

# MODEL-BASED EVENT-TRIGGER PREDICTIVE CONTROLLER WITH SWITCHED SYSTEM APPROACH FOR UNCERTAIN CYBER-PHYSICAL SYSTEM UNDER TIME-VARYING DELAY

Xiaoyi ZHENG<sup>1\*</sup>, Zhen ZHU<sup>2</sup>, Xiaomin FANG<sup>3</sup>

*A model-based event-trigger predictive controller with switched system approach is presented in this paper for uncertain cyber-physical system (CPS) under time-varying delay. Firstly, we propose a common form of CPS with system uncertainties, and some assumptions are stated in order to facilitate further research. Secondly, an event-trigger strategy is designed, meanwhile a time variable and an error variable are introduced, based on these, the CPS model is transformed into a switched time delay system. Thirdly, we discuss the stability analysis of the CPS and the design approach of the event-trigger parameter, and two theorems are presented to give the stability conditions about the switched system. Finally, we conduct a simulation experiment with a ball and beam system to prove the availability of our proposed scheme.*

**Keywords:** CPS, event-trigger, predictive control, time-varying delay

## 1. Introduction

As the information technology develop, its interaction with physical world is becoming more and more prominent, an obvious example is the evolution of the Internet to the Internet of Things (IoT) [1-3]. In fact, any place where Information technology merges with the physical world, it can be called CPS, and the core technology of IoT applications such as intelligent manufacturing and smart city is CPS [4].

CPS requires tight integration and cooperation of computing, communication, and control technologies [5]. Thus from the perspective of control theory, CPS can be generally considered as complex networked control system (NCS), with which communication plays an important role in datum transfer [6]. The same to traditional NCS, CPS also exists time delay and data lose, these factors lead the CPS difficult to be controlled. Up to now, the related research about network-induced time-varying delay is mainly focus on NCS case

<sup>1</sup> Lecturer, Department of Information Engineering, Quzhou College of Technology, China, e-mail: zxy\_xmu@163.com

<sup>2</sup> Engineer, Department of Information Engineering, Quzhou College of Technology, China

<sup>3</sup> Associate Prof., Department of Information Engineering, Quzhou College of Technology, China

[7-9], while the CPS case is still in its infancy [10]. However, traditional time delay handling techniques seem insufficient for CPS case, as they do not take the new features appear in the CPS into full account, such as the CPS structure is more complex than the NCS, and more datum need to be transmitted to keep the CPS work well [11].

For the above reasons, the research of CPS stability control with time-varying delay is necessary, and some related works have been done in last few years. For instance, in literature [12], in order to deal with time delays and impulsive control, a hybrid controller is used to stabilize the networked CPS. In literature [13], a state synchronization problem for TCPS is considered, and the TCPS is suffered from time-varying network delay. In literature [14], a consensus algorithm for processing sensor data of multimissile systems is investigated, and the performance of the proposed algorithm is evaluated. There are also some other results and developments in this domain, see in literatures [10,15-18] and the references therein. We can see that there are few results consider system uncertainties and bandwidth limited constraint, which are the common issues exist in CPS.

Our work is focus on the design of model-based event-trigger predictive controller with switched system approach for CPS under time-varying network delay and system uncertainties. The main contributions are listed below: (i) We put forward a new model-based predictive control method, which is combined with event-trigger idea. By employing this method, the advantages of predictive control is obtained, and network bandwidth is saved. (ii) System uncertainties are considered, and switched system approach is introduced, then the CPS is exponentially stabilized.

The rest of this paper is organized as follows. In Part 2, a common form of uncertain discrete-time CPS model is presented, and some reasonable assumptions are stated. In Part 3, the event-trigger strategy is designed, then a switched system is deduced, based on which two theorems are presented to give the stability condition about the switched system. A simulation example is studied in Part 4, and in Part 5 is the conclusion.

