

## TRANSIENT FLOWS WITH INERTIAL EFFECTS IN WATER SUPPLY SYSTEM OF THE HYDROPOWER PLANTS

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*Se adaptează metoda loviturii de berbec algebrice pentru analiza regimului tranzitoriu pe întregul traseu hidraulic dintre lacul de acumulare și turbine. Modelul permite includerea efectelor inerțiale și de frecare ale coloanei de apă din castelul de echilibru și din puțurile aducțiunilor secundare, precum și efectul isipării de energie în conectorii la galeria de aducție. Se ilustrează cu o aplicație numerică la ramura vest a aducțiunilor din amenajarea Cerna – Motru – Tismana.*

*The algebraic waterhammer method has been adapted to transient flow analysis along the whole hydraulic route between reservoir and plant's turbines. The model allows to include the inertial and friction effects of water column into the surge – tank and secondary intake shaft, and the dissipation energy effect in the connectors to headrace gallery as well. A numerical application for the western branch of galleries system from Cerna – Motru – Tismana development is used to illustrate this formulation.*

**Keywords:** transient flow, algebraic waterhammer, inertial effect.

### 1. Introduction

Usually, unsteady flows in the water supply galleries of a hydropower plant are classified as slowly and suddenly variable flow. The first ones occur into the headrace tunnel and surge tank, being known as mass – oscillations. The last category includes the waterhammer, generated by the valve – closure along the penstock. Because of different time scale of hydraulic processes, their analysis is rather artificially separated, and modeled under distinct hypotheses. The mass oscillations are governed by a mathematical model with ordinary differential equations, while the waterhammer is described by a system of partial differential equations.

However, the present – day computers and numerical methods allows an unitary formulation for the two types of transient flows, with no need for supplementary hypotheses and separate analysis of the two aspects of this simultaneous process, occurring into the same physical system – the water supply galleries between reservoir and plant's turbines.

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Numerical methods for modeling transient flow are well illustrated in bibliography (Wylie and Streeter, 1982; Chaudhry, 1987; Fox, 1977; Watters, 1984; Thorley, 1991; Larock, Jeppson and Watters, 1999, etc.) but practical applications are rather devoted to the transients in water distribution networks (Woods, 2005, Karney and McInnis, 1992, Koelle, 1982, etc.).

In this paper, the unsteady regime caused by the valve closure at hydroplant turbines is analyzed along the whole hydraulic route between reservoir and hydropower plant (including the main headrace tunnel, the secondary intake shaft, the main surge – tank and the penstock). The algebraic waterhammer equations proposed by Wylie and Streeter, 1982, are used for the transient flow modeling in galleries, but some more complex boundary conditions are taken into account (lumped inertia and dissipation models for water column evolution in the shaft/ surge – tank or in the connectors between the main gallery and these devices).

The mathematical model provides to estimate the inertial and friction effects upon the time variation of water level in the surge tank / shaft, an aspect impossible to be modeled under the assumption of slowly variable flow for mass – oscillation.

## 2. Mathematical model

To model the transient flow in pipes, a momentum equation and mass conservation relations are used. With  $x$  and  $t$  – distance along the centerline of the pipe, and time respectively as independent variables, these equations can be written (Wylie and Streeter, 1982):

$$\frac{\partial V}{\partial t} + g \frac{\partial H}{\partial x} + \frac{\lambda V|V|}{2D} = 0 \quad (1)$$

$$\frac{\partial H}{\partial t} + \frac{a^2}{g} \frac{\partial V}{\partial x} = 0 \quad (2)$$

where  $H(x, t)$  is the piezometric head;  $V(x, t) = Q/A$  = mean velocity;  $Q(x, t)$  = flow discharge;  $A = \pi D^2/4$  = cross sectional pipe area;  $D$  = pipe diameter;  $\lambda$  = Darcy – Weisbach friction coefficient;  $a$  = celerity of the shock wave;  $g$  = gravity acceleration. The convective and the slope terms are considered negligible.

In the classical method of characteristics (MOC), the momentum and continuity equations are combined to form the compatibility equations in  $Q$  and  $H$  as follows:

$$dH \pm BdQ \pm \frac{r}{\Delta x} Q|Q|dx = 0 \quad (3)$$

in which  $B = \frac{a}{gA}$ ;  $r = \frac{\lambda\Delta x}{2gDA^2}$ , and these relations are valid only along the  $C^+$  and  $C^-$  characteristic lines defined by  $\Delta x = \pm a\Delta t$  on a computational  $x-t$  grid (as shown in Fig.1).

