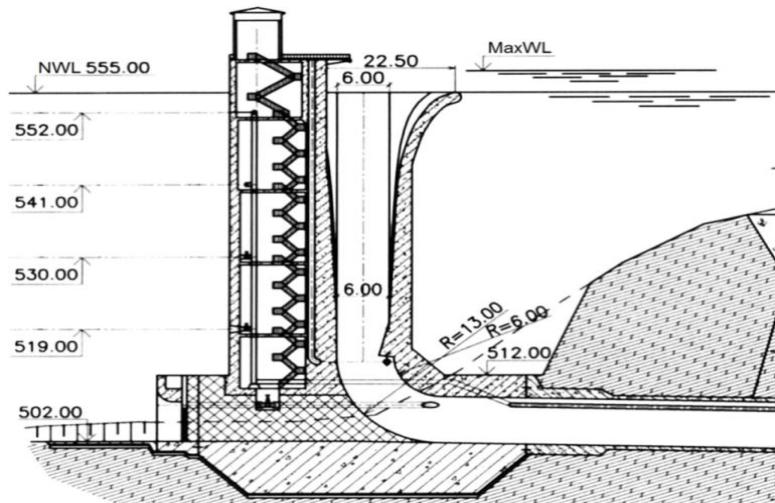


Fig. 3. Longitudinal section of the shaft spillway of the Djedra dam.

## 2.2. Experimental Setup

The spillway model with a polygonal cross section was made in the laboratory of hydroelectric power station of Moscow State University of Environmental Engineering. The experimental set consisted of a large mirror invert with a width of 100 cm and a length of 950 cm with a slope equal to zero, connected to the intake tank of  $1.64 \times 2.0$  m (Fig. 4).

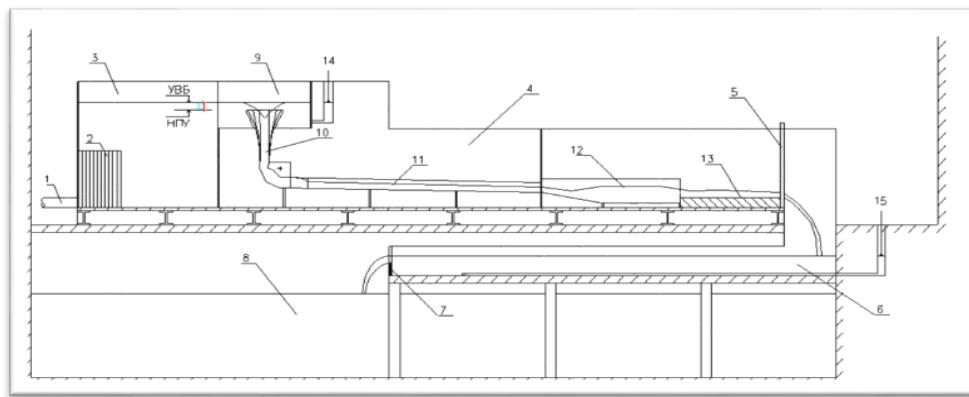


Fig. 4. Experimental setup of the polygonal section model.

The indications of experimentation model are as follows:

1. Supply pipe; 2. Metal grid, 3. Receiving reservoir; 4. Mirror Tray; 5. Shutters to adjust the water level in the trough 6. Measuring weir trough; 7. Dimensional triangular weir; 8. Return water through 9. Simulated ground of the upper side, connecting the spillway; 10. wells, 11. Tunnel; 12. Water well; 13. Simulation plot of the river, 14. Piezometer upstream; 15. Measuring level Piezometer.

The spillway model has been taken as the basis for the design of the shaft spillway with a polygonal cross-section of Djedra dam located in Algeria (Souk-Ahras Wilaya).

The maximum head at the dam makes up 55 m. The model set invert height was 1.6 m that permitted to construct a model with scale 1: 60 of actual size, in accordance with which the crest diameter of the water intake funnel and the crest axis diameter of the water intake funnel is  $D_{cr,mod.} = 22.5 : 60 = 0.375$  m.

According to the feasibility study of Djedra dam project, the maximum head at the crest of the water intake funnel made up  $H_{design} = 3.28$  m at discharge with probability  $P=0.01\%$  with peak discharge  $Q_{peak} = 915 \text{ m}^3/\text{s}$  at  $Z_{upstream water level} = 558.28$  m and transformed discharge  $Q_{transformed} = 740 \text{ m}^3/\text{s}$ .

To ensure kinematic characteristics of the flow in the zone of the water intake funnel impact it was necessary to provide the minimum water depth in the zone of flow approaching to the water intake funnel. According to generally accepted views on the surface weirs operation the sill height should be at least 3 times higher than the maximum head. The maximum head  $H_{\text{design}}=3.28$  m corresponded to the model head  $H_{\text{design,mod.}}=3.28:60=0.055$  m. The shaft height above the chamber bottom of the water intake funnel was adopted equal to  $P=0.19$  m that corresponded to the ratio of  $P: H_{\text{design,mod.}}=0.192: 0.055 = 3.5 > 3$ .