## 2. Problem formulation

We consider the common form of uncertain discrete-time CPS model as below:

$$x(t+1) = (A + \Delta A)x(t) + (B + \Delta B)u(t) \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  denotes the state vector of the CPS plant,  $u(t) \in \mathbb{R}^m$  denotes the control input vector,  $A \in \mathbb{R}^{n \times n}$  and  $B \in \mathbb{R}^{n \times m}$  are two known constant matrices,  $\Delta A \in \mathbb{R}^{n \times n}$  and  $\Delta B \in \mathbb{R}^{n \times m}$  are two unknown matrices denote plant uncertainties.

**Remark 1:**

The CPS model (1) describes a general class of CPS with uncertainties  $\Delta A$  and  $\Delta B$  that satisfying the following requirements:

$$\Delta A = HFD_1 \quad (2)$$

$$\Delta B = HFD_2 \quad (3)$$

where  $H, D_1$  and  $D_2$  are all known constant matrices,  $F$  is an unknown matrix satisfying  $F^T F \leq I$ .

**Remark 2:**

In this paper, we will study the CPS with time-varying delay as shown in Figure 1. The system states sensed by the sensors be installed on the CPS plant and the control datum calculated by the controller are all transmitted by the network. In practical term, there exists time-varying network delay, our goal is to compensate the delay and hold the CPS plant stable meanwhile reduce the communication load with model-based event-trigger predictive control technology.

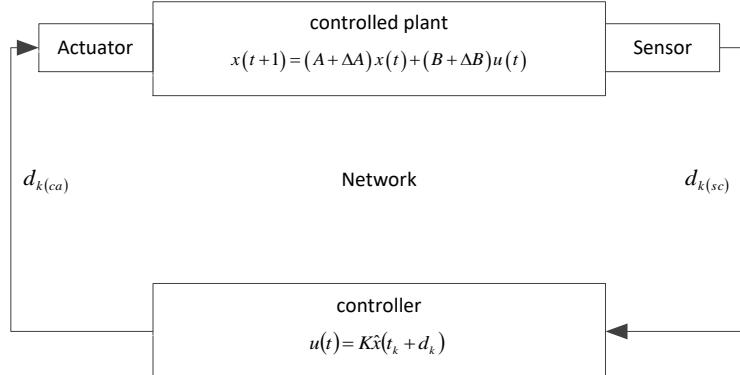


Fig. 1. Architecture of CPS from perspective of control theory

The following assumptions are put forward:

**Assumption 1:** Each control packet transmitted in the network is time-stamped, so the actuator always uses the latest packet while discards the old one.

**Assumption 2:** The round-trip time delay  $d_k$  ( $k = 0, 1, 2, 3, \dots$ ) is bounded, the upper bound is  $d_M$ , the lower bound is  $d_m$ , and when  $k = 0$ ,  $d_0 = 0$ .

**Remark 3:**

The network delay from the sensor to the controller is denoted as  $d_{k(sc)}$ , and the network delay from the controller to the actuator is denoted as  $d_{k(ca)}$ , respectively. Then the round-trip time delay is  $d_k = d_{k(sc)} + d_{k(ca)}$ .

**Assumption 3:** The sampling instants set at the sensor side is described as  $S_1 = \{0, 1, 2, \dots\}$ , the event-trigger instants set at the controller side is described as  $S_2 = \{t_k \mid k = 0, 1, 2, \dots\}$ ,  $t_0 = 0$ , and  $S_2 \subseteq S_1$ .

**Remark 4:**

The sensor gets the measurements from the CPS plant periodically, and only when the event-trigger strategy is triggered, the measurements will be transmitted to the controller. Here we need to notice that  $t_0 = 0$ , which means that event-trigger strategy is always triggered at the initial time, and is a huge simplification for system analysis. By this way, the communication load is reduced effectively.

**Assumption 4:** The predictive step length  $N_u$  satisfies  $N_u \geq d_M$ , and the control input holding strategy is adopted.

