

NUMERICAL STUDY FOR AN ELASTIC-VISCOPLASTIC MODEL IN GEOMECHANICS

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Se studiază variația tensiunilor în jurul unei deschideri circulare verticale excavată în sare. Este utilizat un model elasto-viscoplastic neasociat care descrie compresibilitatea și/sau dilatanta pe durata fluajului tranzitoriu și staționar și care, de asemenea, descrie deteriorarea care poate conduce la rupere. Am folosit metoda elementului finit și o metoda iterativă adaptată problemei rezultate. Soluția numerică obținută este comparată cu soluția simplificată (soluția pentru fluaj) și cu soluția elastică (răspunsul instantaneu). Se studiază variația în timp a razei deschiderii și a stării de tensiune.

The variation of stress during creep convergence of a deep borehole excavated in rock salt is studied. A non-associated elasto-viscoplastic constitutive equation is used to describe both compressibility and/or dilatancy during transient and steady-state creep, as well as evolutive damage possibly leading to failure. An in-house FEM numerical method and iterative method is used for this purpose. The obtained numerical solution is compared with the simplified solution (creep solution), and with elastic solution (instantaneous response). The variation in time of radial convergence of the borehole walls and of the stress state is studied.

Keywords: elastic-viscoplastic model, rock mechanics, numerical methods.

1. Introduction

In this paper we study the distribution of stresses, deformations and displacement around a circular cylindrical borehole. This problem has been studied by several authors analyzed various aspects of them. In some cases it was used together with the linear elasticity assumption of plane state of stress (see [1]). Also with the assumption of plane state of stress the problem has been studied by Massier in [2], but with linear viscoelastic model. To describe the creep of rocks around the borehole, stress relaxation, damage of rocks near the border may be used different constitutive equations. The study of this problem with an elasto-viscoplastic model like (1) is made by Cristescu [3], Cristescu and Hunche [4], Paraschiv and Cristescu [5], considering the plane state of deformation and for determining the creep solution.

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Using standard notation (see [3], [6]), the elasto-viscoplastic model that we consider in this paper is described by equation

$$\dot{\boldsymbol{\epsilon}} = \frac{\boldsymbol{\sigma}^R}{2G} + \left(\frac{1}{3K} - \frac{1}{2G} \right) \dot{\boldsymbol{\sigma}}^R \mathbf{1} + k_T \left(1 - \frac{W^I(t)}{H(\boldsymbol{\sigma})} \right) \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}) + k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}), \quad (1)$$

where K and G are elastic moduli, k_T and k_S are viscosity constants, H is the plasticity function, F is the viscoplastic potential for transitory creep, S is the viscoplastic potential for stationary creep, W^I is the irreversible stress work per unit volume (the internal status parameter) given by

$$W^I(t) = W^{IP} + \int_0^t \boldsymbol{\sigma}(s) \cdot \dot{\boldsymbol{\epsilon}}^I(s) ds,$$

where W^{IP} is the primary value of W^I .

In this paper we are compare the elastic solution with the simplified solution for creep and the numerical solution as in [7] with the difference that in this case we used the functions of the software package MATLAB. The numerical solution obtained can be considered a complete solution because unlike the simplified solution for creep, that solution shall take account of stress variation in time. In most cases, note that near the opening takes place a stress relaxation is comparable to instantaneous response. For short intervals of time, the simplified solution for creep approximates well the behavior of rocks because is very close to the numerical solution.

2. Mechanical problem formulation

We assume that the problem to solve is formulated in cylindrical coordinate (\mathbf{r}, θ, z) . Because it is assumed the plane state of deformation, the domain for the studying problem is a section at depth \mathbf{h} and is represented in figure 1. Suppose that in all orizontal directions the primary stress is the same, $\boldsymbol{\sigma}_h$, and the depth \mathbf{h} is sufficient great to consider that $\boldsymbol{\sigma}_v$, the vertical primary stress, is not variable in the domain. Practically, the problem is not formulated for circular crown because of axial symmetry the variables are not depending on θ , they are depending only on \mathbf{r} . Let a the initial radius of the vertical borehole and $m \in N, m \geq 5$, number of radius which defined the limits of the domain, $[a, ma]$. Also we take consider small deformations. So, we obtain

$$\begin{aligned} \mathbf{u}_\theta &= \mathbf{u}_z = \mathbf{0}; \frac{\partial}{\partial \theta} = \mathbf{0}; \frac{\partial}{\partial z} = \mathbf{0}; \\ \epsilon_{rr} &= \frac{\partial u_r}{\partial r}; \epsilon_{\theta\theta} = \frac{u_r}{r}; \epsilon_{rz} = \epsilon_{r\theta} = \epsilon_{zz} = \epsilon_{z\theta} = \mathbf{0}. \end{aligned}$$

