

A DYNAMIC SYSTEM GOVERNED BY SOMMERFELD EFFECT

Valerica MOȘNEGUȚU¹, Ligia MUNTEANU², Veturia CHIROIU³, Cristina ȘTIRBU⁴

The behaviour of a dynamic system governed by Sommerfeld effect is studied in this paper. We refer to a small building equipped with an electromechanical vibration absorber subjected to harmonic excitation with time dependent frequencies and random vibrations, respectively. This structure is a non-ideal system which acts like an energy sink for which a part of the source energy is spend to deform the system rather than increasing the drive speed. We show that this system exhibits a rich variety of phenomena, including chaotic motion.

Keywords: Non-ideal system, Sommerfeld effect, chaotic behaviour.

1. Introduction

A vibrating system is said ideal when its excitation is not influenced by the response of the system. When the excitation is influenced by the response of the system, the system is said non-ideal. A new degree of freedom is present in the theory for which a new equation has to be added in order to describe how the energy source interacts with the vibrating system. The energy transfer in these dynamic systems is governed by the Sommerfeld effect which appears as a result of the law of energy conservation.

Sommerfeld observed this phenomenon in 1902 while making an experiment with a cantilever beam connected with an energy source at its free end. He observed that the structural response of the system to which an electrical motor is connected may act like energy sink under certain conditions so that a part of the energy supplied by the source is spend to vibrate the structure rather than increasing the drive speed. Sommerfeld had calculated the change of energy between the source and the beam and per se observed that the motor has a speed which remains the same until it suddenly jumps to a much higher value when the driving frequency is closer to the natural frequency of the beam and the drive

¹ Institute of Solid Mechanics, Romanian Academy, Bucharest, Romania

² Institute of Solid Mechanics, Romanian Academy, Bucharest, Romania

³ Institute of Solid Mechanics, Romanian Academy, Bucharest, Romania, e-mail: veturiachiroiu@yahoo.com

⁴ National Meteorological Administration, Bucharest, Romania

power increases further. Its amplitude jumps to a much lower value upon exceeding a critical input power [1-4].

Several papers have analyzed the Sommerfeld effect [5-11]. Depending on the problem, the Sommerfeld effect can be positive and desirable, or can be completely unfavorable, and in this case the effect must be controlled. One of the undesirable effects consists in the additional vibrations send by the source to the system instead of speeding up the machine. The system mimics a disappearance of the energy in the resonance regions which can affect the stability of the system, sending it to chaos [2, 12-13]. Moreover, the class of models governed by the Sommerfeld effect exhibits a rich variety of phenomena, including chaotic motion due to the strong sensitivity to the control parameter [14].

In this paper, we start from the papers [10] and [11], and analyse the response of a small building equipped with an electromechanical vibration absorber to harmonic excitation with time dependent frequencies and random vibrations, respectively. It is shown that the unstable periodic orbits of the structure without the vibration absorber become the source of chaos.

2. Mathematical formulation

We consider the same structure analysed in [10] and [11] equipped with an electromechanical vibration absorber (Fig.1). The second floor is equipped with an absorber. The stiffness and damping considered here are the cubic Duffing, cubic-quintic Duffing or Rayleigh, or another such as the Van der Pol with first, second, third, fourth and fifth term. The stiffness is expressed as a sum of a linear and a cubic Duffing term, while the damping is considered to be linear.

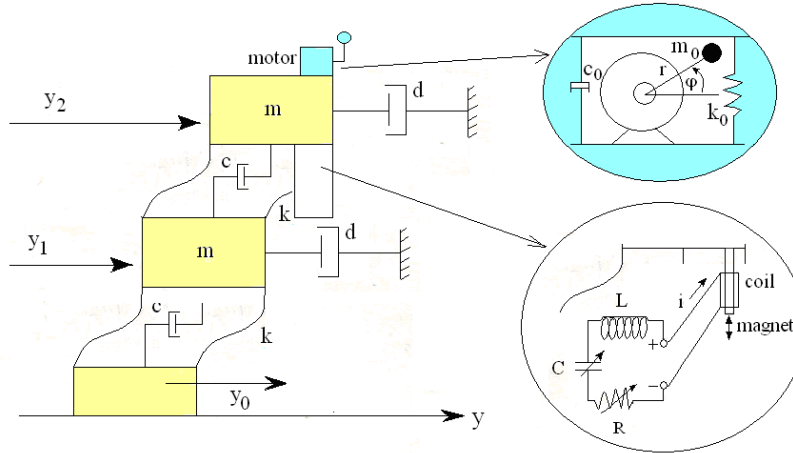


Fig. 1. Scheme of the structure coupled to the absorber device [10], [11].

The coupling between the motion equations of the building and the equations of the energy source is presented next by the following set of equations [10], [11]

$$m\ddot{y}_1 + c(2\dot{y}_1 - \dot{y}_2 - \dot{y}_0) + d\dot{y}_1 + k_1(2y_1 - y_2 - y_0) + k_2(2y_1 - y_2 - y_0)^3 = 0, \quad (1)$$

$$\begin{aligned} m_1\ddot{y}_2 + c(\dot{y}_2 - \dot{y}_1) + d\dot{y}_2 + k_1(y_2 - y_1) + k_2(y_2 - y_1)^3 = \\ = T\dot{q} + m_0 r \ddot{\varphi} \cos \varphi - m_0 r \dot{\varphi}^2 \sin \varphi, \end{aligned} \quad (2)$$

$$m_0 r \ddot{y}_2 \cos \varphi = I \ddot{\varphi} + a \dot{\varphi} - b, \quad (3)$$

