

CONICAL VIBRATING SIEVE

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În lucrarea de față se studiază un tip constructiv original de sită vibratoare conică. Este prezentat un model matematic al mișcării particulelor pe sită, cu ajutorul căruia se obțin ecuațiile diferențiale ale mișcării relative, exprimate în coordonate cilindrice. Aceste ecuații sunt neliniare, iar integrarea lor se face numeric, folosind pentru parametrii de integrare mai întâi valori calculate teoretic, iar apoi date reale obținute în urma unor măsurători. Se face un studiu comparativ între cele două tipuri de rezultate, luând în considerare și valorile limită tehnologice ale mărimilor fizice specifice procesului de separare.

The paper studies an original constructive type of conical vibrating sieve. A mathematical model of the motion of the particles on the sieve is presented, which is used to determine the differential equations of the relative motion, expressed in cylindrical coordinates. These equations are non-linear and they are integrated numerically, using for the integration parameters theoretically calculated values, as well as real data achieved by measurements. A comparative study between the two types of results is performed, taking also into account the technological limit values of the physical quantities specific to the separation process.

Keywords: separation, vibrating sieve, oscillating rotation, limit velocity

List of symbols

A_0 - amplitude of the angular velocity of the sieve (rad/s);
 A_v - amplitude of the velocity of the measuring point (m/s);
 \vec{a}_C - Coriolis acceleration;
 \vec{a}_{tr} - transport acceleration;
 b, l - width and length of the seed, respectively;
 D - diameter of the sieve orifices;
 f - oscillation frequency of the sieve;
 \vec{F}_C - Coriolis force;
 \vec{F}_f - friction force;

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\vec{F}_r - transport force;
 \vec{G} - weight of the seed;
 g - gravitational acceleration;
 m - mass of the seed;
 \vec{N} - normal reaction;
 p - circular frequency of the sieve;
 R - distance between the measuring point and the centre of the sieve;
 r, θ, z - cylindrical coordinates;
 t - time;
 \vec{v}_r - relative velocity of the particle with respect to the sieve;
 v_{lim} - theoretical limit sieving velocity;
 v_{max} - real limit sieving velocity;
 2α - apex angle of the conical sieve;
 β - slope angle of the sieve;
 μ - friction coefficient between the seed and the sieve;
 $\vec{\rho}, \vec{n}, \vec{k}$ - unit vectors of the cylindrical coordinate system;
 Ω - angular velocity of the sieve.

1. Introduction

An important operation in cereal product processing is the separation of the seeds from certain types of impurities (weed, chaff, fractured particles, clay boluses etc.). In order to perform this operation, various types of sieves can be used, such as: shaking sieves, plane sieves with circular motion, aspirating separators with expansion chamber, vibrating sieves, sifting cylinders, vibrating conical centrifugal separators, pneumatic separators [1], [2], [3].

A centrifugal conical separator is presented in reference [4]. The theoretical study of the motion of a particle on this sieve can be found in [5].

In reference [6], the authors establish, by experimental tests, the efficiency of the separation, function of the kinematical index, of the angle of the sieve and of the angle of vibration.

A study of the motion of the particle on a plane sieve, function of the same parameters, is presented in paper [7].

Some interesting experimental results can be also found in reference [8], where the gravitational separation process is considered under the action of air flow, which eliminates the internal friction and separates the particles in the layer.

In the present paper, an original constructive type of sieve is described, called in the following *conical vibrating sieve*. This device is similar to the centrifugal conical separator, but it has opposite motion. A mathematical model of the motion of particles is presented, as well as some numerical results obtained by

theoretical methods and by experimental measurements.

2. Presentation of the conical vibrating sieve

The conical vibrating sieve is a sifting device, used to separate the cereal mixtures for industrialization. The particle separation is performed according to the size.

The constructive model of the sieve is shown in Figure 1.