**Remark 5:**

Base on Assumption 1 and Assumption 4, while the actuator receives a new control packet, firstly, the round-trip time delay is calculated, which is the error value between the time-stamp of the new control packet and current time. Then according to the time delay, an appropriate control value is selected from the new control packet to act on the CPS plant until next packet arrive. So the actuator can always uses the most appropriate control data.

### 3. Main results

#### 3.1 Event-trigger strategy design

The event-trigger strategy is designed as follows:

$$g(t_k + r, t_k) = [x(t_k + r) - x(t_k)]^T \Phi [x(t_k + r) - x(t_k)] - \delta x(t_k + r)^T \Phi x(t_k + r) \quad (4)$$

where  $t_k$  is the current event-trigger instant,  $0 < \delta < 1$  is a given scalar parameter,  $\Phi$  satisfies  $\Phi = \Phi^T > 0$  and need to be calculated. While  $g(t_k + r, t_k) > 0$ , the sensor is triggered and transmits its measurements to the controller, so  $t_k + r$  is the next event-trigger instant denoted as  $t_{k+1}$ . Then the next event-trigger instant can be calculated as below:

$$t_{k+1} = t_k + \min_r \{r \mid g(t_k + r, t_k) > 0\} \quad (5)$$

From (5), we can see that the event-trigger instant depends not only on the error of CPS state between  $t_{k+1}$  and  $t_k$ , but also on  $\Phi$  and  $\delta$ , and we can tune the triggering frequency by adjusting the value of  $\delta$ .

### 3.2 CPS modelling

On the basis of CPS plant described by (1), then consider the event-trigger instant  $t_k$  and the round-trip time delay  $d_k$  together, we divide the time domain  $t$  as  $t \in [t_0 + d_0, t_1 + d_1) \bigcup [t_1 + d_1, t_2 + d_2) \bigcup \dots \bigcup [t_k + d_k, t_{k+1} + d_{k+1}) \bigcup \dots$ .

Then for  $t \in [t_k + d_k, t_{k+1} + d_{k+1})$ , the control input chosen from the control packet is  $u(t) = K\hat{x}(t_k + d_k)$ , where  $\hat{x}(t_k + d_k)$  is the estimated state value at time  $t_k + d_k$  calculated as follows:

$$\text{Step 1: } \hat{x}(t_k + 1) = (A + BK)x(t_k)$$

$$\text{Step 2: } \hat{x}(t_k + 2) = (A + BK)^2 x(t_k)$$

$$\text{Step } j: \hat{x}(t_k + i) = (A + BK)^j x(t_k)$$

$$\text{Step } d_k: \hat{x}(t_k + d_k) = (A + BK)^{d_k} x(t_k)$$

According to the above, for  $t \in [t_k + d_k, t_{k+1} + d_{k+1})$ , we have:

$$x(t+1) = (A + \Delta A)x(t) + (B + \Delta B)K(A + BK)^{d_k} x(t_k) \quad (6)$$

where  $d_k \in P = \{d_m, d_m + 1, d_m + 2, \dots, d_M\}$ .

Let  $i_k = t_k + l$ , where  $l = 0, 1, 2, \dots, t_{k+1} - t_k - 1$ , so  $i_k \in [t_k, t_{k+1} - 1]$ . Then a time variable [19] is defined as:

$$\eta(t) = t - i_k \quad (7)$$

and it is obvious that  $0 < \eta_m \leq \eta(t) \leq h + d_M = \eta_M$ , where  $\eta_m$  is the lower bound of  $\eta(t)$ , and  $\eta_M$  is the upper bound.