$$T(\dot{y}_2 - \dot{y}_1) = -L\ddot{q} + R(1 - i_0^{-2} \dot{q}^2) \dot{q} - C_0^{-1} q - i_0^2 \alpha_3 q^3 - i_0^4 \alpha_5 q^5, \quad (4)$$

$$y_0 = f(t). \quad (5)$$

where y_j is the displacement of j^{th} story, $j=1,2$, m the mass of each story, c the interfloor damping (internal damping), d the external damping constant (velocity damping), k_1 the linear spring constant, k_2 the cubic spring constant, φ the angular displacement of the rotor, r the eccentricity of unbalanced shaft of the electric motor, $m_1 = m + m_0$ with m_0 the mass of unbalanced shaft of the electric motor, I the moment of inertia of the rotor, $T = 2\pi n l B$ the transducer constant, n the number of turns in the coil, l the radius of the coil, B the uniform radial magnetic field strength in the annular gap, L the inductor, C the capacitor, R the resistor, V_{res} the voltage in the resistor, V_{cond} the voltage in the capacitor, i_0 the initial current in the electrical part, C_0 the capacitive characteristic, α_3, α_5 the capacitor coefficients and q the instantaneous electrical charge. The expression for the driving torque of the motor is $b - a\dot{\varphi}$ (linear in the stationary regime) with b related to the voltage applied to the armature of the DC motor, and a is a constant depending of the considered motor.

The electric component of the controller is composed by an inductor L , a capacitor C , and a resistor R . The expression of the voltage in resistor and the capacitor are [9]

$$V_{res} = -Ri_0 \left(\frac{\dot{q}}{i_0} - \frac{1}{3} \left(\frac{\dot{q}}{i_0} \right)^3 \right), \quad V_{cond} = \frac{q}{C_0} + \alpha_3 q^3 + \alpha_5 q^5, \quad (6)$$

In (2) the right side term represents the action of the source of energy. Eq. (3) expresses the supplying of the last story with the source energy. Eqs. (4) and (5) represent the boundary conditions for Eqs. (1)-(3).

When loading is deterministic, the function $f(t)$ from (5) is known as a function of time. When loading is random, the function $f(t)$ is not known a priori and it is defined statistically.

In this paper, the function $f(t)$ can be viewed as a step dependent function $\Omega(t)$

$$\Omega(t) = \omega_1 \left[1 + \frac{\omega_2 - \omega_1}{2} \left(1 + \frac{1}{2} \tanh(\varepsilon(t - t_s)) \right) \right], \quad 0 \leq t \leq T, \quad (7)$$

where ω_1 and ω_2 the initial and final frequencies, t_s is the time at which the frequency is changed, and $\varepsilon \rightarrow \infty$ is a parameter which describes the step function limit. Also, the function $f(t)$ can be viewed as an average of various data y_0 measured from k^{th} earth motions $y_0 = f_k(t)$. This is equivalent to obtain $f_k(t)$ from the average curve $E\{f(t)\}$. The random vibrations are of short duration, so the process is not stationary. From this point of view we can not use a purely random stationary Gaussian process.

We start with 1D expected value $E\{f(t)\}$, and the 2D expected function $E\{f(t_1), f(t_2)\}$ respectively, which are given by [14]

$$\begin{aligned} E\{f(t)\} &= \int_{-\infty}^{\infty} f(t) W_1(f(t)) df(t), \\ E\{f(t_1)f(t_2)\} &= \int_{-\infty}^{\infty} f(t_1)f(t_2) W_2(f(t_1), f(t_2)) df(t_1) df(t_2), \end{aligned} \quad (8)$$

where $W_1(f)$ and $W_2(f(t_1), f(t_2))$ are the 1D probability density function for a given t , and the 2D probability density function, respectively, for given t_1 and t_2 . The n^{th} and m^{th} moments ($m, n = 0, 1, 2, \dots$) are also calculated

$$\begin{aligned} E\{f^n(t)\} &= \int_{-\infty}^{\infty} f^n W_1(f) df, \\ E\{f^m(t_1)f^n(t_2)\} &= \int_{-\infty}^{\infty} f^m(t_1)f^n(t_2) W_2(f^m(t_1), f^n(t_2)) df. \end{aligned}$$

For $m = n = 1$, the correlation function $R_f(t_1, t_2)$ is obtained

$$R_f(t_1, t_2) = E\{f(t_1)f(t_2)\} = \int_{-\infty}^{\infty} f(t_1)f(t_2)W_2(f(t_1), f(t_2))df(t_1)df(t_2). \quad (9)$$

In (7) we can change $f(t)$ with $E\{f(t)\}$ or $R_f(t_1, t_2) = E\{f(t_1)f(t_2)\}$ so that the calculations to remain the same as in the deterministic case. If the correlation function $R_f(t_1, t_2)$ is known, we can calculate the cross correlation functions

$$\begin{aligned} R_{y_j y_k}(t_1, t_2) &= E\{y_j(t_1)y_k(t_2)\} = \\ &= \int_{-\infty}^{\infty} y_j(t_1)y_k(t_2)W_2(y_j(t_1), y_k(t_2))dy_j(t_1)dy_k(t_2), \end{aligned} \quad (10)$$

and the square displacements $E\{y_j^2(t)\}$ for the j^{th} , $j = 1, 2$ story

$$E\{y_j^2(t)\} = \int_{-\infty}^{\infty} y_j^2(t)W_2(y_j(t), y_j(t))dy(t). \quad (11)$$

