

BANDGAPS IN THE DISPERSION OF WAVES ON A STRING OF DUST PARTICLES FLOATING IN PLASMA

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Terahertz (THz) waves are probably set to revolutionize imaging technology and telecommunications. Frequency tuning of THz waves produced in small experimental setups is known to be challenging using current methods. A possible way is to use plasma crystals formed spontaneously from micron size particles with interparticle distances of the order of the wavelength of THz waves. We show that bandgaps are formed in the dispersion relation of electromagnetic waves with sub-millimeter wavelength incident on a string of micron size cylindrical dust particles levitated in a low density plasma.

Keywords: sub-millimeter waves, plasma, dust, microparticles

MSC2010: xxx

1. Introduction

Dust particles with sizes of a few microns can be easily levitated in the sheath of a plasma discharge above an electrode [1]. Usually the plasma is produced by a radio-frequency field at 13.56 MHz in a gas at low pressure (e.g. in argon at a few hundred millitorrs) between electrodes which are plane-parallel [2]. The density of the plasma is of the order of 10^{15} m^{-3} while the electron temperature is a few eV. The plasma screening length $\lambda_D = (\epsilon_0 k_B T_e / n_e e^2)^{1/2}$ is $\approx 330 \mu\text{m}$. The levitation height of dust is a few millimeters above the electrode of the discharge, while the distance between the dust particles is set by the electrostatic repulsion and the ion flow. The dust particles are negatively charged accumulating charges on their surface. The equilibrium charge is reached when the sum of ion and electron currents on the dust surface cancels out [3]. The ions flowing within the sheath towards the electrode, through the dust particles contribute to dust screening and to dust self alignment. Inside the sheath the force of gravity acting on the dust particles is compensated by the electric force of the sheath field. Dust particles released in the plasma self-arrange in the sheath in a 3-D structure with several horizontal layers. In the horizontal plane due to the electrostatic repulsion a state of equilibrium is

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reached for a symmetrical arrangement of the dust particles. It appears that the hexagonal symmetry is the most encountered in experiments [1]. In the vertical direction the dust particles stack on top of each other, forming strings of up to 10-15 particles due to the ion flow. Experimentally it is also possible to obtain only one horizontal layer of dust particles by carefully adjusting the rf power and neutral gas pressure [4]. This means that a large fraction of the dust particles of a crystal are forced to fall to the electrode. Moreover, when the electrode has a particular geometry, e.g. it has a cut in the shape of a narrow strip, dust particles will be confined above this strip and make a dust chain [5]. In this way it is possible to obtain a horizontal chain of dust particles all levitating at the same height. The dust particles can be made of several materials such as plastics or resins, alumina, silica, graphite, etc., however low density materials are preferred which results in a lower dust particle mass [6].

Sub-millimeter electromagnetic waves include for example those with wavelengths of a few hundred microns. The corresponding frequency is a few THz. THz waves are most commonly produced in a biased photoconductive (PC) junction when it is illuminated with a femtosecond laser pulse [7]. The photoelectrons oscillate in the fast varying electric field of the laser beam and emit pulses of THz radiation of a few picoseconds. The power of THz waves is in the nW range, but apparently this level is sufficient for use in diverse applications such as time-domain spectroscopy (TDS) [8]. The THz waves can be detected by focusing them with dedicated optics on a similar PC and by measuring the current flowing in the junction.

THz waves have been lately intensely investigated due to numerous applications in medical imaging, high-speed telecommunications, detection of chemical elements, etc. Manipulation and particularly tuning of THz waves is known to be challenging, especially in small experimental setups [9]. The idea of using plasma crystals for filtering THz waves has been put forward recently in [10] and [11]. An experimental approach on the interaction of THz waves with a crystal formed in plasma which takes into account the particularities of the plasma crystal structure and its dielectric properties in relation with the dispersion equation of electromagnetic waves in this frequency range is however missing. Practical demonstrations of THz waves filtering were carried out using metamaterials such as graphene [12].

