

NUMERICAL SIMULATION OF LASER TECHNIQUES FOR ART CONSERVATION – PART I – FIBER LASER ANALYSIS

Dan SAVASTRU¹, Aurelian VLASE², Ion LĂNCRĂNJAN³, Roxana SAVASTRU⁴, Sorin MICLOȘ⁵

Au fost realizate simulări numerice privind proiectarea unui laser de mare putere cu fibră lucrând în CW în modurile relaxat și comutare pasivă, având ca scop o construcție îmbunătățită a sursei laser dedicate conservării operelor de artă. Rezultatele preliminare prezentate vor fi continuat cu altele privind interacția cu ținta (opera de artă) a fasciculului laser și combinând cele două modele numerice. Ideea principală a modelului de simulare realizat constă în corelarea parametrilor de ieșire ai oscilatorului laser cu caracteristicile specifice ale operei de artă pentru a optimiza întreaga procedură a restaurării operei de artă.

Numerical simulations were performed concerning the design of a high power fiber laser operated in CW free-running and passively Q-switching operation, pointing to an improved design of a laser source dedicated to art conservation. The presented preliminary results will be continued with other concerning the interaction with the target (the art piecework) of the laser beam and combining the two numerical models. The main idea of the developed simulation model consists in correlating the laser oscillator output parameters with the artwork specific characteristics in order to optimize the entire procedure of art restoration.

Keywords: fiber laser, passive Q-switch, art conservation

1. Introduction

Laser applications in art conservation represent a relatively new field of research activity related to the more industrial ones like surface cleaning and paint removal. New laser oscillators operated in free-running and in Q-switching modes as radiation sources for art conservation [1]-[3] are of general interest. This paper, as a part of a series, presents preliminary results obtained in numerical simulation

¹ Eng., National Institute R&D of Optoelectronics-INOE 2000, 409 Atomistilor str., Magurele, Ilfov, RO 077125, Romania, e-mail: dsavas@inoe.inoe.ro

² Prof., University POLITEHNICA of Bucharest, 313 Splaiul Independentei, Bucharest, RO 060042, e-mail: avlase@yahoo.com

³ Advanced Study Center–National Institute of Aerospace Research “Elie Carafoli”, Romania, e-mail: j_j_f_l@yahoo.com

⁴ Ph.D. Eng., National Institute R&D of Optoelectronics-INOE 2000, 409 Atomistilor str., Magurele, Ilfov, RO 077125, Romania, e-mail: rsavas@inoe.inoe.ro

⁵ Eng., National Institute R&D of Optoelectronics-INOE 2000, 409 Atomistilor str., Magurele, Ilfov, RO 077125, Romania, e-mail: miclos@inoe.inoe.ro

of Erbium ions (Er^{3+}) doped fiber laser as a possible radiation source for art piecework cleaning and restoration. The series is pointing to the design of laser oscillators as radiation sources used for art conservation, the phenomena of impurities removal from art piecework surface and to the correlation of radiation source and art piecework characteristics in order to obtain an improved overall system design. The presented preliminary numerical simulation results concern (Er^{3+}) doped fiber laser operated in free-running and passively Q-switching operation regimes.

2. Considerations about Er^{3+} ions doped fiber laser operation

A schematic representation of the electronic energy levels of the investigated Erbium ions (Er^{3+}) doped fiber laser oscillator, in a general form as can be considered correct, is presented in Fig. 1. As would appear in the followings, the most common case, the case of a mono mode optic fiber having the core doped with such ions as the laser active centers is investigated [4]. The pumping transition (denoted as W_p in Fig. 1) and the emission transitions, including the laser one, the laser signal (denoted as W_s in Fig. 1) can be observed in Fig. 1.