The dimensionless version of (1-5) is

$$x_1'' + \beta_1 x_1' + A(\alpha + \delta_1 A^2) + \alpha_1 A' = 0, \quad A = 2x_1 - x_2 - x_0, \quad (12)$$

$$x_2'' + \beta_1 \mu x_2' - \lambda_1 x_4' + B\mu(\alpha + \delta_1 B^2) + \alpha_1 \mu B' = (1 - \mu)(x_3'' \cos x_3 - x_3'^2 \sin x_3),$$

$$B = x_2 - x_1, \quad (13)$$

$$x_3'' - \eta_1 x_2'' \cos x_3 + \mu x_3' = \nu, \quad (14)$$

$$x_4'' - \alpha_2 x_4' + \beta_2 x_4'^3 + \gamma_2 x_4 + \delta_2 x_4^3 + \eta_2 x_4^5 + \lambda_2 B = 0, \quad (15)$$

$$x_0 = g(\vartheta), \quad (16)$$

where $g(\vartheta)$ depends on $f(t)$ which can be viewed as an average of various data y_0 measured from k^{th} earth motions $y_0 = f_k(t)$. In (12-16) prime means the differentiation with respect to τ , and

$$\begin{aligned}
\tau &= \omega_1 t, \quad \omega_1 = \sqrt{\frac{k_1}{m}}, \quad x_0 = \frac{y_0}{r}, \quad x_1 = \frac{y_1}{r}, \quad x_2 = \frac{y_2}{r}, \quad x_3 = \varphi, \quad x_4 = \frac{q}{q_0}, \\
\mu &= \frac{m}{m+m_0} = \frac{m}{m_1}, \quad \alpha = \frac{k_1}{m\omega_1^2} = 1, \quad \alpha_1 = \frac{c}{m\omega_1}, \quad \beta_1 = \frac{d}{m\omega_1}, \\
\delta_1 &= \frac{k_2 r^2}{m\omega_1^2}, \quad \lambda_1 = \frac{Tq_0}{m_1 r \omega_1}, \quad \eta_1 = \frac{rm_0}{I}, \quad u = \frac{a}{I\omega_1}, \quad v = \frac{b}{I\omega_1^2}, \\
\lambda_2 &= \frac{Tr}{q_0 L \omega_1}, \quad \alpha_2 = \frac{R}{L\omega_1}, \quad \beta_2 = \frac{Rq_0^2 \omega_1}{i_0^2 L}, \quad \gamma_2 = \frac{1}{LC_0 \omega_1^2}, \\
\delta_2 &= \frac{i_0^2 \alpha_3 q_0^2}{L\omega_1^2}, \quad \eta_2 = \frac{i_0^4 \alpha_5 q_0^4}{L\omega_1^2}, \quad \vartheta = \frac{\Omega}{\omega_1}.
\end{aligned} \tag{17}$$

In (13) ω_1 is the first natural frequency of the structure. Unknowns of Eqs. (12)-(16) are the dimensionless displacement of j^{th} story x_j , $j=1,2$, the angular velocity of the rotor x_3 , and the dimensionless instantaneous electrical charge x_4 . The resonance condition with the building is given by $x'_3=1$, for which $\dot{\varphi} = \omega_1 \varphi' = \omega_1$.

The only control parameter is $v = b / I\omega_1^2$, where b is related to voltage applied across to the DC motor, I is the moment of inertia of the rotor, and ω_1 is the first natural frequency of the structure. In addition, we take into account the influence of parameters $\alpha_1 = c / m\omega_1$ and $\beta_1 = d / m\omega_1$, where c is the interfloor damping (internal damping) and d is the external damping constant (velocity damping), upon the chaotic behaviour of the system.

Starting from (11) it is interesting to calculate various averages

$$E\{y_j(t)\} = \int_{-\infty}^t K_j(t-\tau) E\{f(\tau)\} d\tau, \tag{18}$$

$$E\{y_j(t_1)y_k(t_2)\} = \int_{-\infty}^{t_1} \int_{-\infty}^{t_2} K_j(t_1-\tau_1) K_k(t_2-\tau_2) E\{f(\tau_1)f(\tau_2)\} d\tau_1 d\tau_2, \tag{19}$$

For $E\{f(\tau)\} = 0$, it results $E\{y_j(t)\} = 0$ from (18). If the autocorrelation function of $f(t)$ is known, we see that $R_f(t_1, t_2)$ is also known. So, we obtain from

(19)

$$R_{y_j y_k} = \int_{-\infty}^{t_1} \int_{-\infty}^{t_2} K_j(t_1 - \tau_1) K_k(t_2 - \tau_2) R_f(\tau_1, \tau_2) d\tau_1 d\tau_2. \quad (20)$$

Calculation of the mean kinetic energy for each story requires the calculation of the integral

$$R_{\dot{y}_j \dot{y}_k} = \int_{-\infty}^{t_1} \int_{-\infty}^{t_2} \dot{K}_j(t_1 - \tau_1) \dot{K}_k(t_2 - \tau_2) \dot{R}_f(\tau_1, \tau_2) d\tau_1 d\tau_2. \quad (21)$$

The shearing force correlation includes the part given by the spring force and the part that is dissipated by the internal damping

$$V_1 = c(2\dot{y}_1 - \dot{y}_2 - \dot{y}_0) + k_1(2y_1 - y_2 - y_0) + k_2(2y_1 - y_2 - y_0)^3.$$

The maximum displacement is observed at this story

$$V_2 = c(\dot{y}_2 - \dot{y}_1) + k_1(y_2 - y_1) + k_2(y_2 - y_1)^3.$$

The expression for the shearing force correlation is

$$R_{V_j V_k}(t_1, t_2) = \int_{-\infty}^{t_1} \int_{-\infty}^{t_2} K_j(t_1 - \tau_1) K_k(t_2 - \tau_2) R_{V_j V_k}(\tau_1, \tau_2) d\tau_1 d\tau_2, \quad (22)$$

where $R_{V_j V_k}(t_1, t_2) = R_{j,k} - R_{j,k-1}$.