2. Dispersion of dust string immersed in plasma

We numerically study the propagation of electromagnetic waves with wavevector \vec{K} along the axis of the dust string, as shown in Fig. 1. The dust particles are rods with a given diameter and a length much larger than their diameter, all parallel to each other and aligned in the same plane [13]. The axis of the dust string is perpendicular on the rods but it is contained in their plane. We consider here only the

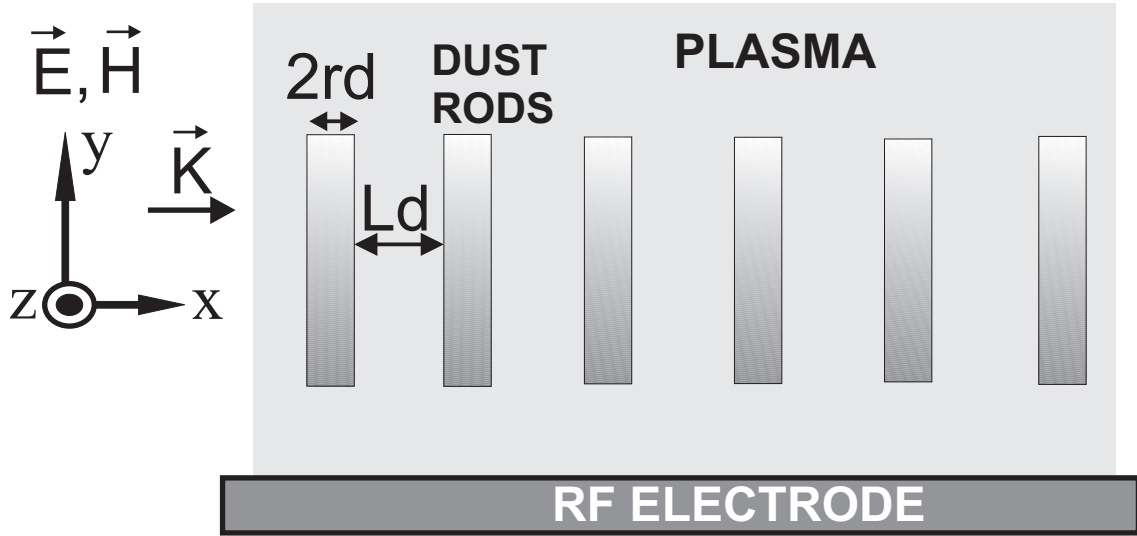


FIGURE 1. \vec{K} and \vec{E} incident on a linear string made of dust rods.

transverse electric (TE) mode of propagation, i.e. the electric field is perpendicular on the rods and on the plane which contains them.

The dust string is characterized by the dust diameter d and inter-particle distance L_d . We can define a periodic cell $\Delta = d + L_d$. The dust particles are immersed in plasma and therefore the interparticle spaces are filled with plasma. The dust particles have a permittivity ε_d while the plasma has a permittivity ε_p which depends of the frequency of EM waves:

$$\varepsilon = \begin{cases} \varepsilon_d & \text{for particles} \\ 1 - \omega_p^2/\omega^2 & \text{for plasma} \end{cases} \quad (1)$$

Here $\omega_p = (n_e e^2 / \varepsilon_0 m_e)^{1/2}$ is the plasma frequency, while collisions of electrons with neutrals in plasma are neglected. As in the case of a photonic crystal the wave equation is solved imposing the following constraints related to the medium of propagation: $E(x + \Delta) = E(x)$ for the electric field and $\varepsilon(x) = \varepsilon[x \pm \Delta]$ for the permittivity [14].

The dispersion relation of the TE mode is found using the transfer matrix method for propagation in the periodically alternating layers of plasma and dielectric [14]:

$$\cos K\Delta = \cos k_p d \cos k_d L_d - \frac{1}{2} \left(\frac{k_d}{k_p} + \frac{k_p}{k_d} \right) \sin k_p d \sin k_d L_d. \quad (2)$$

Here, the index of refraction for the dust and plasma regions are

$n_d = \sqrt{\varepsilon_d}$ and $n_p = \sqrt{\varepsilon_p}$, while k_p and k_d are the wave vectors inside the two regions and are given by:

$$k_d = \left(\frac{\omega^2}{c^2} \varepsilon_d - \beta^2 \right)^{\frac{1}{2}}, \quad (3)$$

$$k_p = \left(\frac{\omega^2}{c^2} \varepsilon_p - \beta^2 \right)^{\frac{1}{2}}, \quad (4)$$

where we introduce the electric field component $E(x, z) = E(x) \exp i\beta z$, where β is the wavevector along z .

