

COMPARATIVE STUDY OF A MILL MACHINE'S DYNAMICS

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Mașinile-unei sunt concepute pentru a atinge un anumit nivel de performanță legată de precizia și rigiditatea statică și dinamică. În acest articol, se prezintă metode de evaluare a dinamicii axelor de mișcare liniare plecând de la faza de dimensionare (model dinamic simplu) și ajungînd până la modelare complexă prin considerarea forțelor perturbatoare (forțe constante și variabile) și a controlerului. Considerațiile ulterioare sunt legate de nelinearitatea elasticității surubului cu bile și influența acesteia asupra comportării sistemului.

Machine tools are designed to reach a given performance level in term of precision and static and dynamic stiffness. In this article, we present methods of evaluating linear axis dynamics starting from the dimensioning stage (simple dynamic model) up to complex modelling taking in consideration the disturbance force (constant and variable cutting force) and the controller. Further considerations are connected with the nonlinearity of the ballscrew stiffness and its influence upon the system behaviour.

Key words: drive, slide, workpiece, disturbance, model, stability, root locus, Bode diagram, Nyquist diagram

1. Introduction

The constant trend in the evolution of machine tools is the increasing of speed and acceleration [1]. The flexibility regarding the structure elements has to be taken in consideration for the study of dynamic behaviour of machine tools. That is why the dynamic model of the axes or of entire machine tool plays a very important role for part dimensioning and also for control design.

The general way of considering machine tool structures regarded historically is that of massiveness. That is why massive structure leads to high structural stiffness desired for reducing deformation under the influence of machining forces and static weight of the machine structure and workpiece. The structure deflection, which can be regarded as a structural loop deformation, leads to errors at the interface between the tool edge and the workpiece.

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Machine tool stiff structures tend to transmit vibration at higher frequencies than compliant (un-stiff) ones [3]. But stiff structures having low internal or external damping will transmit the vibrations caused mainly by the machining process throughout the structure. The transmitted vibrations will cause structure time-varying deflections, which can be amplified in the workpiece, if the vibrations are near an eigenvalue of the machine tool. The conclusion is that a very stiff machine tool is not associated with low deformations.

The solution of compensating for the deflection, which could be a time varying deflection that cannot be easily predicted, is to measure the deflection and compensate for it by using the machine diving systems. But this task is almost impossible at high frequencies.

Besides the stiffness characteristic of a machine tool is also its capacity to dampen vibrations created or transmitted by high stiffness. The performances of the machine tool are influenced by the component materials. The internal damping is different for different materials. For example, steel, cast iron and granite have different damping characteristics [2].

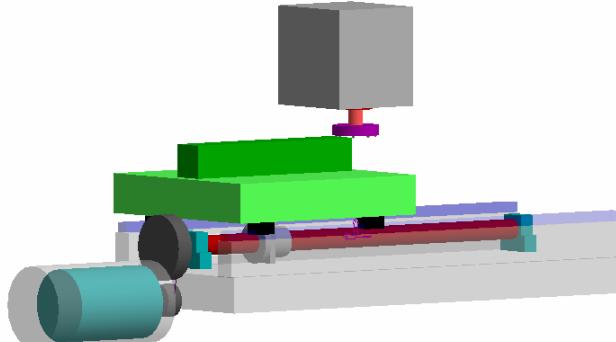


Fig. 1. Feed kinematic chain of a milling machine.

Some materials may also have non-linear damping characteristics. It may dampen the vibration very well for a short time and then amplify or keep the amplitude high.

Having all these in view, one needs to investigate these phenomena and how much has to be the influence of machine tool stiffness and also of damping for the whole structure behaviour. The application of the methods is done for a milling machine feed kinematic chain (Fig. 1).

2. Model of a linear axis

The main primary sources of excitation in a system that require the servo to have a minimum bandwidth are:

- Self-excited structural vibrations, which may be analysed by simulation with step response. Also contouring speed requirements are considered. A simple model can help the designer ensure that sufficient damping is made available and the system is stable in prescribed parameters.
- External disturbance force. It is difficult to determine the effects of system parameters without a complete dynamic simulation including the controller.