The 1D and 2D probability density functions are given by [15]

$$W_1(y_j) = \frac{1}{\sigma_j \sqrt{2\pi}} \exp\left(\frac{-y_j^2}{2\sigma_j^2}\right), \quad (23)$$

$$W_2(y_j, y_k) = \frac{1}{\sigma_j \tau_k \sqrt{2\pi(1-\rho_{jk})}} \exp\left[\frac{1}{(1-\sigma_{jk})} \left(-\frac{y_j^2}{2\sigma_j^2} - \frac{y_k^2}{2\tau_k^2} + \frac{2\rho_{jk} y_j y_k}{\sigma_j \tau_k}\right)\right] \quad (24)$$

with

$$\sigma_j^2 = E\{y_j^2\} - E^2\{y_j\} = R_{y_j}(t, t), \quad \sigma_{jk} = \sigma_j \tau_k R_{y_j y_k}(t, t), \quad \tau_k^2 = R_{y_k}(t, t). \quad (25)$$

From (23) or (24), the correlation functions and various moments become

$$E\{y_j\} = \int_{-\infty}^{\infty} y_j W_1(y_j) dy_j,$$

$$E\{y_j y_k\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y_j y_k W_2(y_j, y_k) dy_j dy_k = R_{y_j y_k}, \quad (26)$$

$$E\{y_j^n\} = \int_{-\infty}^{\infty} y_j^n W_1(y_j) dy_j, \quad E\{y_j^m y_k^n\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y_j^m y_k^n W_2(y_j, y_k) dy_j dy_k. \quad (27)$$

The autocorrelation functions $R_{2,2}$, $R_{1,1}$ and the cross correlation functions $R_{1,2}$ respectively, are divided by $R_0 = \frac{4f_0^2}{441\pi^2}$, where f_0^2 is the spectral density (a constant) of R_f which characterizes a Gaussian processes, for which the correlation function $R_f(t_1, t_2)$ becomes

$$R_f(t_1, t_2) = E\{f(t_1)f(t_2)\} = f_0^2 \delta(t_1 - t_2), \quad (28)$$

with $\delta(t)$ is the Dirac's delta function.

3. Harmonic excitation with time dependent frequencies

Numerical simulations of solutions are carried out by taking the following values for parameters: $\mu = 0.7$, $\gamma_1 = 0.89$, $\delta_1 = 0.83$, $\lambda_1 = 1.2$, $\eta_1 = 0.4$, $\lambda_2 = 2$, $\alpha_2 = 0.3$, $\beta_2 = 0.07$, $\alpha_1 = 0.015$, $\beta_1 = 0.011$, $\gamma_2 = 0.01$, $\delta_2 = 0.06$, $\eta_2 = 0.9$, $g_0 = 15$ and $\vartheta = 1$. For $\alpha_1 = c/m\omega_1$, $\beta_1 = d/m\omega_1$. The time frames are the following: $0 \leq \tau \leq 500$ and $500 \leq \tau \leq 800$ above the resonance, $800 \leq \tau \leq 1400$ inside resonance, and $1400 \leq \tau \leq 1600$, $1600 \leq \tau \leq 2000$ below the resonance, respectively.

The range of the parameter $\mu = \frac{m}{m+m_0} = \frac{m}{m_1}$ varies from 0.7 to 1. The value $\mu = 0.7$ corresponds to the case when the m_0 represents 40% of m while $\mu = 1$ means that the m_0 is negligible compared to m .

The step function (7) is introduced for two cases: (1) $\tau = 2$ in the interval $0 \leq \tau \leq 500$ above the resonance and (2) $\tau = 815$ in the interval $800 \leq \tau \leq 1400$ inside the resonance.

Fig. 2 plots the step function in the interval $0 \leq \tau \leq 500$ for $\omega_1 = 1$, $\omega_2 = 2$ (solid line) and $\omega_2 = 3$ (dash line) and $\varepsilon = 20$, $\tau_s = 2$ [10]. For other intervals, the step function has similar shapes.

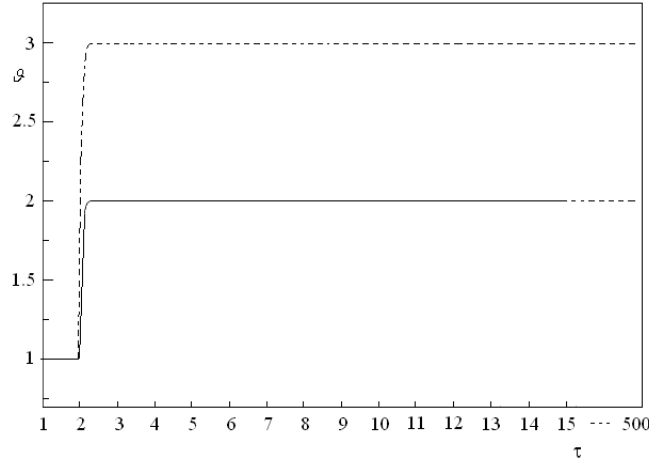


Fig. 2. $\vartheta(\tau)$ as a function of τ above the resonance [10].