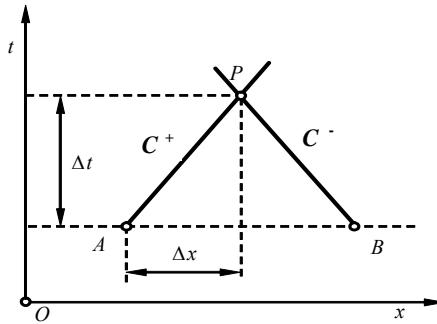


Fig.1. Computational grid for solving transient flow in classical MOC.

Once the flow conditions  $(H, Q)$  are known at time  $t$ , the equation (3) can be integrated along AP and BP to provide two relations for  $H$  and  $Q$  in P, at  $t + \Delta t$ . Karney and McInnis (1992) were proposed for the frictional term integration a form as:

$$\int_A^P Q|Q|dx = \Delta x|Q_A|[Q_A + \varepsilon(Q_P - Q_A)] \quad (4)$$

with  $\varepsilon$  = a linearization constant between 0 and 1 and then

$$H_P = C_P - B_P Q_P; \quad H_P = C_M + B_M Q_P \quad (5)$$

in which:

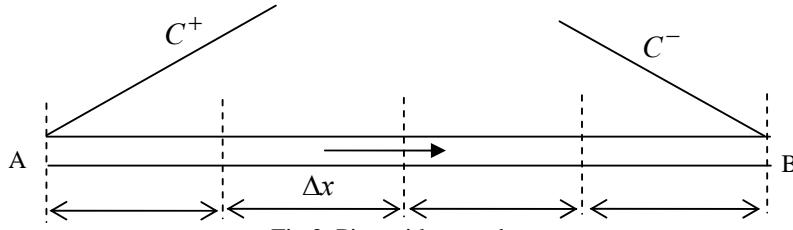
$$\begin{aligned} C_P &= H_A + Q_A [B - r|Q_A|(1 - \varepsilon)]; \quad B_P = B + \varepsilon r|Q_A| \\ C_M &= H_B - Q_B [B - r|Q_B|(1 - \varepsilon)]; \quad B_M = B + \varepsilon r|Q_B| \end{aligned} \quad (6)$$

After elimination of the  $H_P$  value from (5), the  $Q_P$  value is obtained as:

$$Q_P = \frac{C_P - C_M}{B_P + B_M} \quad (7)$$

and then the  $H_P$  value can be easily computed.

The algebraic waterhammer method (AWM) is particularly convenient for transient calculations in piping systems, using the same conception as MOC. The equations for AWM may be applied over several  $\Delta x$  reaches, with no need to compute the transient at the interior sections. For a pipe with  $p$  reaches (fig.2) these equations may be written as follows for time step  $j$ .

Fig.2. Pipe with  $p$  reaches.

$$C^+ : H_B(j) = C_A(j-p) - B_A(j-p)Q_B(j) \quad (8)$$

$$C^- : H_A(j) = C_B(j-p) + B_B(j-p)Q_A(j) \quad (9)$$

in which

$$C_A(j-p) = H_A(j-p) + Q_A(j-p)[B - R|Q_A(j-p)|(1-\varepsilon)] \quad (9)$$

$$C_B(j-p) = H_B(j-p) - Q_B(j-p)[B - R|Q_B(j-p)|(1-\varepsilon)] \quad (9)$$

and

$$B_A(j-p) = B + \varepsilon R|Q_A(j-p)|$$

$$B_B(j-p) = B + \varepsilon R|Q_B(j-p)|$$

where  $R = pr$ .

By starting the  $x-t$  grid with  $p-1$  time steps  $\Delta t$  before the beginning of transient, and storing the steady state values for  $H_A(j)$ ,  $Q_A(j)$ ,  $H_B(j)$ ,  $Q_B(j)$ ,  $j = 1, 2, \dots, p$ , the AWM equations may now be solved for  $j = p+1$ ,  $p+2$ , etc, together with the boundary conditions at ends A and B.

If AWM is used for transient calculations of a piping system as shown in fig.3, the selection of number  $p_1$ ,  $p_2$  and  $p_3$  of the reaches in each pipe must be such that  $\Delta t$  is common, e.g.:

$$\Delta t = \frac{L_1}{p_1 a_1} = \frac{L_2}{p_2 a_2} = \frac{L_3}{p_3 a_3} \quad (10)$$

in which the values of celerity of the shock wave  $a_1$ ,  $a_2$ ,  $a_3$  are to be subjected to small adjustments.