The first task for determining system parameters to prevent self-excited structural vibrations is to model a linear axis, consisting of a driving motor (linear or rotary) that moves a slide as Fig. 2 shows. The model is achieved considering the following components:

- m_1 the mass of the motor that can be linear or rotary motor in kg;
- m_2 is the mass of the slide in kg;
- c_1 is the damping in the linear and rotary bearings;
- c_2 is the damping in the actuator-carriage coupling and the carriage structure (c_1 and c_2 in Ns/m);
- k_2 is the stiffness of the actuator and actuator-carriage-tool structural loop in N/m;

The reflected inertia of the rotor and leadscrew M_{ref} is

$$M_{ref} = \frac{4\pi^2 J}{l^2} \text{ [Nm]}, \quad (1)$$

where J is the moment of inertia of the motor (and leadscrew), in m^4 ;

l – dimension of the elements, in m.

The damping, stiffness and mass effects on the system can be emphasized by simulating the model either as a transfer function (or equivalent) or by simulating as a multibody system.

For the step response, the system model (SolidDynamics [9,10,11]) is excited with a constant driving force that generates the cutting speed of process, in our case $F = 2$ N. The system reaches the constant speed $v_2 = 3.38$ mm/s equivalent to 202.8 mm/min, which means a stable system functioning (Fig. 3). The system parameter values are presented in Table 1.

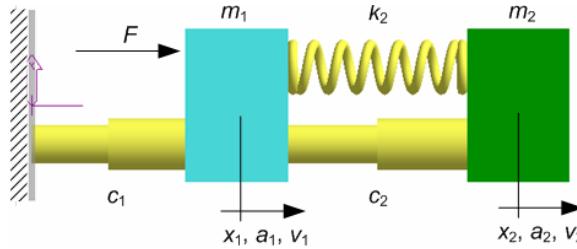


Fig. 2. System model with driving motor and slide.

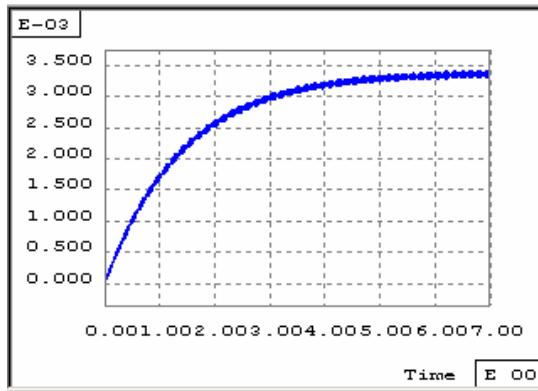
Fig. 3. Model simulation for $F = 1.2\text{N}$ (resulting constant speed $v_2 = 3.38 \text{ mm/s}$).

Table 1

Simulation parameters of the model

Actuator	m_1	m_2	c_1	c_2	k_2
Ball screw	50	450	355	19	3.925×10^8

The equations of motion have the form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix} \quad (2)$$

By considering $\dot{x}_1 = v_1$, $\dot{x}_2 = v_2$, $\ddot{x}_1 = \dot{v}_1$, and $\ddot{x}_2 = \dot{v}_2$ one can reformulate the system in a matricial form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\frac{k_2}{m_1} & \frac{k_2}{m_1} & -\frac{c_1 + c_2}{m_1} & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{c_2}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot F. \quad (3)$$

The transfer function x_2/F – dynamic response of the slide – has the form

$$\frac{x_2}{F} = \frac{k_2 + c_2 s}{c_1 s (k_2 + c_2 s + m_2 s^2) + (m_1 + m_2) s^2 (k_2 + c_2 s) + m_1 m_2 s^4}, \quad (4)$$

The response of the ballscrew driven slide is shown in Fig. 3.

As a general rule, the drive bandwidth of the system is limited by the drive system running without exciting the structural modes. The frequency can be found by drawing a horizontal line 3 dB above the resonant peak to intersect the response curve in the diagram shown in Fig. 4. The frequency can be no higher than this value without special control techniques.

This method is used only for dimensioning, which means to determine the initial sizes of components. To verify performances and achieve an optimization, a detailed control simulation has to be done.

By considering and exciting external force (disturbance) (Fig. 5), which in our case is a cutting force (milling force with a representation as Fig. 6 shows [5,6,7]), it will be difficult to determine the effect of the system parameters without a complete dynamic modelling taking in consideration also the controller.