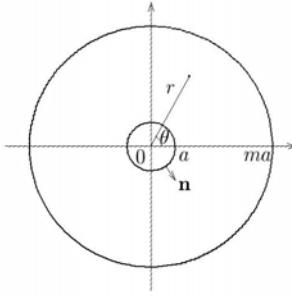


Fig 1. The domain of problem.

Formulation of the problem for determining the stress distribution around a vertical borehole in elasto-viscoplastic rock, as a quasi-static problem with the internal status parameter (see [7], [8]), is:

to determine the displacement function $u_r: \mathbf{R}_+ \times [a, ma] \rightarrow \mathbf{R}$, the stress function $\sigma: \mathbf{R}_+ \times [a, ma] \rightarrow S_3$ and the internal parameter $W^I: \mathbf{R}_+ \times [a, ma] \rightarrow \mathbf{R}$ such that

$$\operatorname{Div} \sigma^R(t, r) = \mathbf{0} \quad \text{in } \mathbf{R}_+ \times [a, ma] , \quad (2)$$

$$\begin{aligned} \dot{\sigma}^R &= 2G\dot{\epsilon} + (3K - 2G)\dot{\epsilon} \mathbf{1} + k_T \left(1 - \frac{W^I}{H(\sigma)}\right) \left[\frac{(2G - 3K)}{3} \frac{\partial F}{\partial \sigma} \mathbf{1} - 2G \frac{\partial F}{\partial \sigma} \right] \\ &\quad + k_S \left[\frac{(2G - 3K)}{3} \frac{\partial S}{\partial \sigma} \mathbf{1} - 2G \frac{\partial S}{\partial \sigma} \right] \quad \text{in } \mathbf{R}_+ \times [a, ma] , \end{aligned} \quad (3)$$

$$\dot{W}^I = k_T \left(1 - \frac{W^I}{H(\sigma)}\right) \frac{\partial F}{\partial \sigma} \cdot \sigma + k_S \frac{\partial S}{\partial \sigma} \cdot \sigma \quad \text{in } \mathbf{R}_+ \times [a, ma] , \quad (4)$$

$$\begin{cases} \sigma_{rr}^R(t, a) = p - \sigma_{rr}^P , \quad \sigma_{r\theta}^R(t, a) = \sigma_{rz}^R(t, a) = 0 , \quad \forall t > 0 , \\ u_r(t, ma) = 0 , \quad \forall t > 0 , \end{cases} \quad (5)$$

$$\begin{cases} \sigma^S(\mathbf{0}, r) = \sigma^P + \tilde{\sigma} \quad \text{or} \quad \sigma^R(\mathbf{0}, r) = \tilde{\sigma} \\ u_r(\mathbf{0}, r) = \tilde{u} \\ W^I(\mathbf{0}, r) = H(\sigma^P) \end{cases} , \quad \forall r \in [a, ma] , \quad (6)$$

where $\tilde{\sigma}$ and \tilde{u} are stress, respectively, radial displacement corresponding to instantaneous response, $\sigma = \sigma^S = \sigma^R + \sigma^P$, $\epsilon = \epsilon^R$, p is the pressure on the inner wall of the borehole.

3. Instantaneous response

The stress distribution after excavation is obtained by exact elastic solution (the instantaneous response). The problem is:

to determine the displacement function $\tilde{u}: [a, ma] \rightarrow \mathbf{R}$, the stress function $\tilde{\sigma}^R: [a, ma] \rightarrow S_3$ such that

$$\begin{cases} \operatorname{Div} \tilde{\sigma}^R(r) = \mathbf{0} & \text{in } [a, ma] , \\ \tilde{\sigma}^R = 2G\tilde{\epsilon} + (3K - 2G)\tilde{\epsilon} \mathbf{1} & \text{in } [a, ma] , \\ \tilde{\sigma}^R(a)\mathbf{n} = p\mathbf{n} - \sigma^P \mathbf{n} , \quad \tilde{u}(ma) = 0 . \end{cases} \quad (7)$$