Here, an error variable is introduced:

$$e(i_k) = x(i_k) - x(t_k) \quad (8)$$

Then, for  $t \in [t_k + d_k, t_{k+1} + d_{k+1})$ , (6) can be rewritten as:

$$x(t+1) = (A + \Delta A)x(t) + (B + \Delta B)K(A + BK)^{d_k} (x(t - \eta(t)) - e(i_k)) \quad (9)$$

and according to the event-trigger strategy (4), for  $t \in [t_k + d_k, t_{k+1} + d_{k+1})$ , we yield:

$$e^T(i_k) \Phi e(i_k) < \delta x^T(i_k) \Phi x(i_k) = \delta x^T(t - \eta(t)) \Phi x(t - \eta(t)) \quad (10)$$

It can be seen that (9) is a switched time delay system. Let  $\sigma_t$ :

$[0, +\infty) \rightarrow P$  be the switching signal, therefore, we can rewrite (9) as a switched system, and for  $t \in [t_k + d_k, t_{k+1} + d_{k+1})$ , the subsystem is given as:

$$x(t+1) = (A + \Delta A)x(t) + (B + \Delta B)K(A + BK)^{\sigma_t} (x(t - \eta(t)) - e(i_k)) \quad (11)$$

### 3.3 Stability analysis

Then, we will discuss the stability analysis of the CPS and the design approach of the event-trigger parameter by using the switching Lyapunov function method.

For simplicity, the following notations are used, where  $i \in P$ :

$$\begin{aligned} M_i &= \begin{bmatrix} A & BK(A+BK)^i & 0 & -BK(A+BK)^i \end{bmatrix} \\ \tilde{M}_i &= M_i - [I \ 0 \ 0 \ 0] \\ \Psi_i &= \begin{bmatrix} D_1 & D_2K(A+BK)^i & 0 & -D_2K(A+BK)^i \end{bmatrix} \\ H\Phi\Psi_i &= \begin{bmatrix} \Delta A & \Delta B(A+BK)^i & 0 & -\Delta B(A+BK)^i \end{bmatrix} \\ \xi(t) &= \begin{bmatrix} x^T(t) & x^T(t-\eta(t)) & x^T(t-\eta_M) & e^T(i_k) \end{bmatrix}^T \\ \zeta^T(t, j) &= \begin{bmatrix} \xi^T(t) & h^T(j) \end{bmatrix} \\ h(t) &= x(t+1) - x(t) = (\tilde{M}_i + \Delta M_i) \xi(t) \end{aligned}$$

#### Theorem 1:

Provided that Assumption 1-4 hold, for given matrices  $A, B, H, D_1, D_2, K$ , and given constants  $\lambda > 1, \alpha > 0, 0 < \delta < 1$ , the Lyapunov functional candidate (14) listed below satisfies  $V_i(t+1) - \lambda^{-\alpha} V_i(t) < 0$  if there exist matrices  $P_i = P_i^T > 0, Q_i = Q_i^T > 0, R_i = R_i^T > 0, Z_i = Z_i^T > 0$ ,  $\Phi = \Phi^T > 0$ ,  $X, Y$  and scalars  $\varepsilon_1 > 0, \varepsilon_2 > 0$  such that the matrix inequations (12) and (13) listed below are hold for all  $i \in P$ .

$$\begin{aligned} & \left[ \begin{array}{cccccc} \Pi_i + \Gamma_i + \Gamma_i^T + \eta_M Z_i & (P_i M_i)^T & 0 & \sqrt{\eta_M} (R_i \tilde{M}_i)^T & 0 & \Psi_i^T & \sqrt{\eta_M} \Psi_i^T \\ P_i M_i & -P_i & \varepsilon_1 H & 0 & 0 & 0 & 0 \\ 0 & \varepsilon_1 H^T & -\varepsilon_1 I & 0 & 0 & 0 & 0 \\ \sqrt{\eta_M} R_i \tilde{M}_i & 0 & 0 & -R_i & \varepsilon_2 H & 0 & 0 \\ 0 & 0 & 0 & \varepsilon_2 H^T & -\varepsilon_2 I & 0 & 0 \\ \Psi_i & 0 & 0 & 0 & 0 & -\varepsilon_1 I & 0 \\ \sqrt{\eta_M} \Psi_i & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I \end{array} \right] < 0 \quad (12) \\ & \left[ \begin{array}{cc} Z_i & X \\ X^T & \lambda^{-\alpha \eta_M} R_i \end{array} \right] \geq 0, \quad \left[ \begin{array}{cc} Z_i & Y \\ Y^T & \lambda^{-\alpha \eta_M} R_i \end{array} \right] \geq 0 \quad (13) \end{aligned}$$