For determining the degree of structural damping the previous method is used. Also a high degree of structural damping is required using:

- an actuator with very high static and dynamic stiffness actuator (it could be a ball screw), or
- an actuator with low or medium dynamic stiffness (linear motor) having a high degree of damping in the interface to the ground.

For the simulation, the cutting conditions were chosen for machining an alloy steel (281 HB) with a cutting tool of milling head type (corner angle 15°) (Fig. 6) [13]. The cutting speed is $v_c = 390$ rpm, $a_p = 4$ mm, $a_e = 100$ mm. The feed speed is considered starting from the feed per tooth $f_z = 0.13$ mm/tooth and is $v_f = 202.8$ mm/min. From simulation consideration we consider $v_f = 3.38$ mm/s. The cutting force acting along the moving direction is called feed force (Fig. 7) and has the value $F_F = 750$ N used for simulation.

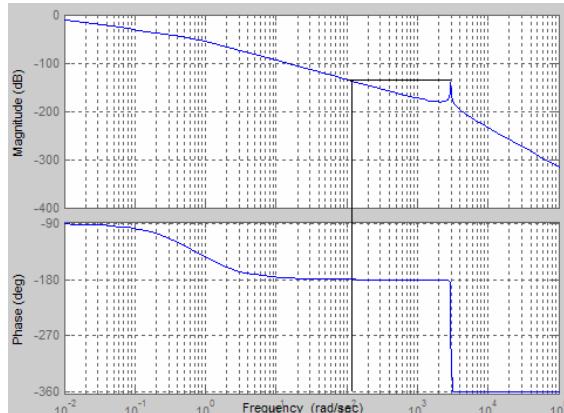


Fig. 4. Response of the ballscrew driven slide (Bode diagram).

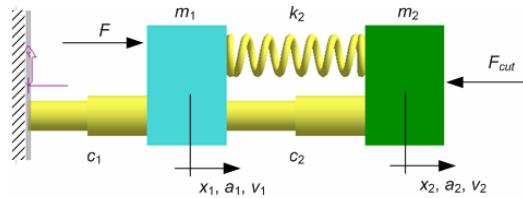


Fig. 5. System model with external disturbance F_{cut} .

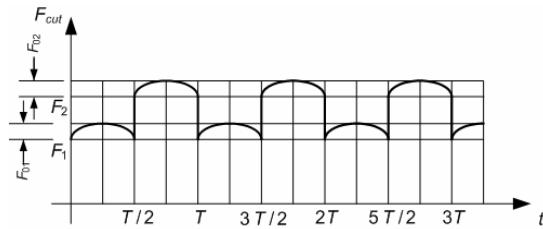


Fig. 6. Cutting force function representation [7,8].

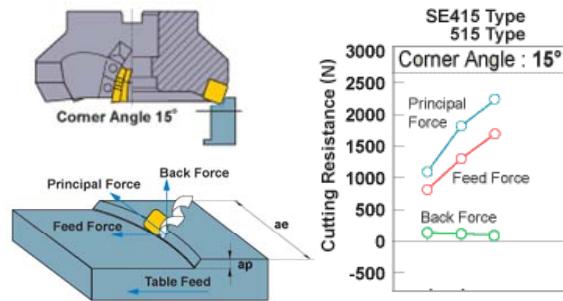


Fig. 7. Choice of cutting conditions used for simulation [13].

3. Complete dynamic modelling considering the controller

3.1. System model without considering disturbance

For system parameters evaluation and optimization a complete dynamic modelling and simulation including the controller is required. The dynamic model control scheme is proposed in Fig. 8. The dynamic model has a proportional regulator of P type. This is also a position feedback control system. The subsystems are transfer functions of second order. The system coefficients are: K_P – amplification coefficient of the regulator; K_{UI} , $K_{I\omega}$, $K_{M\omega}$, $K_{\omega\omega}$, K_{vU} – amplification factors (value transfer or adaptation); T_e , T , T_m – time constants; d_e , d_m – damping coefficients [12].

