

A PHENOMENOLOGICAL UNIVERSALITIES APPROACH TO THE ANALYSIS OF PERINATAL GROWTH DATA

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Metoda abordării prin Universalități fenomenologice (PUN) a fost dezvoltată de P. P. Delsanto și colaboratorii în ultimii 4 ani. Această metodă reprezintă un nou instrument pentru analiza seturilor de date experimentale și transferul de metode între domenii diferite, în particular între fizică/inginerie și medicină, respectiv științele sociale. În fapt, ea permite detectarea similarităților dintre seturi de date din domenii complet diferite, acționând asupra lor ca o lentilă care permite extragerea tuturor informațiilor posibile într-un mod simplu. În cazul problemelor neliniare, ea permite restabilirea invarianței cu ajutorul unor variabile redefinite prin dimensionare fractală. Principalul obiectiv al acestei lucrări constă în extinderea aplicabilității noii abordări la cercetările din domeniul auxologiei (științei despre creșterea organismelor). În particular, lucrarea urmărește o analiză imparțială a seturilor de date privind creșterile perinatale (adică înaintea și după naștere), căutând să răspundă întrebării dacă ele pot fi descrise în ansamblu printr-o singură clasă PUN.

The Phenomenological Universalities (PUN) approach has been developed by P.P. Delsanto and collaborators during the past four years. It represents a new tool for the analysis of experimental datasets and cross-fertilization among different fields, from physics/engineering to medicine and social sciences. In fact, it allows similarities to be detected among datasets in totally different fields and acts upon them as a magnifying glass, enabling all the available information to be extracted in a simple way. In nonlinear problems it allows the nonscaling invariance to be retrieved by means of suitable redefined fractal-dimensional variables. The main goal of the present contribution is to extend the applicability of the new approach to the investigations on the field of auxology. In particular it aims to an unbiased analysis of datasets referring to perinatal growth (i.e. before and after birth) and it queries whether they can be fitted together by a single PUN class curve.

Key words: phenomenological universalities, perinatal growth interdisciplinary approach auxology, endogenous vs. exogenous.

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1. Introduction

Phenomenological Universalities (PUN) have been developed [1] as a new epistemological tool for discovering, directly from the experimental data, formal similarities in different fields ranging from physics to biology to social sciences. The main idea is that in totally different contexts, growth can be described by the same function $y(t)$ of the variable y (mass or length or height, etc.) vs. the time t . The growth rate a may vary in time in a complex nonlinear manner, possibly reflecting different phases exhibiting new and different metabolic stages of interplay with the host.

In a PUN approach relationships among the main variables of the system, e.g. between the growth rate a and its time derivative b , are assumed in the form of power expansions, such as

$$b = \sum_{n=1}^{\infty} \alpha_n a^n \quad (1)$$

If a satisfactory fit of the experimental data is obtained by truncating the above set at the N -th term (or power of a), then we state that the underlying phenomenology belongs to the Universality Class UN . E.g., it can be easily shown [2] that the Universality Class $U0$ (i.e. with $N=0$) represents autocatalytic growth, $U1$ ($N=1$) yields the well known ‘Gompertz’ law, which has been used for more than a century to study all kinds of growth phenomena, and $U2$ is the general form of the West-like growth law [3].

In the present context Phenomenological Universalities are formulated as a method for solving the following problem: given a string of data $y_i(t_i)$ and assuming that they refer to a phenomenology, which can be reduced to a first order ODE of the type

$$\dot{y}(t) = a(y, t)y(t), \quad (2)$$

where a dot denotes a time derivative, we search for a solution $y(t)$, based not on simple numerical fitting (standard Matlab routines), but on an unbiased (i.e. absolutely interdisciplinary or universal) procedure.

It may be useful to introduce the auxiliary variable $z = \ln(y)$, so that Eq. (2) becomes

$$\dot{z} = a(z, t) \quad (3)$$

Starting with the case $a = a(z(t))$, i.e. assuming a nonlinear but not directly time dependent rate of growth, the problem of finding $z(t)$ and $y(t)$ is reduced to the integration first of the equation $\dot{a} = b$, with b given by Eq.(1),

truncated at a given N , and then of Eq. (3). The results are summarized in Table 1, for $N=0,1$ and 2 (U0, U1 and U2, respectively).