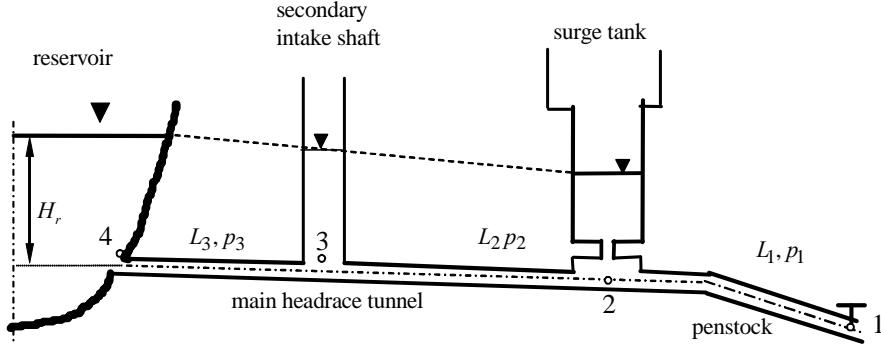


Fig.3. Hydraulic lay – out between reservoir and turbines.

The only sections with transient computations will be then 1, 2, 3 and 4. For section 1, an equation as:

$$C^+ : H_1(j) = C_2(j - p_1) - B_2(j - p_1)Q_1(j)$$

is solved along with a boundary condition for valve closing:

$$Q_1(j) = \frac{Q_0}{\sqrt{H_0}} \sqrt{H_1(j)} \tau(j), \text{ where } \tau = 1 \text{ for initial flow } Q_0, \text{ and } \tau = 0$$

after the valve closure. For section 2, an equation as:

$$C^+ : H_2(j) = C_3(j - p_2) - B_3(j - p_2)Q_2^+(j)$$

and an equation as:

$$C^- : H_2(j) = C_1(j - p_1) + B_1(j - p_1)Q_2^-(j)$$

are to be solved along with the continuity equation:  $Q_2^+ = Q_2^- + Q_c$  (where  $Q^+$ ,  $Q^-$  and  $Q_c$  are the flow discharges upstream, downstream and into the surge tank) and lumped model equations for tank. For section 3 a similar treatment is applied, and for section 4 an equation as

$C^- : H_4(j) = C_3(j - p_3) + B_3(j - p_3)Q_4(j) = H_r$  (if the entrance losses are neglected) is solved. Obviously, in  $C_2$  and  $B_2$  - the  $Q_2^-(j - p_1)$  values must be used; in  $C_3$  and  $B_3$  - the  $Q_3^-(j - p_2)$  values will be used for section 2 and  $Q_3^+(j - p_3)$  values will be used for section 4.

To limit the question under debate only to section 2, a first relation can be written for  $H_2(j)$ :

$$H_2(j) = C_c - B_c Q_c(j) \quad (11)$$

in which

$$B_c = \frac{1}{\frac{1}{B_3(j-p_2)} + \frac{1}{B_1(j-p_1)}}; C_c = B_c \left[ \frac{C_3(j-p_2)}{B_3(j-p_2)} + \frac{C_1(j-p_1)}{B_1(j-p_1)} \right] \quad (12)$$

and where the relations:

$$\begin{aligned} Q_2^+(j) &= \frac{-H_2(j) + C_3(j-p_2)}{B_3(j-p_2)} \\ Q_2^-(j) &= \frac{H_2(j) - C_1(j-p_1)}{B_1(j-p_1)} \\ Q_2^+(j) - Q_2^-(j) - Q_c(j) &= 0 \end{aligned} \quad (13)$$

where used.

When the cross sectional area of the tank/shaft is small, the analysis should account for both head losses and inertia of the water column. The water column is accepted as a lumped element and the equation of motion applied to his displacement give the following relation:

$$H_b(j) - H_w(j) = C'_r + C''_r Q_c(j) \quad (14)$$

in which  $H_b$  = the head at the base of the tank;  $H_w$  = the water surface elevation; and the terms  $C'_r$  and  $C''_r$  = constants over  $\Delta t$  related to inertia and friction effects, i.e.:

$$\begin{aligned} C'_r &= H_w(j-1) - H_b(j-1) - \frac{2l_t(j-1)}{gA_t\Delta t} Q_c(j-1) \\ C''_r &= \frac{2l_t(j-1)}{gA_t\Delta t} + \frac{\lambda_t l_t(j-1)}{gD_t A_t^2} |Q_c(j-1)| \end{aligned} \quad (15)$$