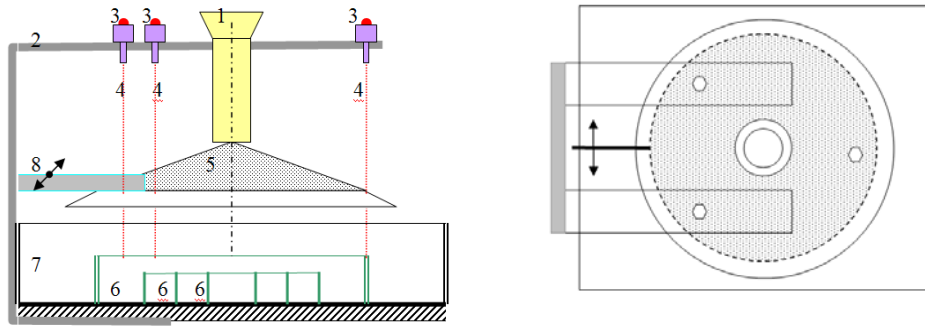


Fig.1 Conical vibrating sieve:

1. filling funnel; 2. fixed frame; 3. screws for cable tensioning; 4. attachment system with three cables at the upper part and three elastic chords at the lower part; the cables and the chords are disposed in horizontal plane at angles of 120° each with respect to the others; 5. conical sieve, with the apex angle 2α , $\alpha=76^\circ=1.33$ rad, made of perforated sheet metal with hole diameter of 4.2 mm; 6. cylindrical collecting pan of the separated material; 7. collecting box with coaxial circular gaps; 8. driving gear connecting the sieve to the electric driven actuator.

The sieve is used for pre-cleaning and cleaning in a single operation, by separating the particles according to their size. The product that must be processed (the seeds which must be separated from fractured particles and other impurities) is delivered on the external surface of the cone 1, at the upper part (Fig. 1) and is submitted to an oscillating rotation motion, in horizontal plane. The conical vibrating sieve is actuated by an electrically driven crank and connecting rod assembly, whose angular velocity is step-controlled. The sieve imparts a complex motion on its surface to the seeds and to the impurity particles. Impurity particles with size less than the diameter of the orifices of the sieve pass through and fall in the collecting pan 6, while the cleaned material falls in the slot 7, from which it is evacuated.

3. Mathematical model

The study is performed under the following assumptions, usually found in the literature [3]:

- the particles are considered material points;
- interaction between seeds (collisions, staggering in layers, dynamic interactions) are negligible;
- since particles move with low velocities (under 1 m/s), the aerodynamical forces are negligible;
- taking into account the shape of the seeds used in tests (corn seeds), the motions of the particles on the sieve take place by sliding, without rolling, while the effect of the pivoting is negligible.

The study of the motion of a material particle makes it necessary first to identify the motion of the external surface of the sieve, which is the transporting element.

The sieve has a conical shape, with vertical symmetry axis and with the apex angle 2α .

The motion of the conical sieve, generated by the actuation system, can be considered a horizontal rotation, about the symmetry axes, with the variable angular velocity Ω . Any vertical displacement is neglected.

In a first order approximation, the expression of the angular velocity can be chosen of the form

$$\Omega = A_0 \sin pt, \quad (1)$$

where A_0 is a constant expressing the amplitude.

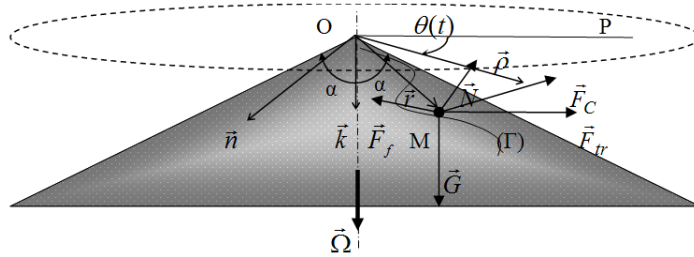


Fig. 2. Forces acting upon the particle on the sieve

A cylindrical coordinate system is used, connected to the sieve (thus mobile), with the origin in the apex O of the cone and with the unit vectors $\vec{\rho}$, \vec{n} and \vec{k} (Fig. 2).