For a more general treatment, if one consider the case

$$a(z, t) = \bar{a}(z) + \tilde{a}(t) \quad (4)$$

in which a is assumed to be the sum of two contributions to the growth rate, one (\bar{a}), that depends only on z (or y), while the other (\tilde{a}) is solely time-dependent.

Then, by writing,

$$y = \bar{y}(t) \tilde{y}(t), \quad (5)$$

it follows

$$\dot{y} = (\bar{a} + \tilde{a}) \bar{y} \tilde{y}, \quad (6)$$

Eq. (2) can consequently be split into a system of two uncoupled equations

$$\dot{\bar{y}} = \bar{a}(z) \bar{y} \quad (7)$$

and

$$\dot{\tilde{y}} = \tilde{a}(t) \tilde{y} \quad (8)$$

Eq. (7) can be solved as before (for the case $a(z)$) giving rise to the classes UN . A general solution of Eq. (8) can be written as

$$\tilde{a} = \dot{\tilde{z}} = \sum_{n=1}^{\infty} A_n E_n \quad (9)$$

where $E_n = \exp[i(n\omega t + \Psi_n)]$. Then, if the sum in Eq.(9) can be truncated to the M -th term, we will state that the corresponding phenomenology belongs to the class UN/TM . It may be interesting to remark that the class $U0/TM$ and its phenomenology, involving the appearance of hysteretic loops and other effects, has been analyzed in detail, in a completely different context (Slow and Fast Dynamics [9]), under the name of Nonclassical Nonlinearity.

Table 1:

Explicit expression of the generating functions $b(a) = \dot{a}$, growth rate $a(z(t))$ and $z(t) = \ln[y(t)]$ for the three classes U0, U1 and U2.

Class	$b(a)$	$a(z(t))$	$z(t)$
U0	0	β	$z = \beta t$

U1	βa	$a_0 \exp(\beta t)$	$z = \frac{a_0}{\beta} (\exp(\beta t) - 1)$
U2	$\beta a + \gamma a^2$	$a_0 \left[\left(1 + \frac{a_0 \gamma}{\beta} \right) e^{-\beta t} - \frac{a_0 \gamma}{\beta} \right]^{-1}$	$z = -\frac{1}{\gamma} \ln \left[1 + \frac{a_0 \gamma}{\beta} (1 - e^{\beta t}) \right]$

2. Results and discussion

Somatic growth is a very important indicator of the health and nutrition conditions of a child or of a specific group of children (to be classified according to the most diverse criteria). For this reason, height, weight and body mass index (BMI) growth chart are very popular diagnostic and investigation tools [4]. As an example of application of the PUN approach discussed in the Introduction, we analyse in the following several datasets including length (or height) and weight data referring to both pre- and post-natal growth [5]. We recall that auxologic data had already been considered in a previous article [6], whose main purpose was to illustrate the “composite” PUN class UN/TM.

In Fig.1 we report the data relative to the mean value of the human embryo length in the very first stage, i.e. from conception up to 60 days. The growth process here can be satisfactorily described by the class U1, with no significant improvement in going to U2. The presence of oscillations, such as described by U1/T1, is plausible but not conclusive, since the improvement of the R^2 might be due to the added parameters.

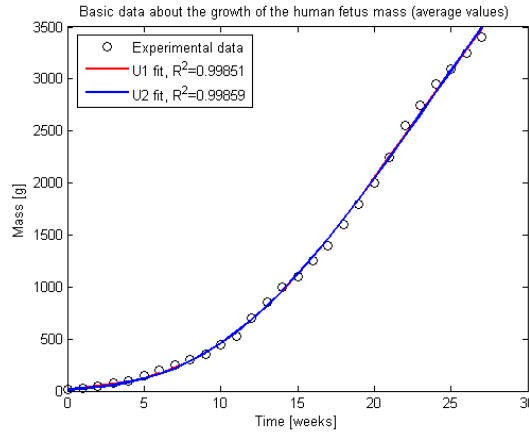


Fig. 1: Length of the embryo vs. time for the first 60 days after conception. The data have been analyzed in the framework of the PUN classes considered (U1, U2, U1/T1 and U2/T1) by means of standard Matlab routines.