From the structure of the root locus (RL), obtained for the variable parameter K (Fig. 9), one can observe that 2 of the 5 branches of the RL are going through the right semiplan C^+ for relative small values of K (the system becoming instable). It results from here that the possibilities of choosing of a regulator which has a proportional component K are limited:

- for $K = 14$, the repartition of the system poles in closed circuit (automatic control system) is: $(-4.9495 + 8.7477i)$, $(-4.9495 - 8.7477i)$, $(-0.0029 + 0.9901i)$, $(-0.0029 - 0.9901i)$, (-0.0952) .
- for $K = 15$, the repartition of the system poles in closed circuit is: $(-4.9495 + 8.7477i)$, $(-4.9495 - 8.7477i)$, $(0.0005 + 0.9901i)$, $(0.0005 - 0.9901i)$, (-0.1020) .

One ascertains that for $K = 14$ the automatic control system (ACS) is internal asymptotic stable because it has all poles with the real part negative (placed in C^-) while for $K = 15$ the ACS is instable having two poles placed in the right semi-plan C^+ : $(0.0005 + 0.9901i)$, $(0.0005 - 0.9901i)$.

Consequence. ACS is stable only for $K \in (0, 14]$.

For the system to be internal asymptotic stable, we propose a regulator of the type proportional P, having $K = 1$ (used also for simulations).

By a double click on a RL point, all data regarding the system performances (gain, poles, damping ratio, percent overshoot, frequency) are obtained (Fig. 10).

The characteristics amplitude-frequency and phase-frequency of the transfer function for $K = 1$ are shown in Fig. 11. One can observe that for the frequency $\omega = 100$ Hz the system is near a resonance frequency!

The Nyquist diagram (hodograph) of the transfer function is shown in Fig. 12 for $K = 1$.

The system response at step input $u(t) = 1(t)$, for $K = 1$ is given in Fig. 13.

After obtaining the equivalent system from the given one as an automatic control system with rigid reaction, the transfer function is:

$$H(s) = \frac{6.12e^{-4}}{10^{-10}s^5 + 101 \cdot 10^{-9}s^4 + 102 \cdot 10^{-6}s^3 + 0.002s^2 + s + 0.6732}. \quad (5)$$

For simulation with Matlab-Simulink, the sampling-time $t = 0.01$ s was used.