The position of the particle is defined by the vector

$$\overrightarrow{OM} = r(t)\vec{\rho}(t) + z(t)\vec{k}. \quad (2)$$

A particle in relative motion with respect to the separation surface (Fig. 2) is acted by

- the weight \vec{G} ,
- the normal reaction

$$\vec{N} = N \cos \alpha \cdot \vec{\rho} - N \sin \alpha \cdot \vec{k}, \quad (3)$$

– the friction force

$$\vec{F}_f = -\mu N \frac{\vec{v}_r}{|\vec{v}_r|} = \frac{\dot{r}\vec{\rho} + r\dot{\theta}\vec{n} + \dot{z}\vec{k}}{\sqrt{\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2}}, \quad (4)$$

– the transport force

$$\vec{F}_{tr} = -m\vec{a}_{tr} = -m \left[\dot{\vec{\Omega}} \times \vec{OM} + \vec{\Omega} \times (\vec{\Omega} \times \vec{OM}) \right] = -m(r\dot{\Omega}\vec{n} - r\Omega^2\vec{\rho}), \quad (5)$$

– the Coriolis force,

$$\vec{F}_C = -m\vec{a}_C = -2m\vec{\Omega} \times \vec{v}_r = -2m(-r\dot{\theta}\Omega\vec{\rho} + \dot{r}\Omega\vec{n}), \quad (6)$$

where the $\vec{\Omega}$ is the angular velocity of the sieve, \vec{v}_r - the relative velocity of the particle, \vec{a}_{tr} - the transport acceleration and \vec{a}_C - the Coriolis acceleration.

The motion of a particle of mass m , without detachment from the sieve surface, is described by the differential equations [9], [10], [11]

$$\begin{cases} m(\ddot{r} - r\dot{\theta}^2) = N\vec{\rho} + F_{tr}^{\vec{\rho}} + F_C^{\vec{\rho}} + F_f^{\vec{\rho}} \\ m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = F_{tr}^{\vec{n}} + F_C^{\vec{n}} + F_f^{\vec{n}} \\ m\ddot{z} = G + N\vec{k} + F_{tr}^{\vec{k}} + F_f^{\vec{k}}. \end{cases} \quad (7)$$

Since the motion of the particle takes place on the conical surface of the sieve, the parameters of the motion are linked by the relation

$$z = r \cdot \operatorname{ctg} \alpha. \quad (8)$$

By replacing relations (3)-(6) and (8) into system (7), after some simple operations, two differential equations are obtained:

$$\begin{cases} \ddot{\theta} = -\frac{2\dot{r}(\dot{\theta} + \Omega)}{r} - \dot{\Omega} - \frac{\mu\dot{\theta} \left[g \sin \alpha - r(\dot{\theta} + \Omega)^2 \cos \alpha \right]}{\sqrt{\frac{\dot{r}^2}{\sin^2 \alpha} + r^2\dot{\theta}^2}} \\ \ddot{r} = g \sin \alpha \cos \alpha + r(\dot{\theta} + \Omega)^2 \sin^2 \alpha - \frac{\mu\dot{r} \left[g \sin \alpha - r(\dot{\theta} + \Omega)^2 \cos \alpha \right]}{\sqrt{\frac{\dot{r}^2}{\sin^2 \alpha} + r^2\dot{\theta}^2}}. \end{cases} \quad (9)$$

Since this system non-linear and it admits no analytical solution, it can be integrated only numerically. The following initial conditions were considered:

$$t = 0 \Rightarrow \theta = \theta_0, \quad r = r_0, \quad \dot{\theta} = \dot{\theta}_0, \quad \dot{r} = \dot{r}_0. \quad (10)$$

4. Limit sieving velocity

The motion of the mass centre of a seed that falls freely into a hole, in radial direction (fig. 3), is described by the equations

$$\begin{cases} x = v_{\text{lim}} t_1 + g \frac{t_1^2}{2} \sin \beta = D - l/2 \\ y = \frac{g t_1^2}{2} \cos \beta = \frac{b}{2}, \end{cases} \quad (11)$$

where the air flow effects have been neglected and where the following notations have been introduced:

β – the slope angle of the conical sieve surface;

l, b – the length and the width of the seed, respectively;

D – the diameter of the sieve orifice;

t_1 – the free fall time.