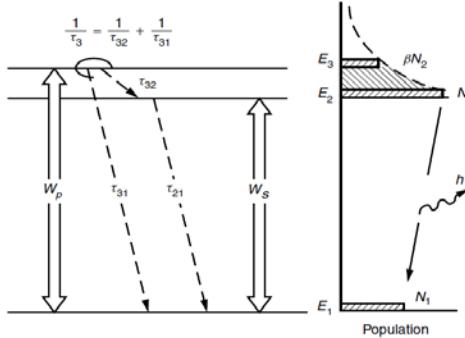


Fig. 1. Schematic representation of the relevant insights of the investigated fiber laser amplifier

As presented in Fig. 1, the Er^{3+} ions have laser active centers of the type “three levels”, having populations in the energy levels implied in the emission of laser beam effect denoted as N_1 , N_2 and N_3 . As functions of time, these populations can be defined as numerical solutions of a coupled differential rate equations as defined in [4]-[16]:

$$\frac{dN_1}{dt} = -W_p(N_1 - N_3) + \frac{N_2}{\tau_{21}} + W_s(N_2 - N_1), \quad (1)$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} - W_s(N_2 - N_1), \quad (2)$$

$$\frac{dN_3}{dt} = W_p (N_1 - N_3) - \frac{N_3}{\tau_3}, \quad (3)$$

As can be observed in Fig. 1, W_p representing the pumping transition is actually the transition probability defined as:

$$W_p(\nu_p) = \sigma_p \frac{I_p}{h\nu_p}, \quad (4)$$

I_p represents the intensity of the pump radiation and ν_p is the frequency of the pump transition.

Similarly, W_s representing the stimulated emission transition is its probability defined as:

$$W_s(\nu_s) = \sigma_s \frac{I_s}{h\nu_s}, \quad (5)$$

I_s represents the intensity of the laser signal emission. ν_s is the frequency of the signal emissive transition.

In equations (4) and (5), σ_p and σ_s are the cross-sections of absorption $E_1 \rightarrow E_3$ and of emission $E_2 \rightarrow E_1$ transitions. The correlation between E_3 (pump) and E_2 (upper laser) levels is defined as a sum of two emissive transition rates:

$$\frac{1}{\tau_3} = \frac{1}{\tau_{32}} + \frac{1}{\tau_{31}}, \quad (6)$$

In equation (6), τ_3 , τ_{32} and τ_{31} denote the total emission transition, the $E_3 \rightarrow E_1$ emissive transition and the $E_3 \rightarrow E_2$ emissive transition from E_3 pump level lifetime values. In Fig.1 τ_{21} represents the laser emission transition $E_2 \rightarrow E_1$ lifetime. The Er^{3+} ions have as an important favorable laser emission characteristic an extremely large value of about 10 ms [4].

For a continuous wave (CW) pumping with a laser diode emitting at a wavelength of 980 nm or 1480 nm, in the stationary case ($d/dt = 0$), the system of coupled rate equations (1), (2) and (3) reduces to an algebraic one which can be easily solved. After some algebraic calculus, several important parameters which characterize the Er^{3+} doped optic fiber can be defined. In their definitions, an important parameter for Er^{3+} ions laser active centers has to be introduced, namely the β factor related to the Boltzmann population ratio of E_3 and E_2 levels and defined as:

$$\beta = \frac{N_3}{N_2} = \exp\left(-\frac{\Delta E}{kT}\right), \quad (7)$$

The β factor is defined in the condition of a low τ_{32} value, of few ps range, which corresponds to a fast $E_3 \rightarrow E_2$ transition of phonon emission. Among the parameters characterizing the Er^{3+} doped optic fiber amplifier and, implicitly, as a CW emitting laser is the relative population inversion ($\Delta N/N$) defined as:

$$\frac{\Delta N}{N} = \frac{N_2 - N_1}{N} = \frac{(1-\beta)W_P\tau_{21} - 1}{(1+\beta)W_P\tau_{21} + 2W_S\tau_{21} + 1}, \quad (8)$$

The maximum value of the relative population inversion is defined as:

$$\frac{\Delta N_{\max}}{N} = \frac{(1-\beta)W_P\tau_{21} - 1}{(1+\beta)W_P\tau_{21} + 1}, \quad (9)$$