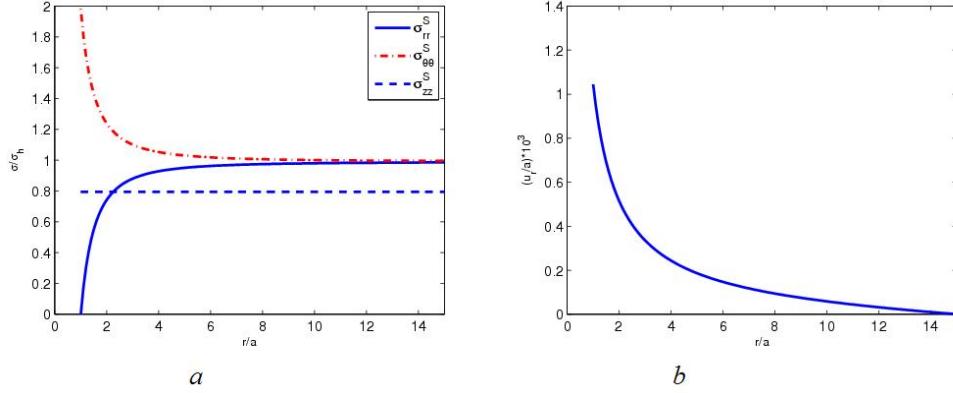


Fig 2. Stress and radial displacement corresponding with the instantaneous response.

Solving the problem (7) is obtained

$$\begin{aligned} \tilde{\sigma}_{rr}^R(r) &= \left[\tilde{N} + (1 - \tilde{N}) \frac{a^2}{r^2} \right] (p - \sigma_h) , \\ \tilde{\sigma}_{θθ}^R(r) &= \left[\tilde{N} - (1 - \tilde{N}) \frac{a^2}{r^2} \right] (p - \sigma_h) , \\ \tilde{\sigma}_{zz}^R(r) &= \frac{3K - 2G}{G + 3K} \tilde{N} (p - \sigma_h) , \quad \tilde{\sigma}_{rθ}^R(r) = \tilde{\sigma}_{rz}^R(r) = \tilde{\sigma}_{θz}^R(r) = 0 , \end{aligned} \quad (8)$$

and

$$\tilde{u}(r) = \frac{1}{2G} \left[\left(1 - \frac{3K - 2G}{G + 2K} \right) \tilde{N} - (1 - \tilde{N}) \frac{a^2}{r^2} \right] r (p - \sigma_h) , \quad (9)$$

where

$$\tilde{N} = \frac{(G + 3K)}{(G + 3K) + 3Gm^2} .$$

Instantaneous response is represented in figure 2.

4. Elasto-viscoplastic creep. Simplified solution

In case of elasto-viscoplastic creep (see [3], [4], [5]) we assume that in $[t_0, T]$ the stress components are constants equal with the instantaneous response given by (8):

$$\sigma^R(t) = \tilde{\sigma}^R , \quad \forall t \in [t_0, T] .$$

Of (1) is obtained

$$\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}^I = k_T \left\langle 1 - \frac{W^I(t)}{H(\boldsymbol{\sigma}^S)} \right\rangle \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) + k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S),$$

where $\boldsymbol{\sigma}^S = \tilde{\boldsymbol{\sigma}}^R + \boldsymbol{\sigma}^P$.

Using (4) it results

$$\left\langle 1 - \frac{W^I(t)}{H(\boldsymbol{\sigma}^S)} \right\rangle = - \frac{k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S}{k_T \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S} + \left(\left\langle 1 - \frac{W^{IP}}{H(\boldsymbol{\sigma}^S)} \right\rangle + \frac{k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S}{k_T \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S} \right) \exp \left[- \frac{k_T}{H(\boldsymbol{\sigma}^S)} \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S (t - t_0) \right],$$

from which is obtained the following equation

$$\dot{\boldsymbol{\epsilon}}^I = k_T \left\{ - \frac{k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S}{k_T \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S} + \left(\left\langle 1 - \frac{W^{IP}}{H(\boldsymbol{\sigma}^S)} \right\rangle + \frac{k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S}{k_T \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S} \right) \exp \left[- \frac{k_T}{H(\boldsymbol{\sigma}^S)} \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S (t - t_0) \right] \right\} \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) + k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S),$$