Equations (2), (3) and (4) are numerically solved for dust particles with $d = 10\mu\text{m}$, $L_d = 500\mu\text{m}$, $n_e = 10^{15}\text{m}^{-3}$, $T_e = 2\text{eV}$ and $\varepsilon_d = 4.2$, which is the permittivity of polyurethane at 1-1.5 THz [15]. The plasma frequency $\omega_p = 1.78 \times 10^9 \text{ rad/s}$ is much lower than the frequency of the electromagnetic waves ω . The results are shown in Fig. 2. The real part of \vec{K} correspond to the transmitted wave while the imaginary part of \vec{K} shows the absorption domains. We can see the existence of several absorption bands at $f = 0.29, 0.57, 0.86, 1.14 \text{ THz, etc.}$, where $f = \omega/2\pi$. The width of the bandgaps appears to increase towards 3 THz and reaches a constant value of about 0.11 THz. The dispersion curve is highly dependent on the material, size, and spacing of rods as well as on the plasma parameters.

3. Conclusions

We show that bandgaps are formed in the THz domain for incident electromagnetic waves on a string of micron size cylindrical dust particles levitated in plasma. The dispersion equation is solved considering the permittivities of dust particle material and of the optically thin plasma surrounding the particles. This study implying dispersion in a 1-D dust string is a first step before generalization to 2-D and eventually 3-D plasma crystals which will show much more complex bandgap structures. The dispersion curve can be in principle used for the design of frequency filters in the THz domain.

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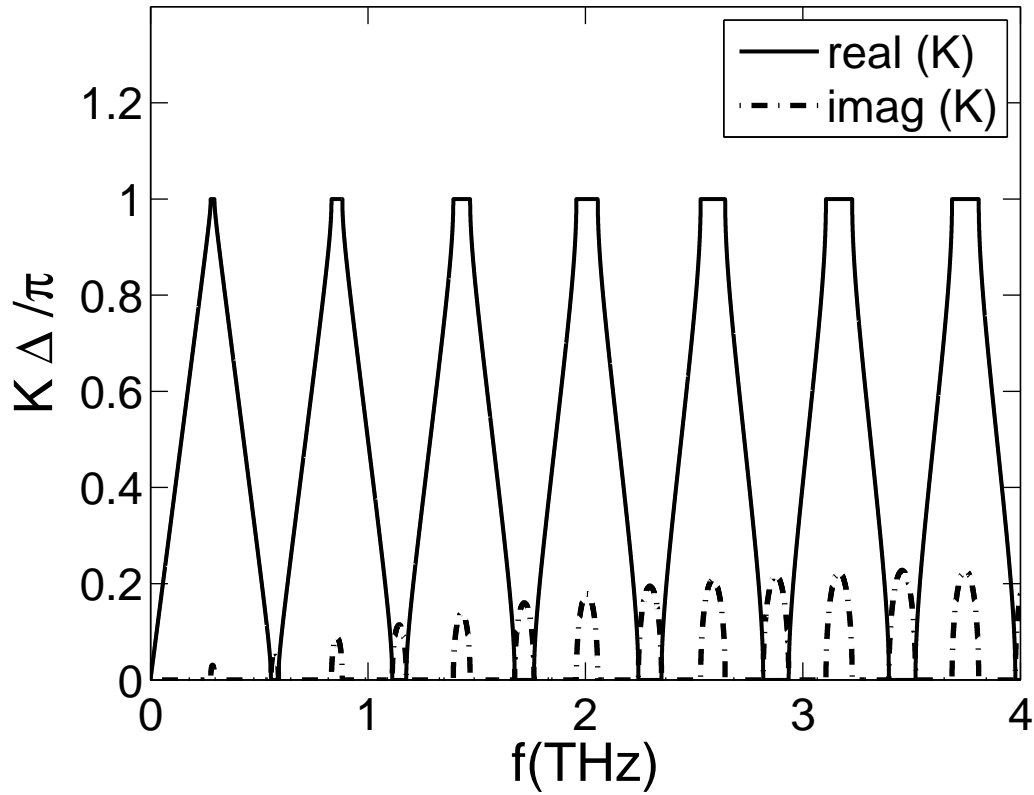


FIGURE 2. Dispersion curve for TE waves at THz frequencies for a string of polyurethane dust rods.

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