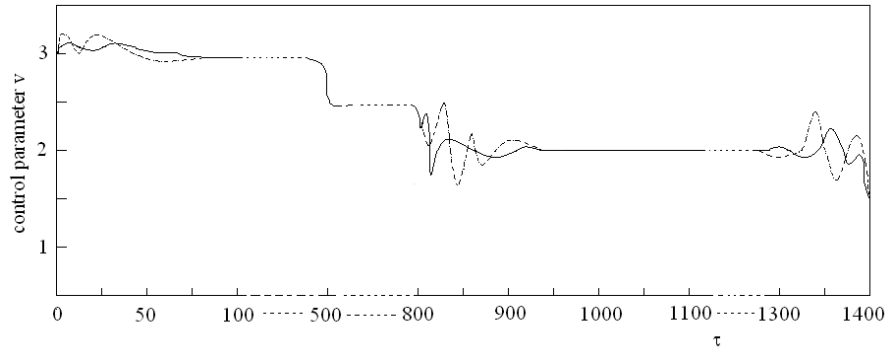


Fig. 3. Variation of control parameter ν with respect to τ above and inside the resonance.

The variation of the control parameter ν with respect to τ is shown in Fig. 3, in the intervals above the resonance and also, inside the resonance, respectively. The control parameter ν knows a sudden variation with respect to τ at the beginning of the interval $0 < \tau < 60$. For $\tau > 60$, ν is constant and then it jumps to 2.5 when $\tau = 500$. Next, ν is constant on $500 < \tau < 800$ and again has a sudden variation at $800 < \tau < 900$. For $900 < \tau < 1315$, ν is constant and then it jumps to the value 1.5 when $\tau = 1400$.

The time variation of the displacement x_2 corresponding to the last story with and without the absorber is plotted in Fig.4 above the resonance in the case of both solid and dash lines (blue and green contours) respectively. The grey region corresponds to the case of no coupling to the absorber, and the red one to the case of coupling.

The time variation of the displacement x_2 corresponding to the last story without/with the absorber, are plotted in Figs. 5 and 6, respectively, inside the resonance in the case of both solid and dash lines (blue and green contours), respectively. According to Fig. 5, the displacements present squeezing and weaving, and an increasing in magnitude for the last story without the absorber inside the resonance. The squeezing and amplitude are enhanced for the dash line (blue and green contours) in Fig. 5. These squeezing and weaving which appear in Fig. 5 characterize large amplitude vibrations and tendency to chaos.

To understand the appearance of the chaos, the stability of trajectories is investigated. Given the trajectories we analyze if their behaviour is chaotic or not with respect to ν . To do this we investigate the behaviour of the Euclidean distance in the phase space [15-17] between a parent trajectory and another trajectory obtained by a slight perturbation of the trajectory parent at $\tau = 0$

$$D(\tau) = \sqrt{\sum_{i=1}^4 (x_i^0 - x_i^1)^2 + \sum_{i=1}^4 (x_i'^0 - x_i'^1)^2 \tau^2}, \quad (29)$$

where the superscript 0 indicates the parent trajectory and 1 the perturbed trajectory, respectively. The initial perturbation applied to the parent trajectory is $D_0 = 2 \times 10^{-3}$.

The motion of the system not coupled with absorber is stable when $D(\tau)$ behaves linearly at short intervals of time. However, for certain values of the control parameter ν , the motion is far away from the linear behaviour and shows an exponential increase in short periods of time. This instability is characteristic of the chaotic motion behaviour at short intervals of time, and can be characterized by the leading Lyapunov exponent λ defined by

$$\exp(\lambda\tau) = \lim_{d_0 \rightarrow 0} D(\tau) / D_0. \quad (30)$$

The initial perturbation applied to the trajectory is denoted by D_0 and its value is $D_0 = 3 \times 10^{-5}$.

The resonant curves of the displacement amplitudes for the last story without the absorber (grey colour) and with the absorber (red colour) are plotted in Fig. 7. We see that in the absence of coupling, the displacements exhibit high values for both damping cases, with increased amplitudes with chaotic aspect over the ranges $\nu > 5.6$.

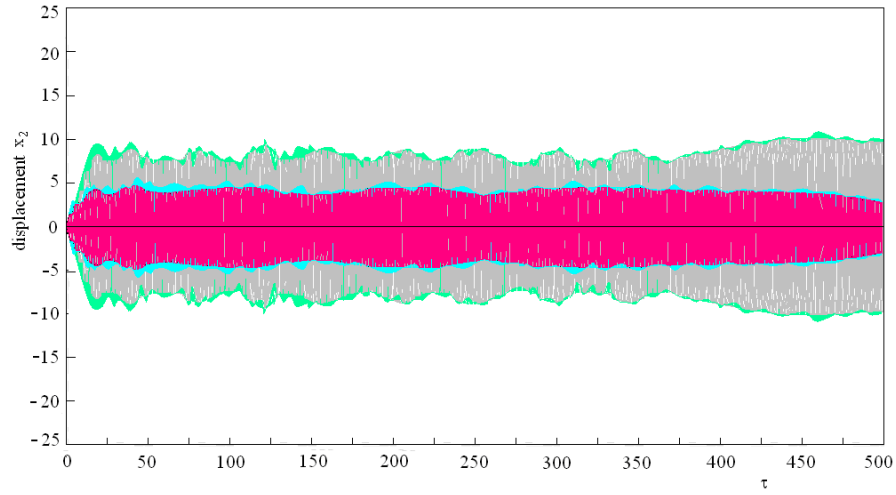


Fig. 4. Time history for x_2 with and without the absorber above the resonance for $\omega_2 = 2$.
The blue and green contours correspond to $\omega_2 = 3$.