that integrate with the condition, $\boldsymbol{\epsilon}^I(t_0, r) = \tilde{\boldsymbol{\epsilon}}(r)$, corresponding with the instantaneous response. By integration we obtain

$$\begin{aligned} \boldsymbol{\epsilon}^I(t, r) = \tilde{\boldsymbol{\epsilon}}(r) &+ \left(- \frac{k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S}{\frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S} \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) + k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \right) (t - t_0) + \\ &+ \frac{H(\boldsymbol{\sigma}^S) \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S)}{\frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S} \left(\left\langle 1 - \frac{W^{IP}}{H(\boldsymbol{\sigma}^S)} \right\rangle + \frac{k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S}{k_T \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S} \right) \\ &\left\{ 1 - \exp \left[- \frac{k_T}{H(\boldsymbol{\sigma}^S)} \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S (t - t_0) \right] \right\}, \quad \text{for } t_0 \leq t \leq t_S, \end{aligned}$$

and

$$\begin{aligned} \boldsymbol{\epsilon}^I(t, r) = \tilde{\boldsymbol{\epsilon}}(r) &+ \left(- \frac{k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S}{\frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S} \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) + k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \right) (t_S - t_0) + \\ &+ \frac{H(\boldsymbol{\sigma}^S) \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S)}{\frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S} \left(\left\langle 1 - \frac{W^{IP}}{H(\boldsymbol{\sigma}^S)} \right\rangle + \frac{k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S}{k_T \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S} \right) \\ &\left\{ 1 - \exp \left[- \frac{k_T}{H(\boldsymbol{\sigma}^S)} \frac{\partial F}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) \cdot \boldsymbol{\sigma}^S (t_S - t_0) \right] \right\} + k_S \frac{\partial S}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma}^S) (t - t_S), \quad \text{for } t \geq t_S, \end{aligned}$$

where t_S is the time of creep stabilization which is obtain for $W^I \rightarrow H(\boldsymbol{\sigma}^S)$, so

$$t_s = t_0 - \frac{H(\sigma^S)}{\frac{\partial F}{\partial \sigma}(\sigma^S) \cdot \sigma^S} \ln \left(\frac{k_s \frac{\partial S}{\partial \sigma}(\sigma^S) \cdot \sigma^S}{\langle 1 - \frac{W^{IP}}{H(\sigma^S)} \rangle \left(k_T \frac{\partial F}{\partial \sigma}(\sigma^S) \cdot \sigma^S \right) + k_s \frac{\partial S}{\partial \sigma}(\sigma^S) \cdot \sigma^S} \right).$$

For radial displacement we have $u_r(t, r) = r\epsilon_{\theta\theta}(t, r)$. In figure 3 are represented deformations and displacement corresponding the solution in case of creep when compared with the instantaneous response given in (8) and (9). It was considered an example of constitutive equation for salt (see [3]).

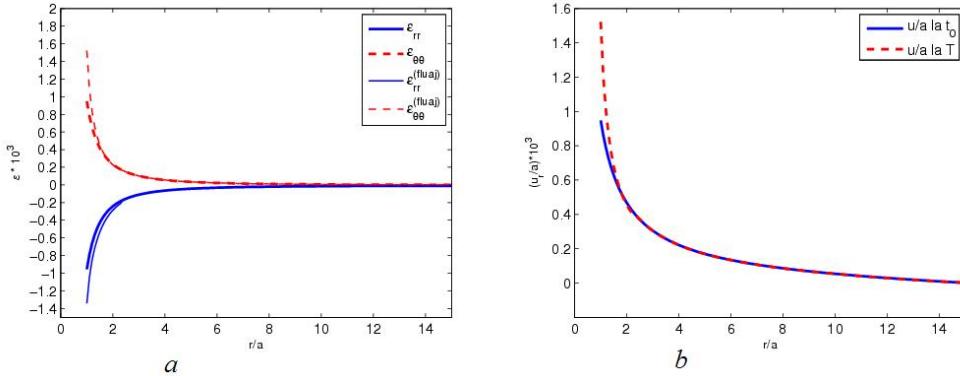


Fig. 3. Comparation between elastic solution and simplified creep solution.