in which  $H_w(j-1) = Z_{bt} + l_t(j-1)$ ;  $Z_{bt}$  = the elevation of the base of the tank;  $l_t(j-1)$  = the length of the water column above  $Z_{bt}$ ;  $D_t$ ,  $A_t$  and  $\lambda_t$  = the tank diameter; cross-sectional area and friction coefficient respectively. Using the continuity equation for the tank with average values over a time step  $\Delta t$ , i.e.:

$$H_w(j) = H_w(j-1) + b_0 [Q_c(j-1) + Q_c(j)]; b_0 = \frac{\Delta t}{2A_t} \quad (16)$$

a relation for  $H_b(j)$  can be written as:

$$H_b(j) = C_b + B_b Q_c(j) \quad (17)$$

with  $C_b = C'_r + H_w(j-1) + b_0 Q_c(j-1)$ ;  $B_b = C''_r + b_0$

If flow passes without any restriction into the tank, then  $H_b(j) \equiv H_2(j)$  and the external flow can be computed using (11) and (17):

$$Q_c(j) = \frac{C_c - C_b}{B_c + B_b} \quad (18)$$

The throttled surge tank (as shown in fig. 3) produces a local head loss and the external flow is related to the head at the junction by the orifice discharge expression:

$$Q_c(j) = s \mu_s \sqrt{s[H_2(j) - H_b(j)]} \quad (19)$$

in which  $s$  takes the sign of the external flow (i.e.,  $s = \text{sign}[Q_c(j)] = \pm 1$ ) and  $\mu_s$  is the orifice parameter (possibly having two values:  $\mu_+$  for flow from the headrace when  $s = 1$ , and  $\mu_-$  for negative flow).

When this connector is modeled only by the  $\mu_s$  term (no as a lumped inertia element), a quadratic equation in  $Q_c(j)$  can be written:

$$Q_c^2(j) + 2\alpha Q_c(j) + \beta = 0 \quad (20)$$

in which  $\alpha = \frac{1}{2} \mu_s^2 s (B_b + B_c)$ ;  $\beta = \mu_s^2 s (C_b - C_c)$

and having as solution:

$$Q_c(j) = -\alpha + s \sqrt{\alpha^2 - \beta} \quad (21)$$

with  $s = \text{sign}(C_c - C_b)$ .

Once the external flow at time step  $j$  is computed, all others interest parameters, i.e.:  $H_b(j)$ ,  $H_w(j)$ ,  $H_2(j)$ ,  $Q_2^+(j)$  and  $Q_2^-(j)$  can also be obtained from the previous relations (17),(16),(11) and (13).

### 3. Numerical application

To illustrate the preceding formulation, a simple hydraulic system having the geometrical data as the western branch of the headrace tunnels in the Cerna – Motru-Tismana development is analyzed. A lay-out as in fig.3, with Pocruiua secondary intake shaft ( $D_t = 3$  m,  $Z_{b_t} = 442$  m,  $\mu_s = 12,9$ ,  $\lambda_t = 0,012$ ), Tismana surge tank ( $D_{t1} = 7$  m - shaft,  $D_{t2} = 17$  m - upper chamber,  $Z_{b_t} = 427,7$  m,  $\mu_s = 4,9$ ,  $\lambda_t = 0,012$ ), ending valve at  $Z = 217$  m and water surface elevation at reservoir between 465 and 480 m were considered. The lengths of the galleries are  $L_1 = 1000$  m;  $L_2 = 3030$  m, and  $L_3 = 5550$  m, all having the diameter  $D = 3,6$  m, and a friction coefficient  $\lambda = 0,018$ .

A time step  $\Delta t = 0,5$  s is used and then:  $p_1 = 2$ ,  $a_1 = 1000$  m/s,  $p_2 = 6$ ,  $a_2 = 1010$  m/s and  $p_3 = 11$ ,  $a_3 = 1009$  m/s. The valve closure is assigned as

$\tau(t) = (1 - t/t_c)^2$ , with closing time  $t_c = 3$  s, and the initial flow discharge was  $Q_0 = 36 \text{ m}^3 \text{s}^{-1}$ .

In fig.4 are shown the time variations over 1000 s of the water surface elevation in surge tank and secondary intake shaft for water level of 468, 474 and 480 m at reservoir.