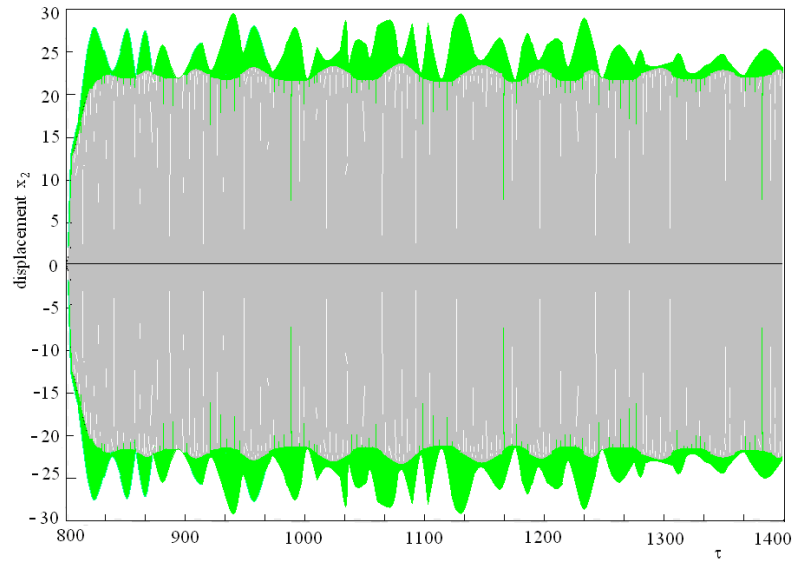


Fig. 5. Time history for x_2 without the absorber during resonance for $\omega_2 = 2$.
The green contour corresponds to $\omega_2 = 3$.

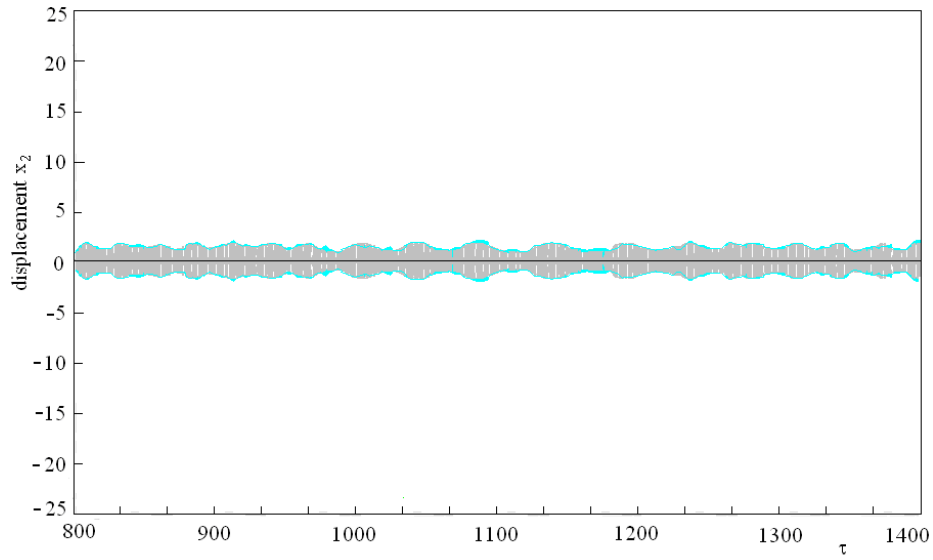


Fig. 6. Time history for x_2 with absorber during resonance for $\omega_2 = 2$.

The green contour **corresponds** to $\omega_2 = 3$.

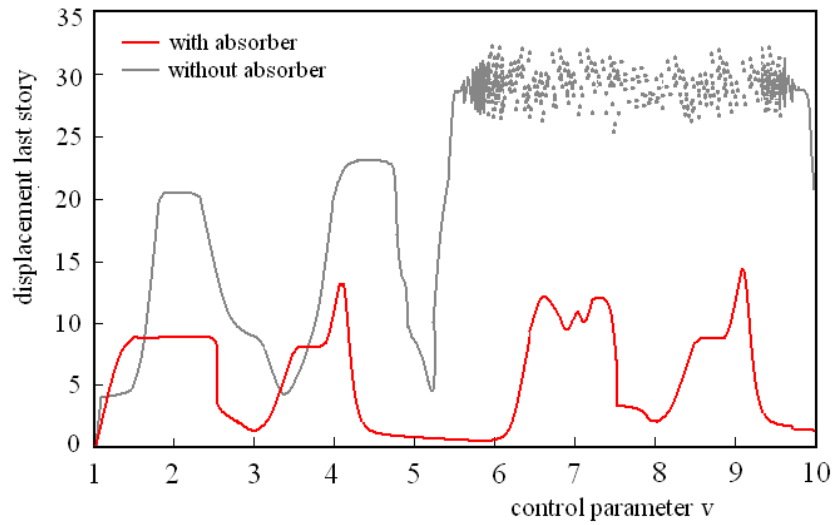


Fig. 7. Resonant curves of the displacement amplitudes for the last story without (grey colour) and with (red colour) coupling.

The initial small deviations amplify exponentially and it takes time to accumulate to substantial amount relative to the small initial swerve. When the structure is not coupled with the absorber, the chaotic aspect is observed over the

ranges $5.6 < \nu \leq 10$. In the case of coupling, the chaotic aspect is absent. The sharp jump phenomenon at $\nu = 2.6$ and 7.55 is a manifestation of the Sommerfeld effect when the energy transfer of the structure to the vibration absorber leads to reduction of the displacements.