5. Numerical solution

If (u, σ^R) is the solution of the problem (2)-(6) and we denote $\bar{u} = u - \tilde{u}$ and $\bar{\sigma} = \sigma^R - \tilde{\sigma}^R$ then we obtain

$$\begin{cases} \text{Div } \bar{\sigma}(t, r) = 0, & \forall t > 0, \quad r \in [a, ma], \\ \bar{u}(t, ma) = 0, \quad \bar{\sigma}(t, a)\mathbf{n} = 0, & \forall t > 0. \end{cases} \quad (10)$$

So, it remains to solve the problem:

to determine the displacement function $\bar{u}: \mathbf{R}_+ \times [a, ma] \rightarrow \mathbf{R}$, the stress function $\bar{\sigma}: \mathbf{R}_+ \times [a, ma] \rightarrow S_3$ and the internal parameter $W^I: \mathbf{R}_+ \times [a, ma] \rightarrow \mathbf{R}$ such that

$$\begin{aligned} \dot{\bar{\sigma}} &= 2G\epsilon(\dot{\bar{u}}) + (3K - 2G)\epsilon(\dot{\bar{u}}) \mathbf{1} + \\ & k_T \langle 1 - \frac{W^I(t)}{H(\bar{\sigma} + \tilde{\sigma} + \sigma^P)} \rangle \left[\frac{2G - 3K}{3} \frac{\partial F}{\partial \sigma}(\bar{\sigma} + \tilde{\sigma} + \sigma^P) \mathbf{1} - 2G \frac{\partial F}{\partial \sigma}(\bar{\sigma} + \tilde{\sigma} + \sigma^P) \right] \\ & + k_s \left[\frac{2G - 3K}{3} \frac{\partial S}{\partial \sigma}(\bar{\sigma} + \tilde{\sigma} + \sigma^P) \mathbf{1} - 2G \frac{\partial S}{\partial \sigma}(\bar{\sigma} + \tilde{\sigma} + \sigma^P) \right], \text{ in } \mathbf{R}_+ \times [a, ma], \\ \dot{W}^I &= k_T \langle 1 - \frac{W^I(t)}{H(\bar{\sigma} + \tilde{\sigma} + \sigma^P)} \rangle \frac{\partial F}{\partial \sigma}(\bar{\sigma} + \tilde{\sigma} + \sigma^P) \cdot (\bar{\sigma} + \tilde{\sigma} + \sigma^P) + \\ & k_s \frac{\partial S}{\partial \sigma}(\bar{\sigma} + \tilde{\sigma} + \sigma^P) \cdot (\bar{\sigma} + \tilde{\sigma} + \sigma^P), \quad \text{in } \mathbf{R}_+ \times [a, ma], \\ \bar{\sigma}(t_0, r) &= 0, \quad \bar{u}(t_0, r) = 0, \quad W^I(t_0, r) = H(\sigma^P), \quad \forall r \in [a, ma]. \end{aligned} \quad (11)$$

According with theory of quasi-static process (see [7], [8]) we denote

$$\mathbf{V}_1 = \{v = (v_1, 0, 0) \mid v_1 \in L^2(\Omega), v_1 = v_1(r), v_1(ma) = 0\}, \quad (12)$$

$$\mathbf{V}_2 = \{\boldsymbol{\sigma} \in [L^2(\Omega)]_S^{3 \times 3} \mid \boldsymbol{\sigma} = \boldsymbol{\sigma}(r), \text{Div } \boldsymbol{\sigma} = 0, \boldsymbol{\sigma}(a)\mathbf{n} = 0\}. \quad (13)$$

Obviously, the solution $(\bar{u}, \bar{\boldsymbol{\sigma}}, W^I)$ for problem (10)-(11) has the properties $(\bar{u}, 0, 0) \in \mathbf{V}_1$, and $\bar{\boldsymbol{\sigma}} \in \mathbf{V}_2$. Initial conditions from (11) are correspond to the difference between elastic response and instantaneous response on $[0, t_0]$. For computation we assume that $t_0 = 0$ because in $[0, t_0]$ changes do not occur.

