

INTRODUCING A NEW ORTHOGONAL SPATIAL TRANSFORM FOR SIGNIFICANT DATA SELECTION

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O nouă transformare ortogonală este introdusă cu aplicații în special pentru tablouri de date. Pornind de la Polinoamele Legendre, s-a dedus o matrice ortogonală, definind noua transformare ce am numit-o „Transformata Legendre Discretă” (DLT). Pentru un proces Markov stochastic de primul ordin, s-a demonstart că (DLT) este asimptotic echivalentă la optimală Transformata Karhunen – Loève (KLT).

Noi am simulațat (DLT) pentru tipărire date codificate și am dedus că performanțele codificate obținute, utilizând (DLT), sunt foarte legate la corespondență, la două dintre cele mai cunoscute transformări până acum (Transformata Cosinus Discretă în variantele lui Ahmed și respectiv Kitajima).

A new orthogonal transform is introduced, with applications especially for picture data compression and for feature selection in pattern recognition. Starting from the Legendre Polynomials, we deduced an orthogonal matrix defining the new transform, that we called the “Discrete Legendre Transform” (DLT). For a first order Markov stochastic process, we proved that the (DLT) is asymptotical equivalent to the optimal Karhunen – Loève Transform (KLT).

We simulated our (DLT) for the picture data coding and deduced that the coding performances obtained using (DLT) are very close to those corresponding to two of the best suboptimal transforms known till now (the Discrete Cosine Transform in the variants of Ahmed and respectively Kitajima).

1. Introduction

A new orthogonal transform is introduced with applications especially for the picture data compressions and for the feature selection in the pattern recognition. Starting from the Legendre polynomials, we deduced an orthogonal matrix defining the new transform, that we called the „Discrete Legendre Transform”, (DLT). For a first order Markov stochastic process, we proved that the *DLT* is asymptotical equivalent to the optimal Karhunen – Loève Transform (KLT). We stimulated our (DLT) for picture data coding and deduced that the coding performances obtained using *DLT* are very close to those corresponding

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to two of the best suboptimal transforms known tell now (the Discrete Cosine Transform in the variants of Ahmed and respectively Kitajima).

2. On the orthogonal transforms for picture data compression and pattern recognition

In recent years there has been an increasing interest concerning the using of the orthogonal transforms for their applications in:

- a) data compression (commonly called „transform coding”) having the most attractive field, the „picture coding”;
- b) pattern recognition (orthogonal transforms are used here to obtain a dimensionality reduction from the pattern space to the feature space).

If X represents a " $N \times 1$ " signal vector

$$X' = (x(0), x(1), \dots, x(N-1)), \quad (1)$$

and T a unitary $N \times N$ matrix, then the transformed vector is given by

$$Y = T \cdot X, \quad (2)$$

so that the signal energy is conserved:

$$\sum_{i=0}^{N-1} |x(i)|^2 = \sum_{i=0}^{N-1} |y(i)|^2. \quad (3)$$

The orthogonal transforms have the following important characteristics:

1) in the transformed signal domain most of the energy is concentrated in relatively few samples (usually in the lower „generalized frequency” samples) and only these samples are sufficient for any subsequent signal processing;

2) the transform being orthogonal the computational effort to obtain the inverse matrix T^{-1} ($T' = T^{-1}$) is avoided.

To the purpose of picture data compression, we use the first " M " samples from Y (those having the most important variances); thus we want to obtain a reproduction of the picture \hat{X} with a small error (Fig.1).