#### Proof:

For  $i \in P$ , we define the Lyapunov functional candidate as [19]:

$$V_i(t) = V_{1i}(t) + V_{2i}(t) + V_{3i}(t) \quad (14)$$

where

$$\begin{aligned}
 V_{1i}(t) &= x^T(t)P_i x(t) \\
 V_{2i}(t) &= \sum_{j=t-\eta_M}^{t-1} x^T(j)\lambda^{\alpha(j-t+1)}Q_i x(j) \\
 V_{3i}(t) &= \sum_{j=-\eta_M}^{-1} \sum_{m=t+j}^{t-1} h^T(m)\lambda^{\alpha(m-t+1)}R_i h(m)
 \end{aligned}$$

and  $P_i = P_i^T > 0$ ,  $Q_i = Q_i^T > 0$ ,  $R_i = R_i^T > 0$ .

$$\begin{aligned}
 \Delta V_{1i}(t) &= V_{1i}(t+1) - \lambda^{-\alpha} V_{1i}(t) \\
 &\leq \xi^T(t) \left[ M_i^T \left( P_i^{-1} - \varepsilon_1 H H^T \right)^{-1} M_i + \varepsilon_1^{-1} \Psi_i^T \Psi_i \right] \xi(t) - \lambda^{-\alpha} x^T(t) P_i x(t) \\
 \Delta V_{2i}(t) &= V_{2i}(t+1) - \lambda^{-\alpha} V_{2i}(t) \\
 &= x^T(t) Q_i x(t) - \lambda^{-\alpha \eta_M} x^T(t - \eta_M) Q_i x(t - \eta_M) \\
 \Delta V_{3i}(t) &= V_{3i}(t+1) - \lambda^{-\alpha} V_{3i}(t) \\
 &\leq \eta_M h^T(t) R_i h(t) - \sum_{j=t-\eta_M}^{t-1} h^T(j) \lambda^{-\alpha \eta_M} R_i h(j) \\
 &= \eta_M \xi^T(t) \left[ \tilde{M}_i^T \left( R_i^{-1} - \varepsilon_2 H H^T \right)^{-1} \tilde{M}_i + \varepsilon_2^{-1} \Psi_i^T \Psi_i \right] \xi(t) - \sum_{j=t-\eta_M}^{t-1} h^T(j) \lambda^{-\alpha \eta_M} R_i h(j)
 \end{aligned}$$

According to the above, we have:

$$\begin{aligned}
 \Delta V_i(t) &\leq \xi^T(t) \left\{ \left[ \left( \Pi_i + \Gamma_i + \Gamma_i^T + \eta_M Z_i \right) + \eta_M \left[ \tilde{M}_i^T \left( R_i^{-1} - \varepsilon_2 H H^T \right)^{-1} \tilde{M}_i + \varepsilon_2^{-1} \Psi_i^T \Psi_i \right] \right] \right. \\
 &\quad \left. + \left[ M_i^T \left( P_i^{-1} - \varepsilon_1 H H^T \right)^{-1} M_i + \varepsilon_1^{-1} \Psi_i^T \Psi_i \right] \right\} \xi(t) \\
 &\quad - \sum_{j=t-\eta_M}^{t-\eta(t)-1} \zeta^T(t, j) \begin{bmatrix} Z_i & Y \\ Y^T & \lambda^{-\alpha \eta_M} R_i \end{bmatrix} \zeta(t, j) - \sum_{j=t-\eta(t)}^{t-1} \zeta^T(t, j) \begin{bmatrix} Z_i & X \\ X^T & \lambda^{-\alpha \eta_M} R_i \end{bmatrix} \zeta(t, j)
 \end{aligned}$$

where  $Z_i > 0$ ,  $\Gamma_i = [X \quad Y-X \quad -Y \quad 0]$ ,

$$\Pi_i = \begin{bmatrix} Q_i - \lambda^{-\alpha} P_i & 0 & 0 & 0 \\ 0 & \delta \Phi & 0 & 0 \\ 0 & 0 & -\lambda^{-\alpha \eta_M} Q_i & 0 \\ 0 & 0 & 0 & -\Phi \end{bmatrix}.$$