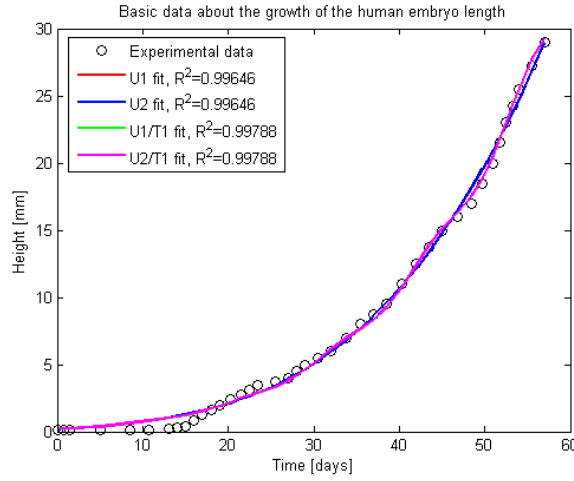


Fig. 2: Mass of the fetus up to the age of 30 weeks. The solid curve represents both U1 and U2 (no appreciable difference between the two descriptions).

Fig. 2 shows that a similar situation can be found also in the case of fetal growth (up to 30 weeks), with the possible presence of oscillations. Unlike Fig.1, Fig.2 refers to weight (or mass) datasets, but the plots for length may be expected to be very similar.

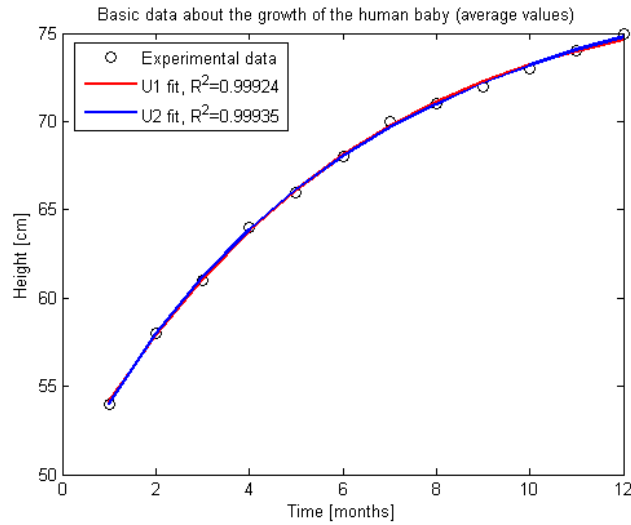


Fig. 3: Height vs. age for a baby up to 12 months

Figs. 3 and 4 refer to the postpartum growth (from birth to 12 months and from 2 to 18 years, respectively). In both cases the results can be satisfactory explained with the framework of the class U1.

More interesting is Fig.5, in which all the length (or height) data from Figs. 1, 3 and 4 are combined. An analysis in terms of U1, U2, U1/T1 and U2/T1 has been performed for the whole dataset of Fig. 5, showing that, also in this case, as in Refs. [5], the effects of the oscillations are not negligible. Indeed, both U1/T1 and (in particular) U2/T1 considerably improve the results obtained by a simple fit without the oscillatory part. It is remarkable that the curve U2/T1 fits very well the entire perinatal curve, which, to our knowledge, is a result never obtained before.

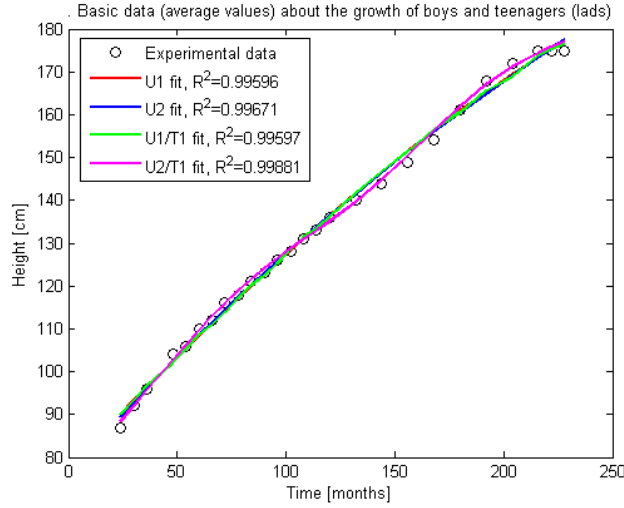


Fig. 4: Height of a child vs. age up to about 18 years.