Determine the numerical solution on the interval $[0, T]$. Let $M \in \mathbf{N}$, $M \geq 2$ and $\Delta t = \frac{T}{M}$. In $[0, T]$ consider the moments of time

$$t_0 = 0, t_{n+1} = t_n + \Delta t, n = \overline{0, M-1}.$$

To determine an approximate solution \bar{u}_h^n for \bar{u} we consider a finite dimensional subspace $\mathbf{V}_h \subset \tilde{\mathbf{V}}_1 = \{v \in L^2(a, ma) \mid v(ma) = 0\}$, $\dim \mathbf{V}_h = I$. Let $\bar{u}_h^0 = 0$ and consider that

$$\exists! (\alpha_j^{n+1})_{j=\overline{1,I}} \subset R \text{ such that } \bar{u}_h^{n+1} = \sum_{j=1}^I \alpha_j^{n+1} \varphi_j, \quad \forall n = \overline{0, M-1}, \quad (14)$$

where $B = \{\varphi_1, \dots, \varphi_I\} \subset \mathbf{V}_h$ is a basis in \mathbf{V}_h .

Assumed that \mathbf{V}_h is a finite dimensional space obtained by finite element method. The coefficients $(\alpha_j^{n+1})_{j=\overline{1,I}}$ are obtained from the following linear system

$$\sum_{j=1}^I R_{ij} \bar{\alpha}_j^{n+1} = \sum_{j=1}^I R_{ij} \bar{\alpha}_j^n - \Delta t T_i, \quad i = \overline{1, I}, \quad (15)$$

where

$$\begin{aligned} \bar{\alpha}_j^n &= \frac{\alpha_j^n}{a}, \quad j = \overline{1, I}, \\ R_{ij} &= \frac{3K + 4G}{3} \left(\int_1^m s \frac{\partial \varphi_j}{\partial s} \frac{\partial \varphi_i}{\partial s} ds + \int_1^m \frac{1}{s} \varphi_j \varphi_i ds \right) \\ &\quad + \frac{3K - 2G}{3} \int_1^m \left(\varphi_j \frac{\partial \varphi_i}{\partial s} + \frac{\partial \varphi_j}{\partial s} \varphi_i \right) ds, \quad i, j = \overline{1, I}, \\ T_i &= k_T \left\{ \int_1^m s \left\langle 1 - \frac{(W^I)_h^n}{H(\bar{\boldsymbol{\sigma}}_h^n + \tilde{\boldsymbol{\sigma}} + \boldsymbol{\sigma}^P)} \right\rangle \left[\frac{2G - 3K}{3} \frac{\partial F}{\partial \boldsymbol{\sigma}} (\bar{\boldsymbol{\sigma}}_h^n + \tilde{\boldsymbol{\sigma}} + \boldsymbol{\sigma}^P) \right. \right. \\ &\quad \left. \left. - 2G \frac{\partial F}{\partial \sigma_{rr}} (\bar{\boldsymbol{\sigma}}_h^n + \tilde{\boldsymbol{\sigma}} + \boldsymbol{\sigma}^P) \right] \frac{\partial \varphi_i}{\partial s} ds + \right. \\ &\quad \left. \int_1^m \left\langle 1 - \frac{(W^I)_h^n}{H(\bar{\boldsymbol{\sigma}}_h^n + \tilde{\boldsymbol{\sigma}} + \boldsymbol{\sigma}^P)} \right\rangle \left[\frac{2G - 3K}{3} \frac{\partial F}{\partial \boldsymbol{\sigma}} (\bar{\boldsymbol{\sigma}}_h^n + \tilde{\boldsymbol{\sigma}} + \boldsymbol{\sigma}^P) \right. \right. \\ &\quad \left. \left. - 2G \frac{\partial F}{\partial \sigma_{\theta\theta}} (\bar{\boldsymbol{\sigma}}_h^n + \tilde{\boldsymbol{\sigma}} + \boldsymbol{\sigma}^P) \right] \varphi_i ds \right\} + \end{aligned}$$

$$k_S \left\{ \int_1^m s \left[\frac{2G - 3K}{3} \frac{\partial S}{\partial \sigma} (\bar{\sigma}_h^n + \tilde{\sigma} + \sigma^P) - 2G \frac{\partial S}{\partial \sigma_{rr}} (\bar{\sigma}_h^n + \tilde{\sigma} + \sigma^P) \right] \frac{\partial \varphi_i}{\partial s} ds + \right. \\ \left. \int_1^m \left[\frac{2G - 3K}{3} \frac{\partial S}{\partial \sigma} (\bar{\sigma}_h^n + \tilde{\sigma} + \sigma^P) - 2G \frac{\partial S}{\partial \sigma_{\theta\theta}} (\bar{\sigma}_h^n + \tilde{\sigma} + \sigma^P) \right] \varphi_i ds \right\}, \\ i = \overline{1, I}.$$