According to (12), finally we can conclude that:

$$\left( \Pi_i + \Gamma_i + \Gamma_i^T + \eta_M Z_i \right) + \eta_M \left[ \tilde{M}_i^T \left( R_i^{-1} - \varepsilon_2 H H^T \right)^{-1} \tilde{M}_i + \varepsilon_2^{-1} \Psi_i^T \Psi_i \right] + \left[ M_i^T \left( P_i^{-1} - \varepsilon_1 H H^T \right)^{-1} M_i + \varepsilon_1^{-1} \Psi_i^T \Psi_i \right] < 0$$

and together with (13), we have  $V_i(t+1) - \lambda^{-\alpha} V_i(t) < 0$ .

**Theorem 2:**

Provided that Assumption 1-4 hold, for given constants  $\lambda > 1, \alpha > 0, 0 < \delta < 1, \mu > 1$ , if there exist matrices  $P_i = P_i^T > 0, Q_i = Q_i^T > 0, R_i = R_i^T > 0, Z_i = Z_i^T > 0, \Phi = \Phi^T > 0$ ,  $X, Y$ , and scalars  $\varepsilon_1 > 0, \varepsilon_2 > 0$  for all  $i \in P$  satisfying (12), (13) and

$$P_i \leq \mu P_j, Q_i \leq \mu Q_j, R_i \leq \mu R_j \quad (\forall i, j \in P) \quad (15)$$

then the CPS plant described by (1) is exponentially stabilized for any switching signal with average dwell time  $T_a > T_a^* = \frac{\ln \mu}{\alpha \ln \lambda}$ , and the decay rate is  $\sqrt{\lambda^\rho}$ ,

$$\text{where } \rho = -\frac{\ln \mu}{\alpha T_a \ln \lambda} + 1.$$

**Proof:**

Similar to the above, we define the Lyapunov functional candidate as [19]:

$$V_{\sigma_t}(t) = V_{1,\sigma_t}(t) + V_{2,\sigma_t}(t) + V_{3,\sigma_t}(t) \quad (16)$$

where

$$V_{1,\sigma_t}(t) = x^T(t) P_{\sigma_t} x(t)$$

$$V_{2,\sigma_t}(t) = \sum_{j=t-\eta_M}^{t-1} x^T(j) \lambda^{\alpha(j-t+1)} Q_{\sigma_t} x(j)$$

$$V_{3,\sigma_t}(t) = \sum_{j=-\eta_M}^{-1} \sum_{m=t+j}^{t-1} h^T(m) \lambda^{\alpha(m-t+1)} R_{\sigma_t} h(m)$$

and  $P_{\sigma_t} = P_{\sigma_t}^T > 0, Q_{\sigma_t} = Q_{\sigma_t}^T > 0, R_{\sigma_t} = R_{\sigma_t}^T > 0$ , which can be calculated according to matrix inequations (12) and (13). From (15), we have:

$$V_{\sigma_{t_k}}(t_k) \leq \mu V_{\sigma_{t_{k-1}}}(t_k)$$

Noticing that  $\sigma_{t_{k-1}} = \sigma_{t_{k-1}}$ , we can obtain that:

$$V_{\sigma_t}(t) \leq \mu^{N_{\sigma_t}[t_0, t]} \lambda^{-\alpha(t-t_0)} V_{\sigma_{t_0}}(t_0) = (\lambda^\rho)^{-\alpha(t-t_0)} V_{\sigma_{t_0}}(t_0) \quad (17)$$

where  $N_{\sigma_t}[t_0, t] \leq N_0 + \frac{t-t_0}{T_a}$  denotes the switching times of  $\sigma_t$  over  $[t_0, t]$ ,

