

## NOTES ON VOLTAGE DROP ANALYSIS

P. VLADU<sup>1</sup>

*În această lucrare sunt propuse formule de calcul pentru căderile de tensiune în sisteme cu sarcini distribuite. Conceptul de sursă ideală de curent permite o analiză tabelară ce poate fi folosită în practica inginerescă.*

*Using the concept of current sink to represent distributed loads, formulae for voltage drop analysis are drawn. Suitable to spreadsheet treatment this approach should be interesting for the practicing engineer.*

**Keywords:** voltage drop, current sink, ideal current generator.

### 1. Introduction

The modern highways represent a sizeable amount of investment dollars for public authorities. The lighting systems associated with highways represent a fraction of the roadway construction costs, in relative terms, but the absolute numbers may be not that trivial. The detailed analysis for voltage drop of lighting feeders should help in optimizing the cost structure of these systems.

Certainly, the voltage drop analysis is an elementary topic in electrical engineering. So elementary that sometimes is neglected. This paper is intended to show that basic electricity methods are still productive.

### 2. Modeling and analysis

Let's assume a simple lighting system, with  $T$  luminaires, of different wattages, spaced at various lengths  $d$  ( $k, k-1$ ),  $k=1,2\dots T$  and distributed on three phases A, B, C. The maximum number of luminaires on a phase is  $M$ , with  $M=1, 2\dots$ . Then it is obvious that

$$T = 3 * M \pm 1 \quad [1]$$

Each luminaire can be represented as a current sink. That is, luminaire  $k$  will drain the current

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<sup>1</sup> Senior Engineer, AG Consulting Engineers, New York, NY USA, psv@ix.netcom.com

$$i_k = \frac{P_k^{nom}}{E_{knom}}, \quad [2]$$

where  $P_{knom}$  is the nominal wattage of luminaires  $k$  lamp and  $E_{knom}$  is its nominal voltage. The fact that present Electrical codes {1} allow a maximum voltage drop of 3% at the far end luminaires permits this substitution without too much loss of precision.

As a matter of fact this is a good start for feeder optimal sizing.

Due to the multi-phase connection, the current thru the phase wire is different in value from the current thru the neutral wire, as is well known.

The simple model shown in Fig.1 is good enough to carry on the classical Kirkhoff analysis. For the last node, T, the voltage loop equation is:

$$0 = i_{T,T-1}^W * (Z_{T,T-1}^W + Z_{T-1,T-2}^W + Z_{T-2,T-3}^W) - (i_{T,T-1}^N * Z_{T,T-1}^N + i_{T-1,T-2}^N * Z_{T-1,T-2}^N + i_{T-2,T-3}^N * Z_{T-2,T-3}^N) \quad [3]$$

The first term in [3] is the phase voltage drop. The second term is the neutral voltage drop. A spreadsheet can be arranged to iterate [3] for the whole system

The Kirkhoff analysis current equation, for nodes  $T, T-1, T-2, T-3$  shows that

$$i_{T,T-1}^W = i_{T,T-1}^N = i_T \quad [4]$$

$$i_{T-1,T-2}^N = i_{T,T-1}^N + i_{T-1} \quad [5]$$

$$i_{T-2,T-3}^N = i_{T-1,T-2}^N + i_{T-2} \quad [6]$$

By substituting [4] and [5] in [6] we show that

$$i_{T-2,T-3}^N = i_T + i_{T-1} + i_{T-2} \quad [7]$$

This result is banal, but equations [3] and [7] are useful for tabulating partial voltage drop values, for a variety of wire sizes and luminaire wattages. In practice the luminaires have the same wattages and are almost equidistant located, for various illumination and construction reasons. Also the practice of using different wire gauges for different feeder sections is not recommended. If so,

$$\sum_{k=1,2,3,\dots}^T i_{K,K-1}^N = 2 * i_K \quad [8]$$

(for unbalanced load) or zero (balanced load).

### 3. Results

Repeating [3]  $M$  times, for all luminaries on same phase and using the simplification used in practice, the phase voltage drop can be approximated as

$$\Delta V^W = 3 * Z_{K,K-1} * \sum_{K=3,\dots,N} i_K = 3 * Z_{K,K-1} * i_K * (1 + 2 + 3 + \dots + M) \quad [9]$$

that is

$$\Delta V = 3 * Z_{K,K-1} * i_K * \frac{M * (M + 1)}{2}, \quad [10]$$

The neutral voltage drop will be maximum

$$\Delta V^N = 2 * Z_K * i_K * \left( \sum_t 1 \right) = 2 * T * i_K * Z_K \quad [11]$$

Formulae [7] and [11] allow a rapid evaluation of voltage drop for distribution systems with uniform loads.

### 4. Conclusions

The results have no special theoretical value but are rather useful for practical design. The substitution [2] allows for piecewise classic analysis which can be iterated to yield system wide results. Spreadsheets based upon the above considerations have been used recent public works.

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### R E F E R E N C E S

- [1]. National Electrical Code ed. 2002, NEC, Quincy, MA-used as NYC electrical code:
- [2]. M. Iordache, V. Iorache, P. Ene, Construcția și exploatarea instalațiilor de iluminat public, Editura Tehnică, București, 1977.

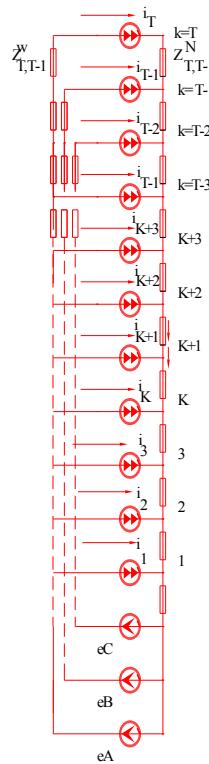


Fig.1. Analysis model

## Legend

$Z_{K,K-1}^w$  = Phase wire impedance between nodes  $K$  and  $K-1$

$Z_{K,K-1}^n$  = Neutral wire impedance between nodes  $K$  and  $K-1$

$i_{K,K-1}^w$  = Phase current flowing between nodes  $K$  and  $K-1$

$i_K$  = Load at node  $K$ , presented as an ideal current generator