At the width of the model headrace channel 1.0 m where the model of spillway is installed the distance between the crest axis of the water intake funnel and invert wall made up 0.313 m that corresponded to  $B: H_{\text{design,mod.}}=0.313: 0.055=5.7$ . The surface spillway head is measured at a distance  $B: H_{\text{design}}=3$ , so modelling was right according to this parameter too. Downstream along the flow the upstream face of the dam was modelled as a vertical wall also installed at a distance 0.313 m from the crest axis of the water intake funnel of the spillway.

At the height of the water intake funnel crest  $P=0.192$  m the minimum relative height is

$P / H_{\text{max}} = 3.68$ . For the funnel sill height  $P / H_{\text{design}} > 3$  its effect on discharge capacity is absent according to the data of studies Berezinskiy [21].

In order to provide the possibility of visual observation over the flow processes, the model of shaft spillway with polygonal cross-section and the chamber of the water intake funnel were made of plexiglass.

### 2.3. Main components of the model

The main components of the model are represented by the open-type water intake funnel (fig. 5a) and lower section of the shaft with transition and initial section of the tunnel. (fig. 5b). the height of the funnel head varies from 11 to 0.5 cm. For closed spillway the hydraulic radius value equals to its depth  $R = H$ .

Specific discharge at the water intake funnel crest is determined from equation,

$$q = 44.3 \times m \times H^{1.5} \quad (3)$$

For the preliminary value of discharge coefficient  $m = 0.45$  and the value of coefficient of kinematic viscosity of water  $\nu = 0.01$  cm<sup>2</sup>/s we obtain:

$$Re \approx 1995 \times H^{1.5} \quad (4)$$

Model heads corresponded to Reynolds numbers  $Re = 72783 \dots 705$ .

For model heads 11.0...0.5 (cm) actual hydraulic radius  $R$  makes up 6.6 to 0.3 (m) which corresponds to the coefficients of hydraulic friction  $\lambda = 0.0094 \dots 0.0264$ , that is slightly below the model values. As a result, the data of the model

studies will be slightly underestimated leading to margin for hydraulic calculations.

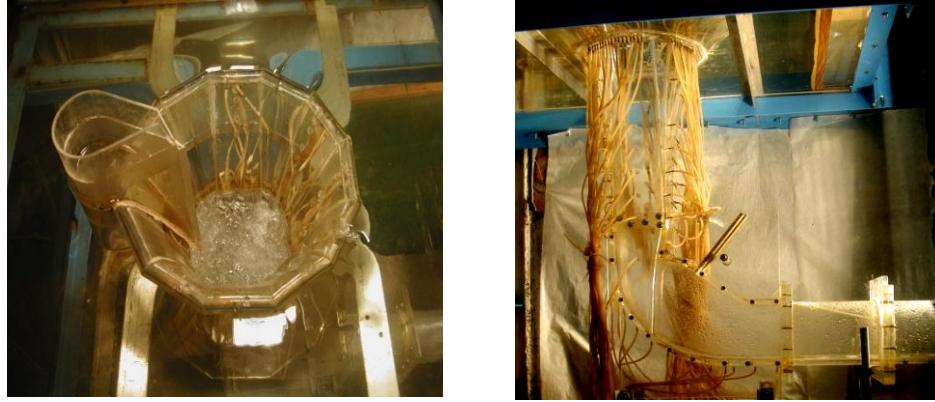


Fig.5a. Funnel of reception of water with a free entrance

Fig 5b. Shaft of the well connecting the bend and the initial section of the tunnel.

Fig. 5. Photo of Polygonal Transverse Spillway Model.

### 3. Results and Discussion

The results of studies for 2 alternatives of outlet section of the shaft with dimensions

(a)  $\times$  (b) = 5 x 6 (m) and (a)  $\times$  (b) = 5.5 x 6 (m) are shown in figures (6) and (7).

Dependence of the upstream water level on discharge  $U_{UWL}=f(Q)$  is shown in fig. (6), and dependence of the discharge coefficient on the relative head values ( $H: H_{\text{profile}}$ );  $m=f(H: H_{\text{profile}})$  is shown in figure (7).

According to figures (6) and (7), it can be seen that the maximum discharge coefficient of the water intake funnel in free-flow conditions depends on dimensions of the outlet section of the shaft. At the same time, when the flooding regime of operation is started, the discharge capacity decreases dramatically and the discharge curve takes a shape close to the vertical line.