$N_0 \geq 0, T_a > 0$  denotes the average dwell time of switching signal  $\sigma_t$ ,

$$\rho = -\frac{\ln \mu}{\alpha T_a \ln \lambda} + 1.$$

In addition, there exists constants  $a = \min_{i \in P} \{\lambda_{\min}(P_i), \lambda_{\min}(Q_i), \lambda_{\min}(R_i)\}$  and  $b = \max_{i \in P} \{\lambda_{\max}(P_i), \lambda_{\max}(Q_i), \lambda_{\max}(R_i)\}$ . According to (17), we have:

$$a\|x(t)\|^2 \leq V_{\sigma_i}(t) \leq b(\lambda^\rho)^{-\alpha(t-t_0)}\|x(t_0)\|^2$$

Then, the following inequation is obtained:

$$\|x(t)\| \leq \sqrt{\frac{b}{a}} \sqrt{\lambda^\rho}^{-\alpha(t-t_0)} \|x(t_0)\| \quad (18)$$

$T_a > T_a^* = \frac{\ln \mu}{\alpha \ln \lambda}$  keeps that  $\sqrt{\lambda^\rho} > 1$ , then the inequation (18) implies that (1) is exponentially stabilized and the decay rate is  $\sqrt{\lambda^\rho}$ .

#### 4. Illustrative example

In this part, we consider a ball and beam system presented in [20], to prove the availability of our proposed scheme.

The nonlinear model of the system is described in [20] in detail, and it is omitted here. Thus, we give the linear discrete-time approximation about the ball and beam system with sampling period  $h = 0.02s$  as below directly:

$$\begin{aligned} x(t+1) &= (A + HFD_1)x(t) + (B + HFD_2)u(t) \\ \text{where } A &= \begin{bmatrix} 1.0000 & 0.0200 & -0.0014 & -0.0000 \\ 0.0000 & 1.0000 & -0.1401 & -0.0014 \\ 0.0000 & 0.0000 & 1.0000 & 0.0200 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}, \quad B = \begin{bmatrix} -0.0000 \\ -0.0000 \\ 0.0002 \\ 0.0200 \end{bmatrix}, \\ H &= \begin{bmatrix} -0.1 & 0 & -0.3 & 0 \end{bmatrix}^T, \quad F = 10^{-3} * I, \quad D_1 = \begin{bmatrix} 0 & -0.1 & 0 & 0.15 \\ 0.2 & -1.1 & 0 & 0.3 \end{bmatrix}, \\ D_2 &= [-2.1 \ 0.2]^T. \end{aligned}$$

The controller gain  $K$  is set directly as  $K = [2.8954 \ 4.3013 \ -15.4463 \ -4.3781]$  to guarantee the local certain model  $x(t+1) = Ax(t) + Bu(t)$  is Schur-stable.

Here, we consider the case that  $d_M = 3h$ . Solving the inequations (12), (13) and (15) in Theorem 2 with  $\lambda = 1.0001$ ,  $\alpha = 2$ ,  $\delta = 0.36$  and  $\mu = 1.2$ , then the event-trigger matrix  $\Phi$  is obtained:

$$\Phi = \begin{bmatrix} 880.2 & 510.8 & -967.4 & -464.9 \\ 510.8 & 1810.2 & -757.2 & -1533.0 \\ -967.4 & -757.2 & 1652.6 & 689.0 \\ -464.9 & -1533.0 & 689.0 & 1358.2 \end{bmatrix}$$

On the basis of the above matrix and parameters, simulation results are displayed as follows. Fig. 2 shows the state responses of CPS plant, Fig. 3 shows the event-trigger instants. It can be seen that the CPS state can be exponentially stabilized, and the communication load is effectively reduced, only 22.4% sampled datum are transmitted.