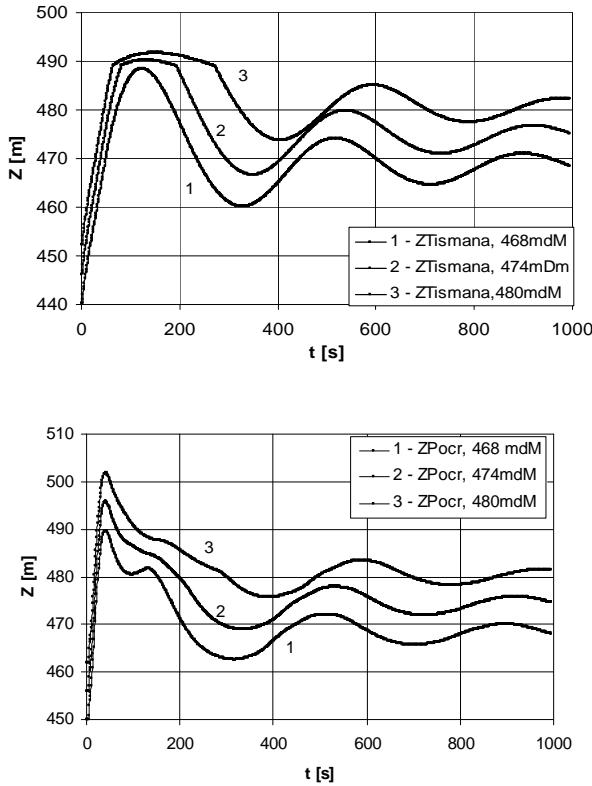


Fig. 4 – Time variations of water surface elevation at Tismana and Pocruiua for 3 water levels in reservoir.

In fig. 5 a comparison between the two cases: with and without ( $C'_r$  and  $C''_r$  equal to zero in the model) inertial and friction effects in the shaft and surge tank may be observed – for water level in reservoir at 468m.

The surge tank oscillations are damped and delayed, but maintain the same form, while these oscillations are strongly influenced by inertial effects in the shaft.

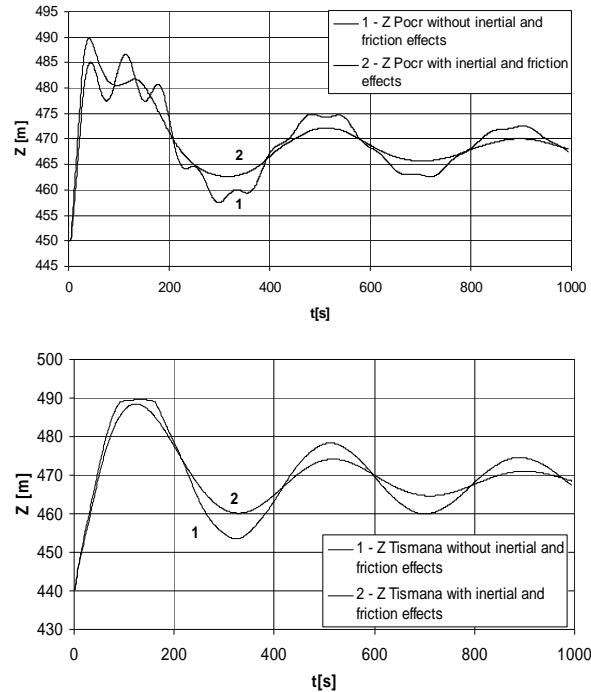


Fig.5. Inertial and friction effects on the water column oscillation in Pocrui shaft/Tismana tank.

The discharge time-variation into the surge-tank and secondary intake shaft are shown in fig. 6, for water level in reservoir at 468m and without inertial effects.

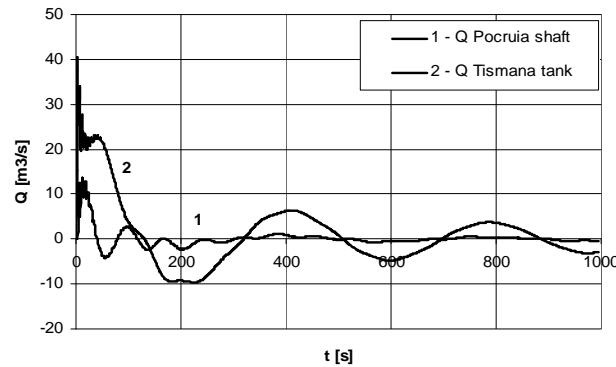


Fig.6. Time-variation of the discharge into the surge-tank and Pocrui shaft (468m at reservoir, without inertial effects).

#### 4. Conclusions

The aim of this paper was to provide a unified and systematic formulation of transient flow in hydrotechnical galleries connected to various other devices (surge-tank, secondary intake shaft, valve, energy dissipator element, etc)

An algebraic waterhammer method has been adapted to such a hydraulic system, and used to illustrate the inertial and friction influence upon the water column evolution in surge-tank /shaft after the valve-closure at hydropower plant.

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