The theoretical limit sieving velocity results from relations (11):

$$v_{\text{lim}} = \frac{\sqrt{bg \cos \beta}}{2} \left(\frac{2D - l}{b} - \tan \beta \right). \quad (12)$$

The real limit sieving velocity can be greater than the value given by formula (12), due to the collision phenomenon that can occur between the particle and the edge of the hole, as well as to the existence of a tangent component of the velocity. Therefore, according to [12], [13], a greater value for the limit velocity can be considered:

$$v_{\text{max}} = 1.47 v_{\text{lim}}. \quad (13)$$

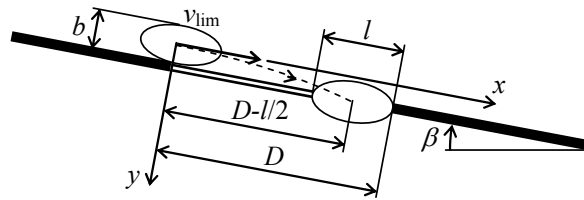


Fig. 3. Trajectory of the seed, corresponding to the limit passing through the orifice

5. Experimental determinations

5.1. Determination of the dynamic parameters

Experimental determinations have been made with a measuring chain, consisting in an acquisition card, four accelerometers and a personal computer.

The accelerometers were positioned as shown in Fig. 4.

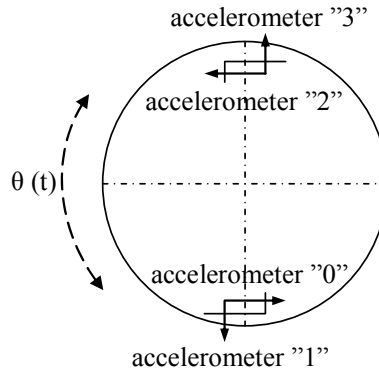


Fig. 4. Position of the accelerometers

The variation of the acceleration measured by accelerometer "0" and its amplitude spectrum are illustrated in Figure 5 and 6, respectively. The variation of the velocity resulted by integrating the measured acceleration and its amplitude spectrum are presented in Figures 7 and 8, respectively.

The analysis of the curve in Figure 7 and of the spectrum in Figure 8, leads to the conclusion that the motion of the sieve can be considered a periodical rotation, with the fundamental harmonic component at the frequency of 12.5 Hz, which corresponds to the perturbation circular frequency