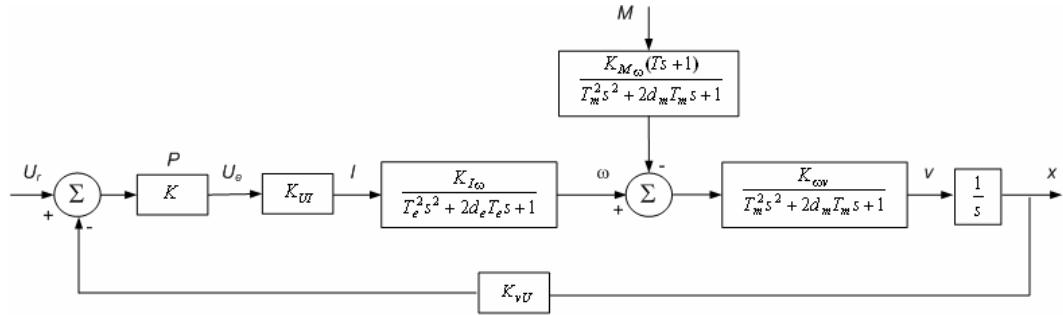


Fig. 8. Complete dynamic model of the axis including the controller

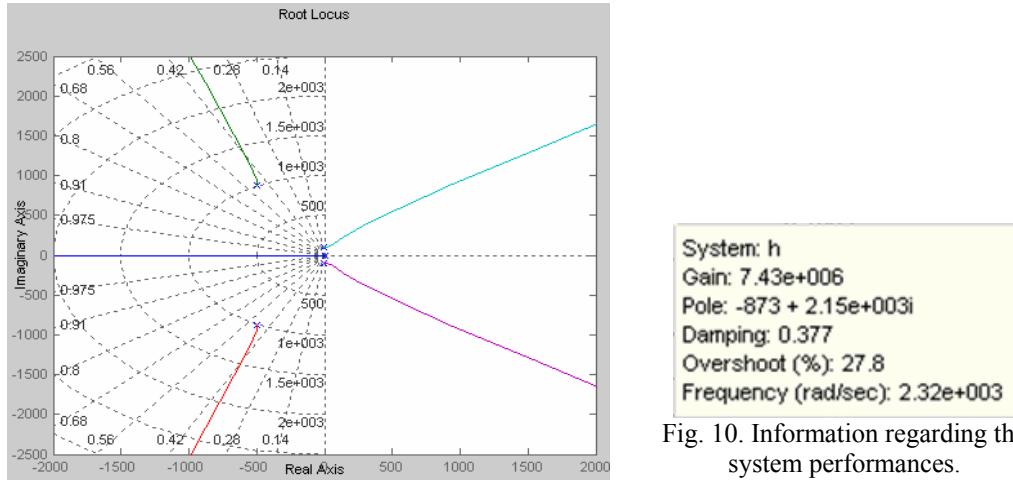


Fig. 9. Root Locus of the transfer function.

Fig. 10. Information regarding the system performances.

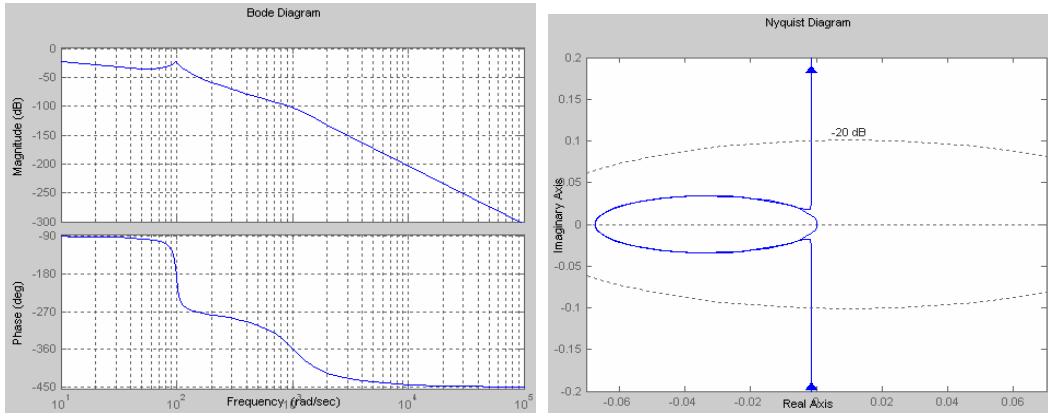


Fig. 11. Amplitude and phase-frequency diagrams.

Fig. 12. The Nyquist diagram.

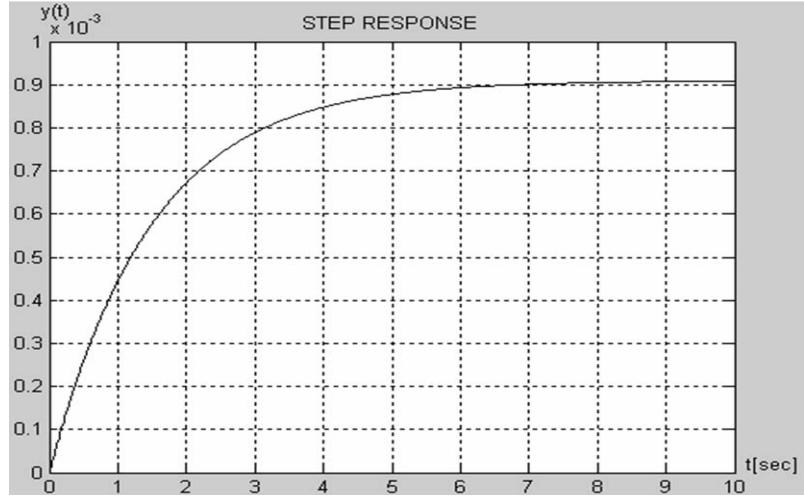


Fig. 13. The system response at unitary step input.

3.2. System with disturbance

The transfer function a disturbance H_{Mx} has the form:

$$H_{Mx}(s) = \frac{r(s)}{p_1(s) \cdot p_2(s)} \quad (6)$$

where

$$r(s) = 912 \cdot 10^{-12} s^3 + 958 \cdot 10^{-9} s^2 + 957 \cdot 10^{-7} s + 457 \cdot 10^{-4},$$

$$p_1(s) = 0.0001s^2 + 0.001s + 1,$$

$$p_2(s) = 10^{-10}s^5 + 101 \cdot 10^{-9}s^4 + 102 \cdot 10^{-6}s^3 + 0.002s^2 + s + 0.6732.$$

The answer of the position feedback control system (Fig. 14) at disturbance ($u(t) = 1(t)$, for $K = 1$) has at the beginning of the transitory regime some oscillations (Fig. 15) generated by the derivative component placed at the numerator of the transfer function of disturbance.