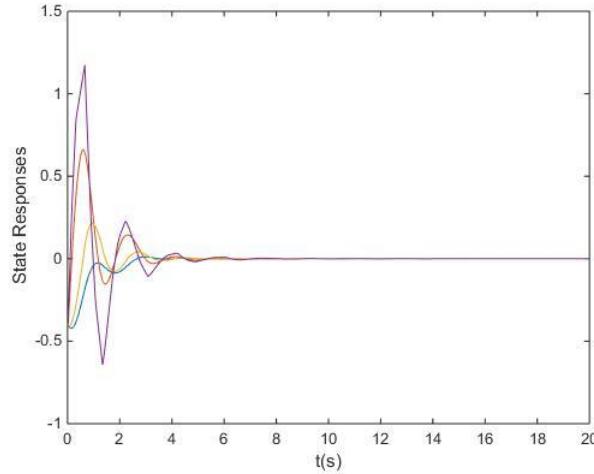


Fig. 2 the state response of CPS plant

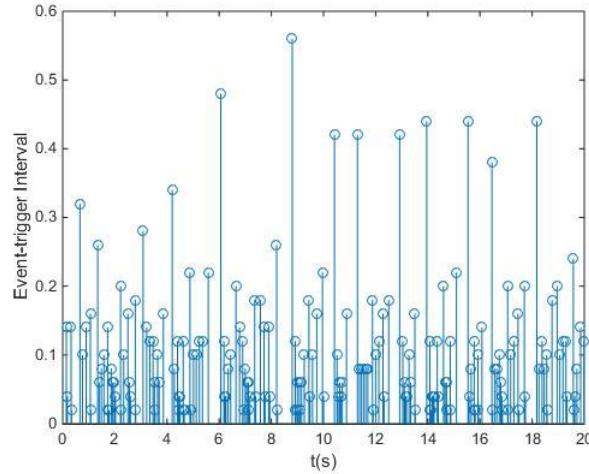


Fig. 3 the event-trigger instants

Next, we investigate the influence of the event-trigger parameter  $\delta$  and the upper bound of round-trip time delay  $d_M$  on the number of event-trigger times. Let  $\delta = 0.06, 0.12, 0.18, 0.24, 0.30, 0.36$  and  $d_M = 2h, 3h, 4h, 5h$ , respectively, other parameters are the same as before. The results are shown in Table 1. We can see that the number of event-trigger times increases as  $d_M$  increases, decreases as  $\delta$  increases, which shows that a decreased  $d_M$  and an

increased  $\delta$  lead to fewer communication load. This is easy to explain, for  $\delta$ , we can make it out directly from the event-trigger strategy (5). And for  $d_M$ , when the upper bound of round-trip time delay increases, the system performance will degrades, then the event-trigger strategy will be triggered more frequently.

Table 1

Event-trigger times for different  $d_M$  and  $\delta$ 

| event-trigger times | $\delta = 0.06$ | $\delta = 0.12$ | $\delta = 0.18$ | $\delta = 0.24$ | $\delta = 0.30$ | $\delta = 0.36$ |
|---------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $d_M = 2h$          | 417             | 337             | 299             | 252             | 222             | 196             |
| $d_M = 3h$          | 456             | 383             | 310             | 287             | 252             | 224             |
| $d_M = 4h$          | 490             | 404             | 349             | 311             | 270             | 243             |
| $d_M = 5h$          | 514             | 422             | 365             | 336             | 300             | 271             |

From the above, it is obvious that the method proposed in this paper can still keep the controlled plant stable while reduce the communication load effectively.

## 5. Conclusions

In this paper, we investigate the design of model-based event-trigger predictive controller with switched system approach for uncertain cyber-physical system under time-vary delay and system uncertainties. A novel method is adopted to achieve the goal of less control data transmitted in communication network while still keep the CPS plant stable, and in order to strength the universality about the method, system uncertainties are considered. The proposed scheme is verified effective by the illustrative example. However, for simplicity, the present work only considered CPS with time-varying network delay case, while data dropout is absent, so our future study will focus on the CPS with both sufferings.

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