$$p = 2\pi f = 78.5 \text{ s}^{-1}. \quad (14)$$

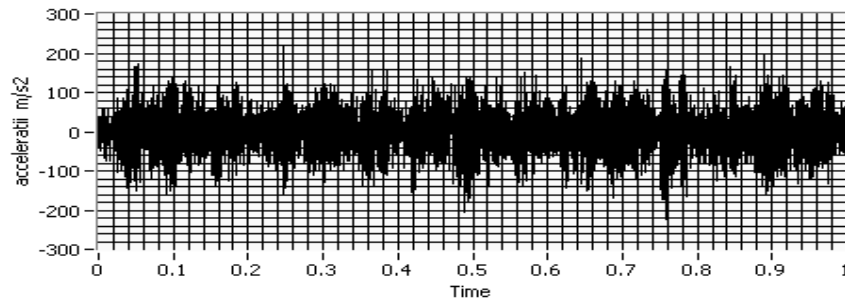


Fig. 5. Variation of the measured sieve acceleration

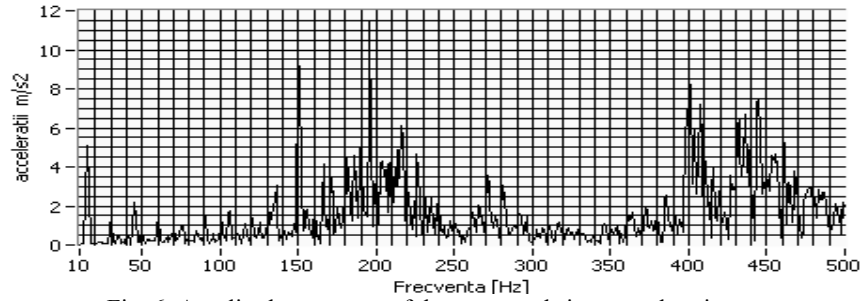


Fig. 6. Amplitude spectrum of the measured sieve acceleration

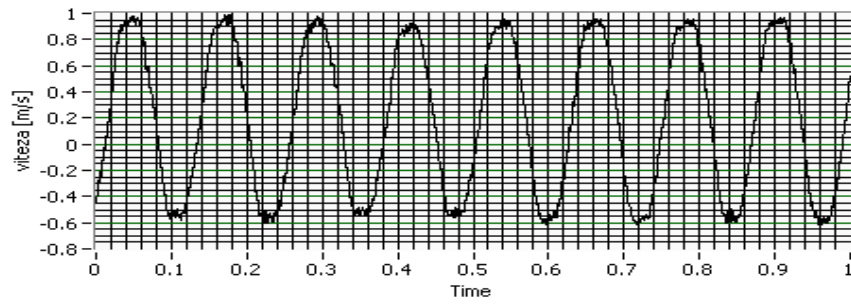


Fig. 7. Variation of the sieve velocity

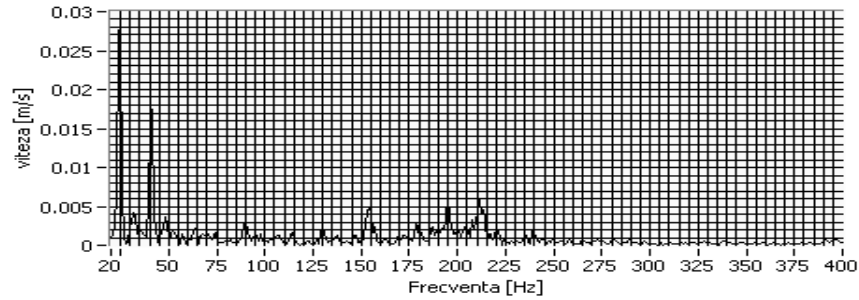


Fig. 8. Amplitude spectrum of the sieve velocity

The amplitude of the fundamental component of the velocity in the point “0” is $A_v = 0.55\sqrt{2} = 0.775 \text{ m/s}$, where the factor $\sqrt{2}$ has been introduced in order to compensate a normalization operation performed by the computer program. Amplitude A_v corresponds to the angular velocity amplitude of the sieve $A_0 = \frac{A_v}{R}$, where $R = 0.5 \text{ m}$ is the distance between the measuring point and the centre of the sieve. It results $A_0 = 1.55 \text{ rad/s}$.

5.2. Determination of the technological parameters

A series of measurements have been performed, intending to determine the influence of the oscillating frequency over the efficiency of the separation of a mixture with the total mass of 500 g, containing 420 g corn seeds and 80 g fractured particles, able to pass through the sieve orifices.

The results in Table 1 have been obtained.

Table 1

Oscillating frequency [Hz]	Mass of the separated material, corn / fractured particles [g]	Mass m_1 of the separated fractured particles [g]	Mass m_2 of the non-separated fractured particles [g]	Separation efficiency $\frac{m_1}{m_1 + m_2}$ [%]
4	420/80	67	13	84%
6	420/80	72	8	90%
8	420/80	75	5	94%
10	420/80	73	7	91%
12	420/80	72	8	90%
13	420/80	69	11	86%
15	420/80	42	38	52%

These results show that the separation process is the most efficient if the sieve oscillating frequency is situated in the interval 6-13 Hz.

6. Numerical results

Differential system (9), of two equations of the second order, can be integrated numerically by using fourth order Runge-Kutta method [14].

In order to apply this method, the system is transformed into an equivalent system of four equations of the first order,

$$\left\{ \begin{array}{l} \dot{\theta} = \omega \\ \dot{r} = v \\ \dot{\omega} = -\frac{2v(\omega + \Omega)}{r} - \dot{\Omega} - \frac{\mu\omega \left[g \sin \alpha - r(\omega + \Omega)^2 \cos \alpha \right]}{\sqrt{\frac{v^2}{\sin^2 \alpha} + r^2 \omega^2}} \\ \dot{v} = g \sin \alpha \cos \alpha + r(\omega + \Omega)^2 \sin^2 \alpha - \frac{\mu v \left[g \sin \alpha - r(\omega + \Omega)^2 \cos \alpha \right]}{\sqrt{\frac{v^2}{\sin^2 \alpha} + r^2 \omega^2}}, \end{array} \right. \quad (15)$$

where auxiliary variables ω and ν have been introduced.