This mode of operation of the shaft spillway is unacceptable because at small exceedance of discharge above the design one the spillway discharge over the dam crest may occur, that is unacceptable in accordance with the dam operation mode.

The discharge coefficient ( $m$ ) of the shaft spillway was determined by formula:

$$m = \frac{Q}{B\sqrt{2g} H^{3/2}} \quad (5)$$

where:  $Q$  – shaft spillway discharge;  
 $B$  – length of discharge perimeter of the water intake funnel;  
 $H$  – head at the water intake funnel crest;  
 $H_p$  – profile head at the water intake funnel crest;  
 $g = 9.81 \text{ m.s}^{-2}$  – gravity acceleration.

The profile head at the water intake funnel crest adopted equal  $3.4r$  of radius of the funnel crest, which corresponds  $H_{\text{profile}}=2.55 \text{ m}$  and this head corresponds to the maximum discharge of the spillway with straight weir  $m_{\text{max}} = 0.56$ . According to the graph in figure 7, it can be seen, that the maximum coefficient of the water intake funnel of the shaft spillway with polygonal cross-section and circular cylindrical crest has reached the value of  $m_{\text{max}}=0.52$  at relative head

$$H: H_{\text{profile}}=1.1.$$

The Wagner spillway shaft coefficient versus US Bureau of Reclamation curve is plotted on the graphs of figure 7. The results of model studies of the shaft spillway differ significantly from the data of Wagner and US Bureau of Reclamation due to the following reasons. The reduced values of the discharge coefficients are given for the sharp-crest circular weir; which inner diameter was equal to the crest diameter. Therefore, the maximum discharge coefficient was obtained at the minimum head when a jet flows on the inner circular surface and curvature of the jet has the maximum value. The maximum flow coefficient value can attain  $m_{\text{max}} = 0.527$  with a relative load  $H / H_{\text{profile}} = 1.12$ . However, the flow coefficient decreases and its maximum value is between 0.44 and 0.46, in the case of the well weir models studies (Wagner and US Bureau of Reclamation), which take into account a ring weir with a sharp edge whose internal diameter is equal to the diameter of the crest. The obtained value of the discharge coefficient of the shaft spillway of Djedra dam exceeds these values by (11...12) %. At the same time, this discharge coefficient is lower than the maximum value of the straight aerated weir by 7.7%. This can be explained by small amount of vacuum at the top of the water intake funnel.

According to the calculations of the hydrographic transformation of the flood with a peak flow of peak discharge value  $Q_{\text{peak}}^{P=0,01\%} = 915 \text{ (m}^3.\text{s}^{-1}\text{)}$ , taking into account the results of model studies, the maximum discharge made up  $Q_{\text{max}} = 638 \text{ (m}^3.\text{s}^{-1}\text{)}$  at maximum elevation

$Z_{\text{maximum water level}} = 557.608 \text{ m}$  which corresponds to head at the water intake funnel crest  $H_{\text{max}}=2.61 \text{ m}$ , which makes it possible to reduce the dam crest by 0.68 m.

The maximum flood discharge with check discharge  $Q_{\text{peak}}^{P=0,01\%} = 915 \text{ (m}^3.\text{s}^{-1}\text{)}$  is less than the design value by  $183 \text{ (m}^3.\text{s}^{-1}\text{)}$ . Decreasing the maximum discharge simultaneously with decreasing the dam body volume permits to reduce the

dimensions of the tailrace tunnel and stilling basin at simultaneous simplification of transition sections of the flow downstream the stilling basin.

The characteristics of the shaft spillway in accordance with the preliminary design and the results of hydraulic model studies taking into account transformation of the flood hydrograph with peak discharge  $Q_{peak}^{P=0.01\%} = 915 \text{ (m}^3\text{s}^{-1}\text{)}$  are given in table 1 below.

**Table 1**  
**Comparison of discharges of the shaft spillway preliminary design and the shaft spillway of polygonal cross-section.**

Upstream Water Level, (m)	Head, H, (m)	Discharge according to feasibility study data( $\text{m}^3\text{s}^{-1}$ )	Flow of studied models, Q( $\text{m}^3\text{s}^{-1}$ )
556.0	1	115	135.5
557.0	2	386	415
557.61	2.61	569	638
558.0	3	687	799
<b>558.28</b>	<b>3.28</b>	<b>740</b>	<b>923</b>