4. Random vibrations

The autocorrelation functions $R_{2,2}$, $R_{1,1}$ and the cross correlation functions $R_{1,2}$ respectively, are plotted versus the correlation interval in Figs. 8-10. The parameters values were selected as $R_0 = \frac{4f_0^2}{441\pi^2}$, $\frac{k}{m} = 50$, $c = d = 0.2$.

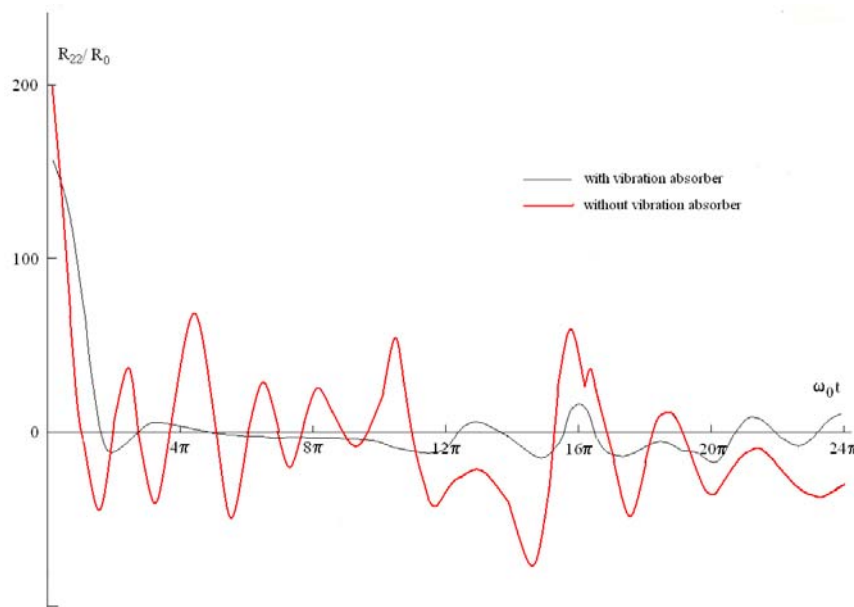


Fig. 8. Displacement autocorrelation functions for the second story with and without the vibration absorber.

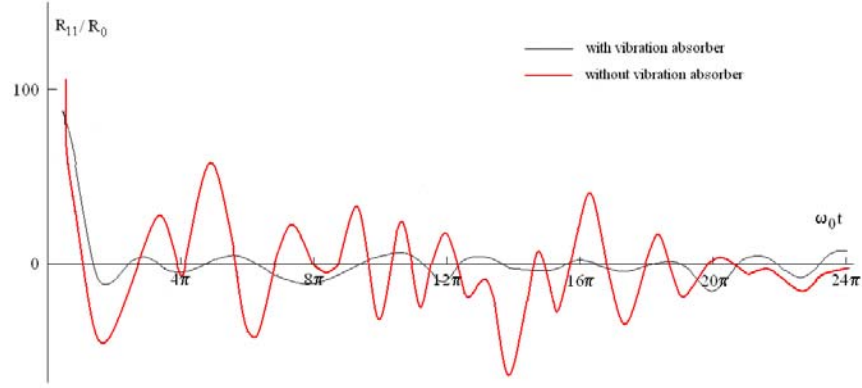


Fig. 9. Displacement autocorrelation functions for the first story with and without the vibration absorber.

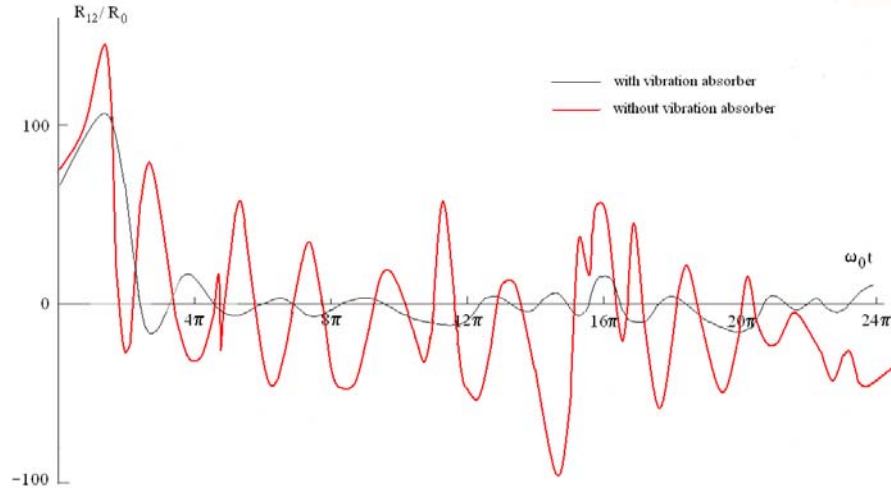


Fig. 10. Displacement cross correlation functions with and without the vibration absorber.

It is clear that *the general case evaluation* presented in this Section is difficult. Eringen [6] has simplified these calculations by considering the purely random stationary Gaussian processes. In this case (20) becomes

$$R_{y_j y_k} = \frac{f_0^2}{4\pi^2} \int_{-\infty}^{t_1} \int_{-\infty}^{t_2} K_j(t_1 - \tau) K_k(t_2 - \tau) d\tau. \quad (31)$$

where f_0 is defined in (28). Eringen [18] has observed a peculiar dependence on τ in the correlation functions which indicate that in the random response perhaps a period is present. This dependence becomes important in a damped fashion. This periodicity is not present in the autocorrelation function for the shearing force. The maximum mean square shearing force occurs at the first floor.