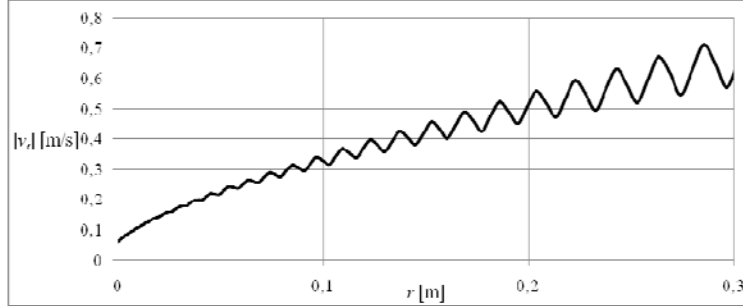


Fig. 9. Variation of the particle relative speed with the radius, for the harmonical motion

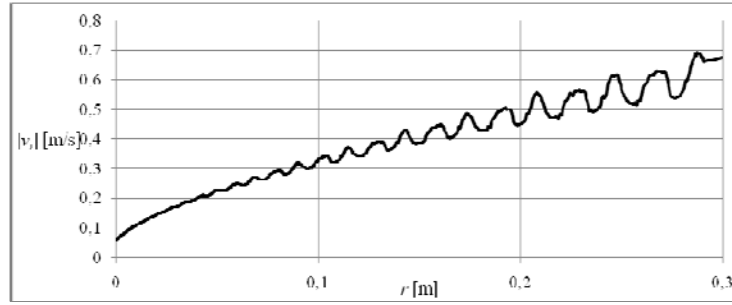


Fig. 10. Variation of the particle relative speed with the radius, for the real motion

The method has been implemented in a computer code written by the authors.

The following values have been assumed: gravitational acceleration $g = 9.81 \text{ m/s}^2$, the dynamic friction coefficient of the corn seeds and of the fractured particles $\mu = 0.23$ [15]

Initial conditions $\theta_0 = 0$, $r_0 = 0.00004 \text{ m}$, $\dot{\theta}_0 = 0$, $\dot{r}_0 = 0.06 \text{ m/s}$, have been considered.

The variation with the radius r of the relative speed of the particles

$$|\vec{v}_r| = \sqrt{\dot{r}^2 + (r\dot{\theta})^2 + \dot{z}^2} = \sqrt{\frac{\nu^2}{\sin^2 \alpha} + r^2 \omega^2} \quad (22)$$

has been determined for two computational cases:

- 1) considering the harmonic motion (1) of the sieve (Fig. 9);
- 2) considering the real motion of the sieve, determined from the measured values (Fig. 10).

In both cases, fourth order Runge-Kutta method has been used.

7. Conclusions

Technological tests performed for various oscillating frequencies lead to the following conclusions:

- a good agreement was found between the values predicted by the theoretical model and the values provided by experimental data;
- albeit the measured signal shows that the motion of the sieve is not perfectly harmonic, the influence of the high order harmonics components does not affect significantly the separation phenomenon;
- the separation of the cereal mixtures is best achieved if the oscillating frequency of the sieve is situated in the interval 6-13 Hz, which is compatible with results reported in other papers [16];
- for lower values of the oscillating frequency, the speed of the material submitted to the separation process has a small value, hence the delivery rate is small and the efficiency is reduced; these conclusion can be also found for other sieve configurations [7], [17];
- for higher values of the oscillating frequency, the material submitted to the separation process moves on the sieve surface with bounces, which leads to a low quality sieving;
- parameter v_r , which determines the sieving quality, does not exceed the maximum accepted value indicated by the literature.

Based on the ideas of the authors, a limited number of tests have been performed, in order to verify the theoretical model, as well as the functioning of the presented device. Further studies are intended, by using different materials and functioning parameters, in order to get more general information.

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