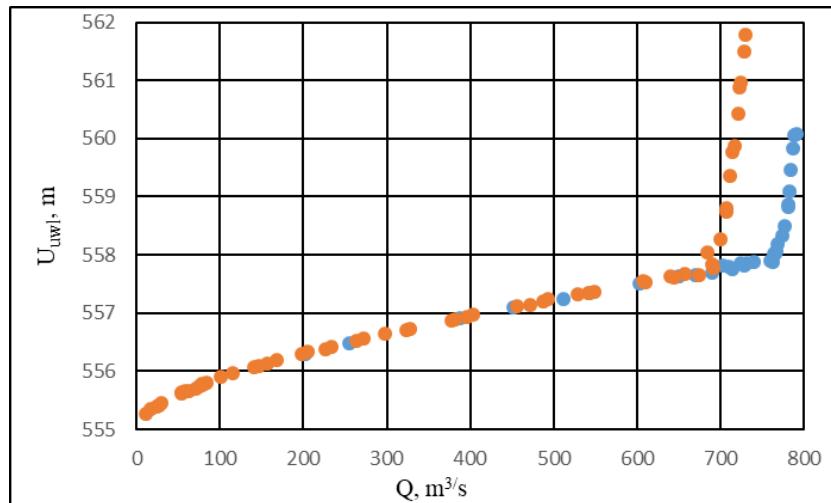
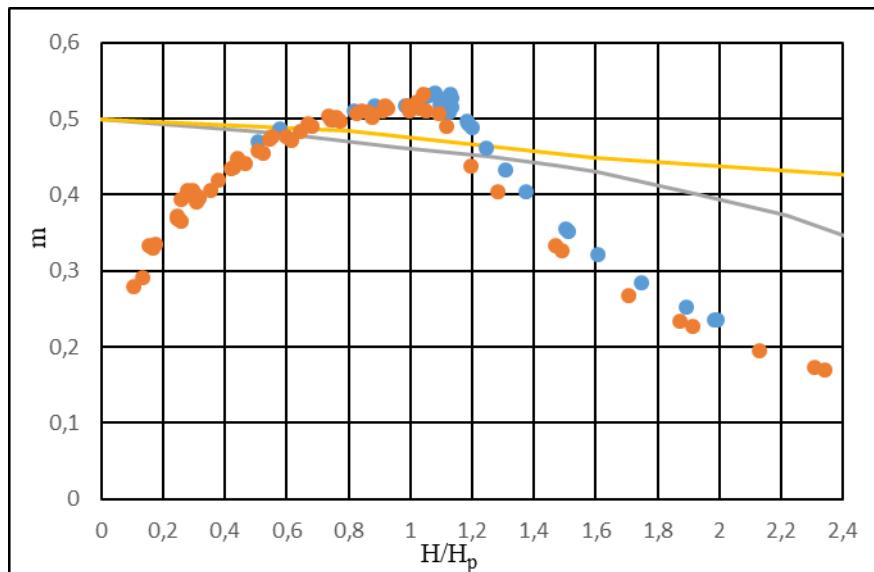


Fig. 6. Flow relationship versus top rating  $U_{\text{Upstream water level}} = f(Q)$ :  
● section of the tree (a)  $\times$  (b) =  $5 \times 6 \text{ m}$  ; ● section (a)  $\times$  (b) =  $5.5 \times 6 \text{ m}$ .



**Figure 7.** Coefficient of flow versus relative load  $m = f(H / H_p)$ :

- section of the tree (a)  $\times$  (b) = 5 x 6 m;
- section of the tree (a)  $\times$  (b) = 5.5 x 6 m.
- Wagner results;
- Results from US Bureau of Complaint

#### 4. Conclusion

A new form of the cross-section of the shaft spillway with circular cylindrical profile of the water intake funnel crest is an effective alternative for traditional structure of the nappe-shaped shaft spillway, significantly reducing the risk of cavitation erosion of concrete. The advantages of this described structure can be listed as follows:

1. The described weir design allows forming the longitudinal continuous surface of the weir edge of the shaft providing continuous contact of the jet. Removal of bends on the shaft surface greatly reduces the risk of cavitation erosion of concrete and simplifies the formwork structure, thus reducing the cost of construction of the dam.
2. The replacement of the circular cross-section of the shaft by a polygonal cross-section simplifies its structure, simultaneously improving the quality of the work and the reliability of functioning during operation.
3. The reinforcement is greatly simplified because transverse reinforcement can be manufactured of linear elements.
4. Using the circular cylindrical cross-section of the water intake funnel can significantly reduce the maximum discharge of the spillway due to more effective transformation of the flood hydrograph.

5. The use of the circular cylindrical cross-section of the water intake funnel allows decreasing the level of the dam crest and height of the tailrace tunnel and stilling basin simultaneously with reducing the maximum discharge.

7. Reducing the maximum discharge increases the reliability and safety of the dam operation in general.

8. The increase of discharge capacity of the shaft spillway with polygonal cross-section permits to decrease the maximum water level (MWL) by 0.68 m during passing the maximum discharge, which makes operation of Djedra Dam more reliable.

9. The well of the shaft spillway operates in free flow regime, which testifies the absence of pulsations and the danger of occurrence of cavitation phenomena.

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