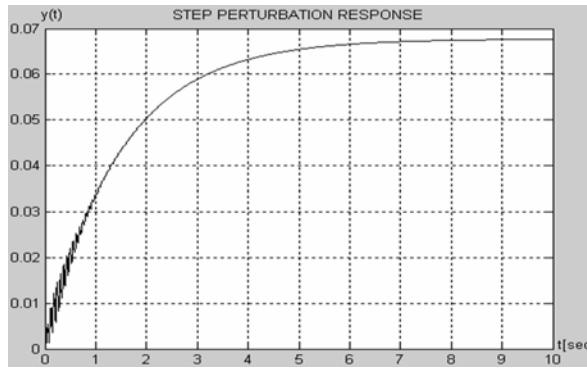


Fig. 14. The system response at unitary step input.

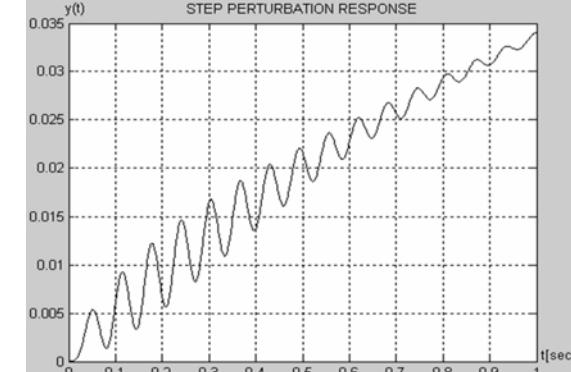


Fig. 15. Oscillations at the beginning of system response

4. Variable ballscrew stiffness

A linear axis system of a milling machine, namely a feed kinematic chain is considered for the study. The slide is driven by a rotary motor and a ballscrew-nut mechanism (Fig. 16).

For a long ballscrew of $1.5 - 2$ m and more, its stiffness characteristic is variable. It is considered an initial length l_0 . The total stroke of the slide is h . The

machining is done by a milling head and the cutting force component that is involved along the axis is the feed force F_{cut} as disturbance in the system.

In this case the crew stiffness becomes nonlinear due to the dependence of the elastic constant to the length

$$k_x = \frac{EA}{l_x}, \quad (6)$$

where k_x is the ball screw stiffness, E – elasticity modulus of steel, $E = 2 \cdot 10^{11}$, A – screw cross section area, having expression $A = \frac{\pi d^2}{4}$ with d – ball screw diameter in m.

The variable frequency of the system has the expression:

$$\omega_x = \sqrt{\frac{EA}{m} \cdot \frac{1}{l_x}}, \quad (7)$$

where $l_x \in [l_0, l_0 + h]$.

If one considers an initial length $l_0 = 0.13$ m, for four ball screw sizes ($d_1 = 25$ mm, $d_2 = 40$ mm, $d_3 = 50$ mm, and $d_4 = 63$ mm) the variation of spindle stiffness with length for a total length of 2 m is presented in Fig. 17.

The force as disturbance is considered for a mill with four inserts ($N = 4$) of diameter $D = 100$, in the same cutting conditions as previous. Depending on the ratio a_e / D , the force could have different variation forms [4]. In our case (roughing), the average force is about 750 N and the variation forms are presented in Fig. 18.