It is also to be noted that the averaging among a large number of individuals washes away some evidence of well-defined oscillations in transversal data sets. For this reason it is suggested that longitudinal data sets (referring to a single individual or group of individual) be used instead for more focussed studies.

3. Conclusions

The problem of finding a single PUN class describing the whole perinatal growth process is of particularly interest, since it represents a crucial test of the range of validity of PUN's. In fact, while it is obvious that any kind of PUN class can only satisfactorily describe endogenous phenomenologies, in which no new extraneous factors disturbing the system intervene during the time range

considered, the distinction between endogenous and exogenous processes is not always clear

As a typical example, all the mechanisms for the postpartum survival and continued growth are preordained in the fetus' DNA, so that one could argue that the whole perinatal process should be considered endogenous. On the other side, however, the birth itself, with a transition from a liquid to a gaseous environment (without external and pressure constraints), represents such a shock for the newborn, that a discontinuity in the curve (hence a change of PUN class or at least of its parameters) is unavoidable, thus supporting the "exogenous" assumption.

The results discussed in the previous Section seem to favor the separation of the perinatal growth curve in two stages (before and after birth), since the R^2 values are much closer to one in Figs 1 to 4 than in Fig.5. However, also the fitting of the whole perinatal curve in Fig.5 by means of U1/T1 (or U2/T1) does not yield an unacceptable value for R^2 . Therefore a more comprehensive analysis, based on a much large number of datasets, is required before an unambiguous conclusion can be reached.

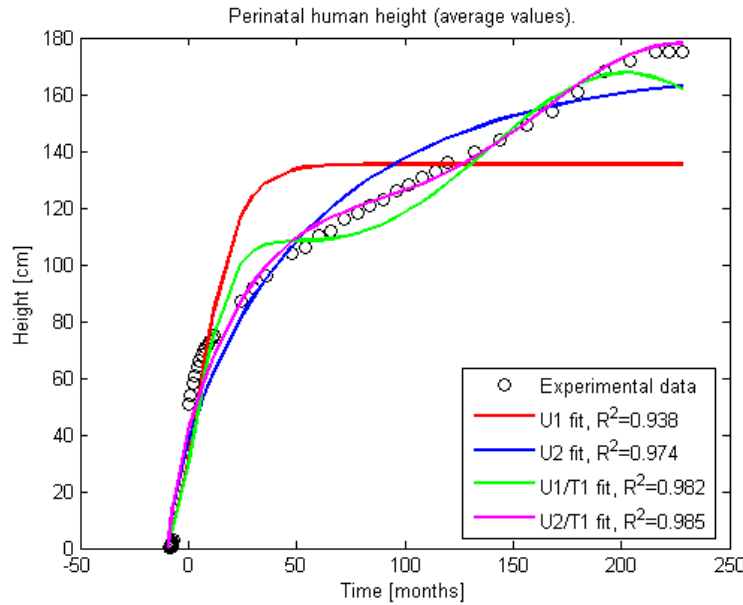


Fig. 5: Combined plot of height (or length) for the whole perinatal period (from Figs. 1,3 and 4).

As an example, we report the U2/T1 parameters:

$$\beta = -0.012, \gamma = -1.52, a_0\gamma / \beta = 1.03 \times 10^4, \omega = 0.028$$

It is important to remark that the purpose of the present paper was to search for an "integral" description of the whole perinatal curve, which is very

important from the point of view of a PUN approach, in order to emphasize the interdisciplinary nature of the growth processes. For practical applications, however, it is sometimes more useful to find a piecemeal description, in which the various growth phases are identified and analyzed separately. Such an approach is carried on in Ref.[5], in which not only some specific growth stages are discovered (such as a “germination” and an “inflation” phase in the first two weeks and next ten days of embryo growth, respectively), but also an intriguing analogy between the Universe and the human growth is presented.

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