Using the maximum value of the relative population inversion in eq. (8), a relation using the saturation value of signal transition probability is defined as:

$$\frac{\Delta N}{N} = \frac{\Delta N_{\max}}{N} \frac{1}{1 + \frac{W_S}{W_S^{Sat}}}, \quad (10)$$

The saturation value of signal transition probability is defined as:

$$W_S^{Sat} = \frac{1}{\tau_{21}} \left(1 + \frac{1+\beta}{1-\beta} \frac{W_P}{W_P^{th}} \right), \quad (11)$$

The threshold value of the pumping transition probability is defined as:

$$W_P^{th} = \frac{1}{(1-\beta)\tau_{21}}, \quad (12)$$

The above defined parameters are used for analyzing the operation of continuous wave emitting Er^{3+} optic fiber lasers.

3. Considerations about passively Q-switched fiber laser operation

In Fig. 2 a schematic of the investigated Er^{3+} doped fiber laser operated in passively Q-switched mode is presented. The starting point of the numerical investigation of model shown in Fig. 2 is a system of differential coupled laser rate equations [12], [17]-[23]. This system of differential coupled laser rate equations is valid for a quasi-three level active medium such as Er^{3+} ions doped optic fiber and a generalized two-level saturable absorber (in this case, $\text{Co}^{2+}:\text{ZnSe}$ crystal). The system of differential coupled laser rate equations is defined as:

$$\frac{d\phi_a}{dt} = S + \frac{\phi_a}{t_r} \left[2\sigma_p n_a l_a - 2\sigma_s n_s l_s - \ln \left(\frac{1}{R_1 R_2} \right) - \alpha \right], \quad (13)$$

$$\frac{dn_a}{dt} = -\gamma c \sigma_p n_a \phi_a + \Delta(N_a - n_a) - \frac{n_a + N_a(\gamma-1)}{\tau_{21}}, \quad (14)$$

$$\frac{dn_s}{dt} = -c K \sigma_s n_s \phi_a - \frac{n_s - N_s}{\tau_{21}}, \quad (15)$$

As presented in Fig. 2, it has to be observed that the three-level system of the Er^{3+} doped optic fiber active medium can be considered as a “reduced two-

level" one, mainly because the upper laser level (with concentration density N_2) is many order of magnitude larger than emitted laser pulse time scale. In the above defined system of differential coupled laser rate equations the following parameters are used:

- ϕ_a is the photon density inside the laser resonator at the position of active medium (erbium doped optic fiber);
- n_a is the instantaneous population inversion density in the active medium;
- N_a is the initial value of population inversion density in the active medium;
- σ_a is the laser stimulated emission cross section of the Er^{3+} ions;
- l_a is the length of AM;
- γ is the factor representing reduction in the population inversion in the active medium (the parameter γ accounts for the effects of level degeneracy and relaxation or thermalization rates in the laser medium, defined by β factor [24]);
- τ_{21} is the lasing lifetime of erbium ions, the same as for continuous laser emission;
- S is the factor accounting for the rate at which spontaneous emission is added to the laser emission;
- n_s is the instantaneous population density of absorbing centers (Co^{2+} ions) in saturable absorber ground state;
- N_s is the total density of the saturable absorption centers in the crystal;
- σ_s is the cross section of saturated resonant absorption of the $\text{Co}^{2+}:\text{ZnSe}$ passive optical Q-switch;
- l_s is the length of the passive optical Q-switch;
- c is the speed of light. The above defined parameters are used for analyzing the operation of continuous wave emitting Er^{3+} optic fiber lasers.