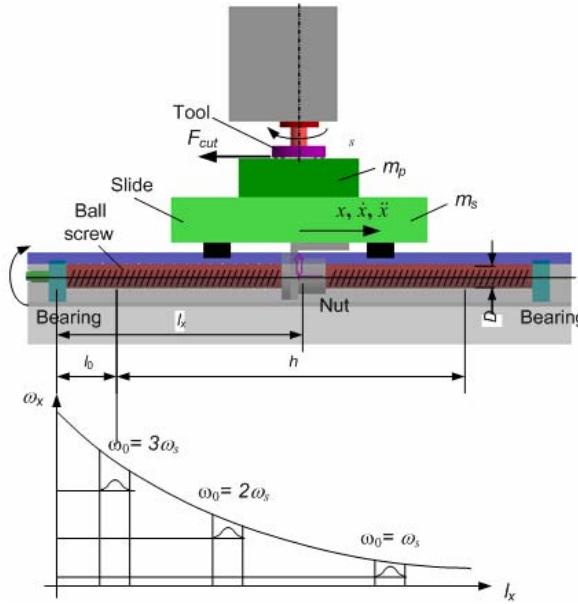


Fig. 16. Feed kinematic chain model characteristics.

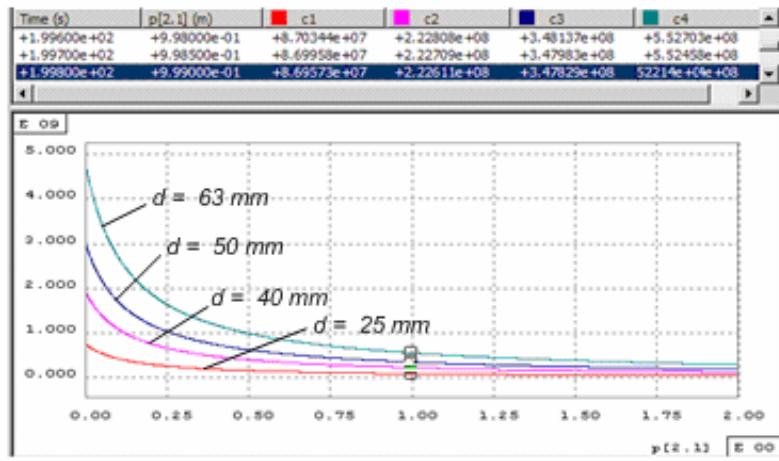


Fig. 17. Variation of ballscrew stiffness with length.

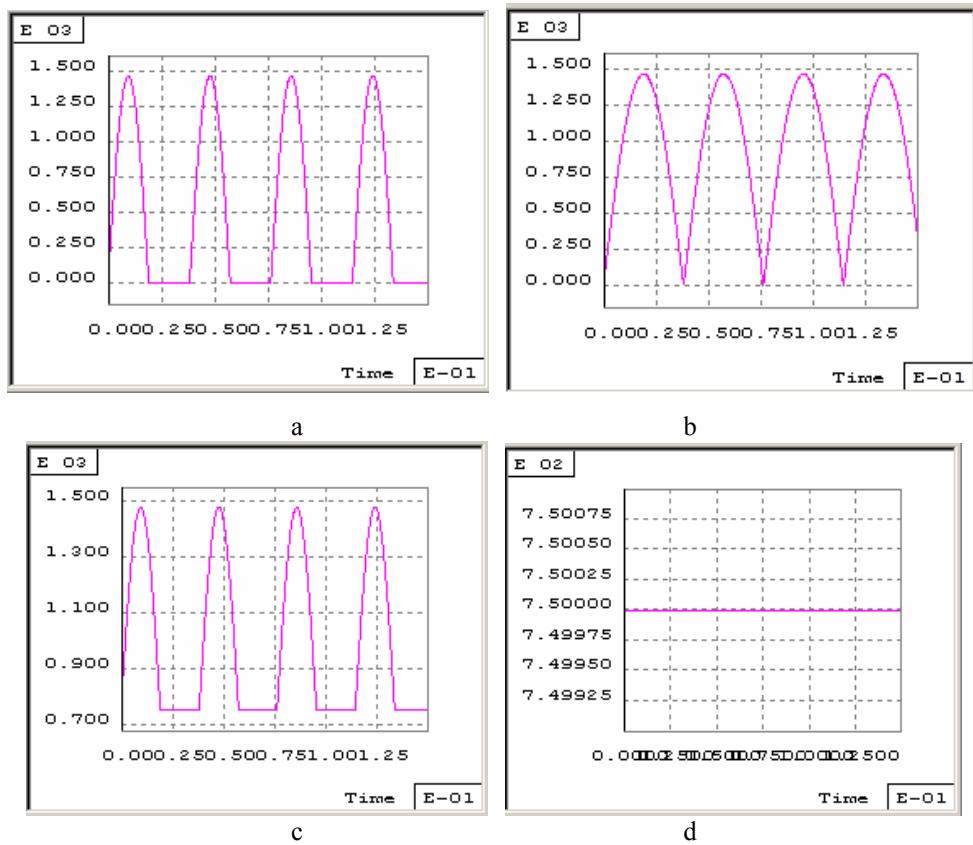
Fig. 18. Cutting force variation diagrams in up-milling (number of cutting teeth $N = 4$) for varying immersion ratios a_e/D : a – $a_e/D = 0.25$; b – $a_e/D = 0.5$; c – $a_e/D = 0.75$; d – $a_e/D = 1$.