5. Conclusions

This work is dealt with the analysis of the response to vibrations of a small building equipped with an electromechanical vibration absorber. The interaction between the vibrating system and the energy source is investigated for step dependent frequencies and for random vibrations, respectively. This interaction corresponds to a system with non-ideal excitation explained by the Sommerfeld effect. The Sommerfeld effect is a universal phenomenon which appears as a result of the law of energy conservation. It is due to direct and feedback coupling between the vibration absorber and the vibrational loads. The Sommerfeld effect concerns the jump induced due to the influence of the unbalance response on a non-ideal drive around the critical speed of the excited structure. As the motor accelerates to reach near resonant conditions, a considerable part of its output energy is consumed to generate large amplitude motions of the forced structure which can counteract the vibrations of this structure, without increasing the angular speed of the motor. Therefore, the reduction of vibrations for a resonant structure is made by coupling it with a vibration absorber.

Acknowledgement

This research was elaborated through the PN-II-PT-PCCA-2011-3.1-0190 Project nr.149/2012 UEFISCSU, Romania. The authors acknowledge the similar and equal contributions to this article.

REFERENCES

- [1]. *A. Sommerfeld*, Beitrage Zum Dynamischen Ausban Der Festigkeitslehe, Physikal Zeitschr, **vol. 3**, 1902, pp. 266-286.
- [2]. *A. K. Samantaray, S. S. Dasgupta, R. Bhattacharyya*, Bound graph modeling of an internally damped non-ideal flexible spinning shaft, Journal of Dynamical System, Measurement and Control, Trans. ASME, **vol. 132**, no. 6, 2010, pp. 061502-9.
- [3]. *V. O. Kononenko*, Vibrating Systems with a Limited Power Supply (Iliffe Books Ltd, London) 1969.
- [4]. *M. Eckert*, The Sommerfeld effect: theory and history of a remarkable resonance phenomenon, European Journal of Physics, **vol. 17**, no. 5, 1996, pp. 285-289.
- [5]. *J. L. P. Felix, J. M. Balthazar, R. M. L. R. F Brasil*, On saturation control of a nonideal vibrating portal frame foundation type shear-building, Journal of Vibration and Control, **vol. 11**, 2005, pp. 121-136.

- [6]. *J. L. P. Felix, J. M. Balthazar*, Comments on a nonlinear and nonideal electromechanical damping vibration absorber. Sommerfeld effect and energy transfer, *Nonlinear Dynamics*, **vol. 55**, 2009, pp. 1–11.
- [7]. *J. M. Balthazar, B. I. Chesankov, G. T. Ruschev, L. Barbanti, H. I. Weber*, Remarks on the passage through resonance of a vibrating system with two degrees of freedom, excited by a non-ideal energy source, *Journal of Sound and Vibration*, **vol. 239**, no. 5, 2001, pp. 1075–1085.
- [8]. *Balthazar, J. M., Mook, D. T., Weber, H. I., Brasil, R. M. L. R. F., Fenili, A., Belato, D., Felix, J. L. P.*, An overview on non-ideal vibrations, *Meccanica*, **vol. 38**, no. 6, 2003, pp. 613–621, 2003.
- [9]. *K. V. Frolov, T. S. Krasnopolskaya*, Sommerfeld effect in systems without internal damping, *International Applied Mechanics*, **vol. 23**, no. 12, 1987, pp. 1122–1126.
- [10]. *L. Munteanu, V. Chiroiu, T., Sireteanu, T.*, On the response of small buildings to vibrations, *Nonlinear Dynamics*, **vol. 73**, no. 3, 2013, pp. 1527–1543.
- [11]. *L. Munteanu, C. Brişan, V. Chiroiu, D. Dumitriu, R. Ioan, R.*, Chaos-hyperchaos transition in a class of models governed by Sommerfeld effect, *Nonlinear Dynamics*, **vol. 78**, no. 3, 2014, pp. 1877–1889.
- [12]. *T. S. Krasnopolskaya, A. Y. Shvets*, Chaos in vibrating systems with a limited power-supply *Chaos: An Interdisciplinary Journal of Nonlinear Sciences*, **vol. 3**, 1993, pp. 387.
- [13]. *T. S. Krasnopolskaya*, Acoustic chaos caused by the Sommerfeld effect, *Journal of Fluids and Structures*, **vol. 8**, no. 8, 1994, pp. 803–815.
- [14]. *S. Staicu*, *Mecanică analitică şi vibraţii*, Editura MATRIX ROM, Bucureşti, 1998.
- [15]. *F. H. Moraes, Jr., B. R. Pontes, M. Silveira*, Influence of ideal and non-ideal excitation sources on the dynamics of a nonlinear vibro-impact system, *Journal of Theoretical and Applied Mechanics*, **vol. 51**, no. 3, 2013, pp. 763–774.
- [16]. *Y. Li, E. Blaisten-Barojas*, Nonlinear coupling between rotation and internal vibration in simple molecular system, *Journal of Physics B: Atomic, Molecular and Optical Physics*, **vol. 30**, 1997, pp. 309–318.
- [17]. *H. E. Kandrup, M. E. Mahon*, *Three-Dimensional Systems*, eds. H.E. Kandrup, S.T. Gottesman and J.R. Ipsen, *Annals of the New York Academy of Sciences*, 751, 1995.
- [18]. *A. C. Eringen*, *Response of tall buildings to random earthquakes*, Purdue University, Division of Engineering Sciences, 1957.