To compute stress and internal parameter we will consider that $\bar{\sigma}_h^0 = 0$ and $(W^I)_h^0 = H(\sigma^P)$ and determine the $\bar{\sigma}_h^{n+1}$ and $(W^I)_h^{n+1}$ for $n = \overline{0, M-1}$ from following equalities

$$\bar{\sigma}_h^{n+1} = \bar{\sigma}_h(t_{n+1}), \quad (W^I)_h^{n+1} = (W^I)_h(t_{n+1}), \quad (16)$$

where $\bar{\sigma}_h$ and $(W^I)_h$ verify the following problem with initial conditions:

$$\left\{ \begin{array}{l} \dot{\bar{\sigma}}_h = 2G[\epsilon(\bar{u}_h^{n+1}) - \epsilon(\bar{u}_h^n)] - (3K - 2G)[\epsilon(\bar{u}_h^{n+1}) - \epsilon(\bar{u}_h^n)] \mathbf{1} + \\ \Delta t k_T \langle 1 - \frac{(W^I)_h}{H(\bar{\sigma}_h + \tilde{\sigma} + \sigma^P)} \rangle \left[\frac{2G - 3K}{3} \frac{\partial F}{\partial \sigma} (\bar{\sigma}_h + \tilde{\sigma} + \sigma^P) \mathbf{1} - \right. \\ \left. 2G \frac{\partial F}{\partial \sigma} (\bar{\sigma}_h + \tilde{\sigma} + \sigma^P) \right] + \\ \Delta t k_S \left[\frac{2G - 3K}{3} \frac{\partial S}{\partial \sigma} (\bar{\sigma}_h + \tilde{\sigma} + \sigma^P) \mathbf{1} - 2G \frac{\partial S}{\partial \sigma} (\bar{\sigma}_h + \tilde{\sigma} + \sigma^P) \right], \\ (W^I)_h = \Delta t k_T \langle 1 - \frac{(W^I)_h}{H(\bar{\sigma}_h + \tilde{\sigma} + \sigma^P)} \rangle \frac{\partial F}{\partial \sigma} (\bar{\sigma}_h + \tilde{\sigma} + \sigma^P) \cdot (\bar{\sigma}_h + \tilde{\sigma} + \sigma^P) \\ + \Delta t k_S \frac{\partial S}{\partial \sigma} (\bar{\sigma}_h + \tilde{\sigma} + \sigma^P) \cdot (\bar{\sigma}_h + \tilde{\sigma} + \sigma^P), \quad t \in [t_n, t_{n+1}], \\ \bar{\sigma}_h(t_n) = \bar{\sigma}_h^n, \\ (W^I)_h(t_n) = (W^I)_h^n. \end{array} \right. \quad (17)$$

The numerical solution is obtained based on following algorithm:

Input: the functions H, F, S and corresponding constants ([7]);

σ^P = primary stress;

T = length of time;

M = number of steps in time;

m = number of radius;

I = dimension of space \mathbf{V}_h ;

$B = \{\varphi_1, \dots, \varphi_I\}$ = a basis in \mathbf{V}_h ;

Initializations: $\Delta t = \frac{T}{M}$, $\Delta r = \frac{m-1}{I-1}$;

$t_0 = 0$, $t_{n+1} = t_n + \Delta t$, $n = \overline{0, M-1}$;

$\bar{\alpha}_j^0 = 0$, $j = \overline{1, I}$;

$(\epsilon_{rr})_h^n(j) = 0$, $(\epsilon_{\theta\theta})_h^n(j) = 0$, $j = \overline{1, I}$;

$\bar{\sigma}_h^0 = 0$, $(W^I)_h^0 = H(\sigma^P)$;

Compute the instantaneous response $\tilde{\sigma}$, \tilde{u} using (8) and (9) ;

For $n = \overline{0, M-1}$ do

Compute $(\bar{\alpha}_j^{n+1})_{j=\overline{1,I}}$ from system (15);