As can be noticed in Fig. 2, two fibers having Bragg gratings induced in the core (FBG), denoted as **1** and **2**, are used as coupling output mirrors of the investigated erbium passively Q-switched fiber laser. The reflection coefficients of these two coupling output mirrors are defined as R_1 and R_2 . The initial value of population inversion density in the active medium is defined as:

$$N_a = \frac{N_0(\gamma - 1) \left(\exp\left(-\frac{h\nu}{kT}\right) - 1 \right)}{1 + (\gamma - 1) \exp\left(-\frac{h\nu}{kT}\right)}, \quad (16)$$

In the above equation N_0 is the concentration of doped Er^{3+} ions in the optic fiber, while ν is the frequency of the laser transition.

In the system of coupled laser rate equations (13), (14) and (15), the density of saturable absorption centers (Co^{2+} ion doped in the ZnSe crystal -

$\text{Co}^{2+}:\text{ZnSe}$) is defined as depending on the initial small signal, unbleached, transmittance of the passive optical Q-switch, T_{Lin} , by the relation:

$$N_s = -\frac{\ln(T_{Lin})}{\sigma_s l_s}, \quad (17)$$

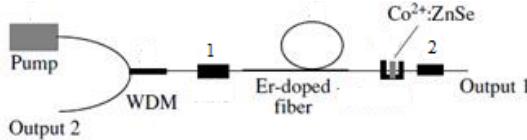


Fig. 2. The schematic of the analyzed passively optical Q-switched Er^{3+} doped optic fiber laser

As can be noticed in Fig. 2, the pump radiation is injected into the erbium doped fiber active medium through a wavelength demultiplexer (WDM) [25] being supplied by a laser diode. The considered pumping radiation wavelength is 980 nm. The value of the pump power is defined as:

$$P_{PUMP} = \frac{\Delta N_0 V h \nu}{\eta}, \quad (18)$$

In equation (18) V is the erbium doped optic pumped volume, η represents the factor accounting the Stokes losses and non-whole volume pumping of the erbium doped fiber.

The optical length of the investigated passively Q-switched Er^{3+} doped fiber laser is defined as:

$$l = l_0 + l_a(\eta_a - 1) + l_s(\eta_s - 1), \quad (19)$$

where η_a and η_s are the refractive indices of the erbium doped optic fiber and $\text{Co}^{2+}:\text{ZnSe}$ crystal, l_0 is the air gaps in the cavity.

4. Numerical simulation results for the free-running fiber laser

As mentioned in Section 2, there are several issues of interest for characterization of Er^{3+} fiber laser, issues which are investigated by numerically solving the system of three coupled rate equations (1), (2) and (3). In the following, graphs of these parameters variations versus the pumping rate are presented.

The relative population inversion of Er^{3+} ions variations as functions of normalized pumping rate are presented in Fig. 3. The situations of using different pumping radiation wavelengths were investigated by considering different β factor values. The upper curves represent the situation of using a pump radiation with a wavelength of 980 nm. The lower curves represent the case of using 1480 nm pumping radiation. $W_{s\tau}$ represents the value of laser signal intensity propagating through the analyzed system.

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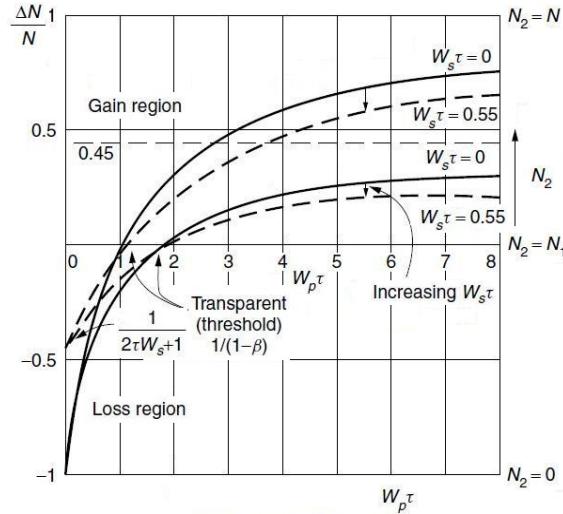


Fig. 3. The relative population inversion variations versus normalized pumping rate

The relative population inversion of Er^{3+} ions variations as functions of normalized pumping rate are presented in Fig. 4. The situations of using different pumping radiation wave-lengths were investigated by considering different β factor values.