Table 2

Cases of simulation with variable stiffness

Case	F [N]	F_{cut} [N]	l_0 [m]	m_2 [kg]	c_1 [Ns/m]	c_2 [Ns/m]	k_2 [N/m]
1	991	Var.	0.3	450	355	19	9.132×10^8
2	991	Var.	0.87	450	355	19	3.925×10^8
3	991	Var.	1.15	450	355	19	3.066×10^8

For end milling, the cutting force peaks could vary between 10% and 24% of the cutting force [7,8].

In machining, for some system frequency variable with ballscrew length given by (7), multiple of the exciting frequency of the tool (for example $\omega_0 = 3\omega_s$, $\omega_1 = 2\omega_s$, and $\omega_3 = \omega_s$), unexpected cutting phenomenon could appear that can be seen on the machined surface as zones with less quality, especially in finishing. The required roughness is increasing (Fig. 16).

It is interesting to investigate the dynamic system behaviour at different lengths of the ballscrew (Table 2). The case of machining is that presented in Fig. 18,c with $a_e/D = 0.75$.

The model simulation reveals that the position diagrams have magnitudes of 25 μm in the first case (Fig. 19), 35 μm in the second one, and 50 μm in the last one.

In the case where the shock of the first contact, impact and variation of the cutting force will be minimum, then the magnitude of vibrations that appear during milling will be minimum, also the tool life will increase and the surface quality (in finishing) will be superior. For reducing the cutting force [6], the first parameter that influences the vibrations, it is possible to chose an optimum geometry of the cutting tool active part, to give the optimum position a_e/D of the workpiece, or placing the cutting inserts after a certain law. It is also recommended to avoid the frequency ω_i in finishing.

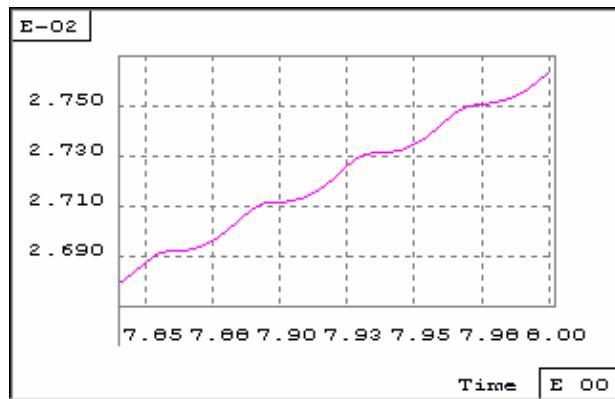


Fig. 19. First case position diagram with magnitude (25 μm).

5. Conclusions

The paper discusses the methods of evaluation of linear axes in machine tools. The first step is dedicated to the dimensioning stage in which a simple dynamic model is created and simulated for dynamic conditions (masses, stiffness, damping, and forces). The step response and amplitude-frequency responses are considered. For considering the effect of external disturbances (*constant and variable cutting force*), a complex model was proposed taking in consideration the controller. From this point of view of complete model with regulator one can conclude:

- the automatic control system is internal asymptotic stable;
- automatic control system answer is non-periodic, namely without oscillations;
- automatic control system answer has a null stationary error at unitary step input because the transfer function of the open circuit has a pole in origin, or in other words, contains the intern model of the extern variables.

Further considerations are connected with the nonlinearity of the ballscrew stiffness and its influence upon the system behaviour.

Original contributions:

- computer aided designing of the linear system by using of the root locus and Matlab program
- computer aided analysis of the mill machine's dynamics
- comparative study of the system behaviour by modelling with Solid Dynamics System and Matlab-Simulink.

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