Compute

$$(\epsilon_{rr})_h^{n+1}(j) = \frac{\bar{\alpha}_{j+1}^{n+1} - \bar{\alpha}_j^{n+1}}{\Delta r}, \quad j = \overline{1,I-1};$$

$$(\epsilon_{rr})_h^{n+1}(I) = \frac{\bar{\alpha}_I^{n+1} - \bar{\alpha}_{I-1}^{n+1}}{\Delta r};$$

$$(\epsilon_{\theta\theta})_h^{n+1}(j) = \frac{\bar{\alpha}_{j+1}^{n+1}}{1 + (j-1)\Delta r}, \quad j = \overline{1,I};$$

For $j = \overline{1,I}$ do

Compute $(\bar{\sigma}_h, (W^I)_h)$ from (17) using ODE ([9],[10]);

$$\bar{\sigma}_h^{n+1}(j) = \bar{\sigma}_h(t_{n+1});$$

$$(W^I)_h^{n+1}(j) = (W^I)_h(t_{n+1}),$$

End for

End for

Output: $\bar{\alpha}_j^M$, $\bar{\sigma}_h^M(j)$, $(W^I)_h^M(j)$, $j = \overline{1,I}$.

The results for the numerical solution using the equations and algorithm above are presented in figure 4 for the components of deformation and stress, and in figure 5 for the displacement and the internal parameter. In the numerical computation we considered $T = 7.8 \cdot 10^2 s$, $m = 15$, $I = 140$, $M = 100$.

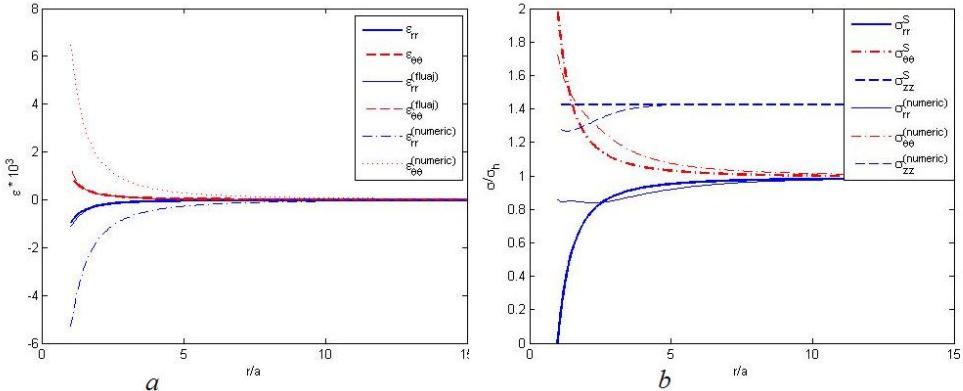


Fig. 4. Comparation between elastic solution, simplified solution and numerical solution.

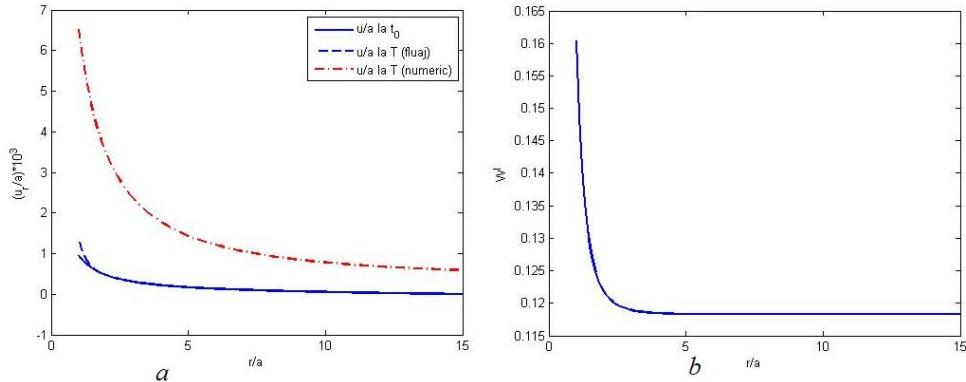


Fig. 5. a) Displacement for elastic, simplified and numerical solutions; b) Internal parameter.

6. Conclusions

This study shows that the elasto-viscoplastic model (1) describes well the behavior of salt rock around underground openings. For problem (2) - (6) is proposed an algorithm to determine the numerical solution that is compared with the elastic solution and the creep solution. For short period of time, the creep solution and the numerical solution are very close (see figures 4b and 5a). Large variations are observed for stress and displacement near the boundary $r = a$.

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