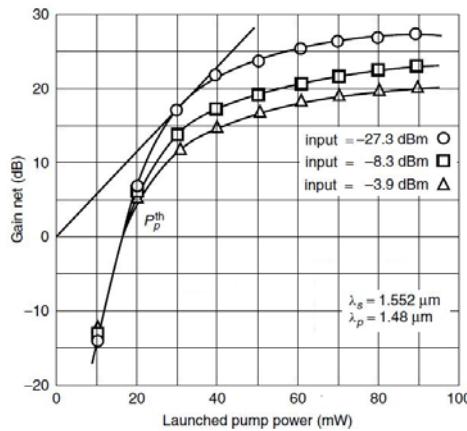


Fig. 4. Gain variations versus pump power launched in the fiber optic laser

The upper curves represent the situation of using a pump radiation with a wavelength of 980 nm. The lower curves represent the case of using 1480 nm pumping radiation. W_{st} represents the value of laser signal intensity propagating through the analyzed system.

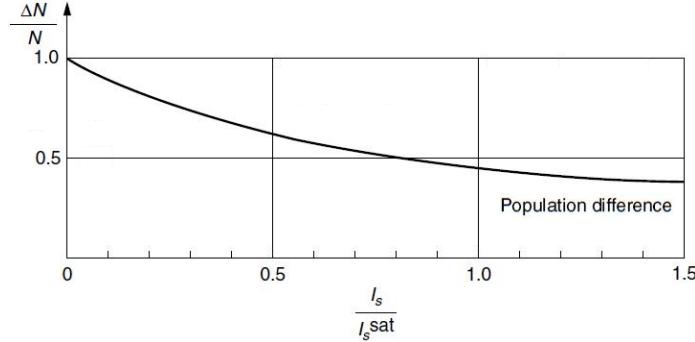


Fig. 5. Relative population difference versus normalized signal power intensity

In Fig. 5 the variation of relative population inversion in the erbium doped optic fiber amplifier as depending, as a function of the normalized laser signal power propagating through the optic fiber is presented. The laser signal intensity is normalized at its saturation value which is proportional to W_s^{Sat} .

5. Results obtained in the case of passively Q-switched fiber laser

The system of differential coupled laser rate equations (13), (14) and (15) is numerically solved using a developed program based on an adaptive integration step Runge-Kutta-Fehlberg (RKF45) algorithm [16],[26]. The numerical solving of this system of differential equations is based on the use of constant parameters of the investigated passively Q-switched laser. The accuracy definitions of these constant parameters proved, as expected, to be of importance [16],[23],[26].

More precisely, there were used reflection coefficients of the two coupling output mirrors are defined as $R_1 = 90\%$ and $R_2 = 95\%$. For defining the Er^{3+} active medium parameters there were used the values: $\sigma_P = 0.7 \cdot 10^{-20} \text{ cm}^2$, $\gamma = 1.8$, $\tau_{2I} = 1 \cdot 10^{-2} \text{ s}$ [27], $\eta_a = 1.5$ [27]. The optical fiber core diameter was $\mathcal{O}_a = 6.5 \mu\text{m}$, this value being considered as the laser generation spot size. For defining the passive optical Q-switch cell parameters, the $\text{Co}^{2+}:\text{ZnSe}$ crystal, we used the values: $\sigma_s = 5.3 \cdot 10^{-19} \text{ cm}^2$; $\tau_s = 0.29 \cdot 10^{-3} \text{ s}$; $\eta_s = 2.45$ [26]; $l_s = 0.05 \text{ cm}$.

The generation spot size was considered to be $\mathcal{O}_s = 12-15 \mu\text{m}$. For defining the laser cavity parameters there were used the values: ratio of the laser beam spot-sizes in the active medium (erbium fiber) and saturable absorber, the $\text{Co}^{2+}:\text{ZnSe}$ crystal, $K \approx 0.6$; overall optical length $l = 3000.06 \text{ cm}$; $\alpha = 0.7$ (this parameter is evaluated to be a sum of passive losses in the erbium fiber and the

losses owing to coupling of the laser radiation passing the $\text{Co}^{2+}:\text{ZnSe}$ crystal). The value of pumped optical fiber volume is considered to be $V = 7.5 \cdot 10^{-4} \text{ cm}^3$.

The pump power was chosen in the range 0-125 mW. The initial, unbleached value of the passive optical Q-switch is considered in the range 0.90-0.95.

In Fig. 6 and 7 the results of numerical simulation of photon number and passive optical Q-switch cell transmittance performed considering a pumping power of 100 mW and an initial value of 0.94 for passive optical Q-switch transmittance are presented.

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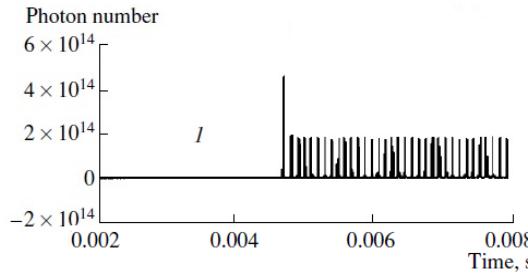


Fig. 6. Photon number time variation

As expected, in Fig. 6 the first laser burst corresponds to a drain of the pumping energy stored in the active medium. It can be noticed that the rest of the train of laser pulses is of lower amplitude and regular as shape and pulse full width at half maximum.

In Fig. 7 the observed variation of passive Q-switch transmittance is in biunivocal correspondence to the laser pulses, including the delay between the variations of each parameter imposed by the second threshold passive Q-switching condition.

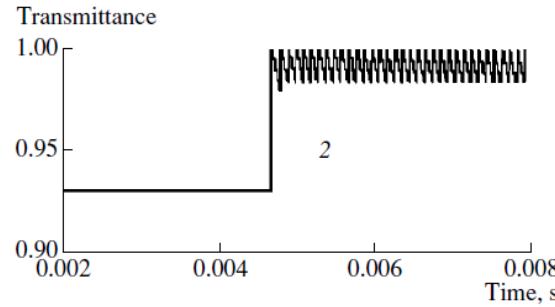


Fig. 7. The passive optical Q-switch transmittance time variation

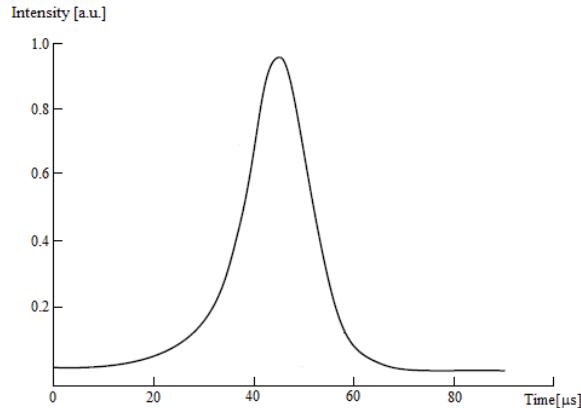


Fig. 8. The single laser pulse time shape

In Fig. 8 the time simulated photon variation corresponding to a single pulse emission is presented. The single pulse variation is in agreement to the theoretical prediction of CW pumped laser operated with high reflectivity mirrors.

6. Conclusions

Numerical simulations were performed concerning the design of a high power fiber laser operated in CW free-running and passively Q-switching operation, pointing to an improved design of a laser source dedicated to art conservation. The presented preliminary results will be continued with other concerning the interaction with the target (the art piecework) of the laser beam and combining the two numerical models. The main idea of the developed simulation model consists in correlating the laser oscillator output parameters with the artwork specific characteristics in order to optimize the entire procedure of art restoration.

The presented preliminary numerical simulation results are perfectible and will be further developed for improved designing purposes by comparison with experimental results.

R E F E R E N C E S

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