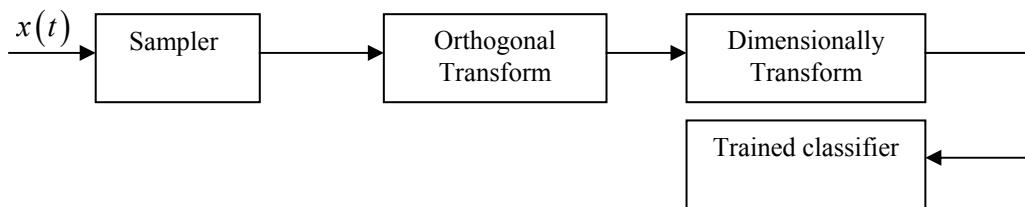


Fig.1. A picture data compression system with orthogonal transforms

The following relations are true:

$$\|\varepsilon\| = \|X - \hat{X}\|; \quad (4)$$

$$\hat{X} = T^{-1}\hat{Y} = T'\hat{Y}; \quad (5)$$

$$Y' = (y(0), y(1), \dots, y(N-1)); \quad (6)$$

$$\hat{Y}' = (y(0), y(1), \dots, y(M-1), \underbrace{0, 0, \dots, 0}_{N-M}). \quad (7)$$

In a pattern recognition system (Fig.2), using an orthogonal transform for feature selection, only a subset of the all transformed samples are retained to be processed for pattern discrimination.

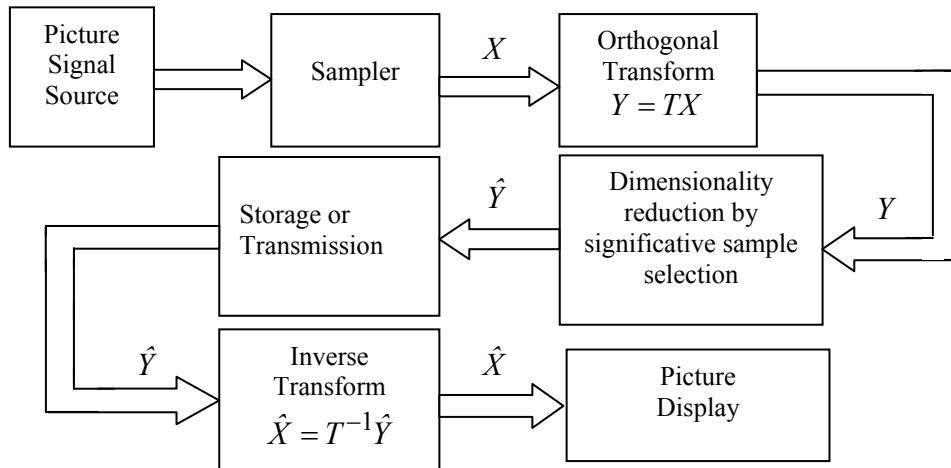


Fig.2. A pattern recognition system with orthogonal transforms

For a given class of signals having the same second – order statistics, the Karhunen – Loève Transform (*KLT*) is shown to be optimal with respect to the following performance measure: variance distribution, estimation using the mean square error and the rate distribution function. Although *KLT* is optimal, it has dimensionality difficulties. First, *KLT* is unique for a class of signals, therefore it has to be computed for that particular class. Second, even if a closed form analytical expression for *KLT* is known, the transformation calculations do not, generally, have a fast algorithm available. Suboptimal transforms such as Discrete Fourier Transform (*DFT*), Walsh-Hadamard Transform (*WHT*), Cosine Discrete Transform (*CDT*), Haar Transform (*HT*), Slant Transform (*ST*), Discrete Sine Transform (*DST*), which do not depend on the particular image class statistics, are used instead of *KLT*, with performances close to it. We introduce now a new orthogonal transform starting from the Legendre

Polynomials. In another paper we used these polynomials in a modulation system for multiple transmission of information, having an optimal character [2].

3. Legendre Polynomials and their properties. Introducing a new orthogonal transform based on the Legendre Polynomials

Denoting by $P_j(x)$ the Legendre Polynomial of the „j” th degree, for $x \in [-1, 1]$, we have:

$$P_j(x) = \frac{1 \cdot 3 \cdot 5 \cdots (2j-1)}{j!} \left[x^j - \frac{j(j-1)}{2(2j-1)} x^{j-2} + \frac{j(j-1)(j-2)(j-3)}{2(2j-1)(2j-3)} x^{j-4} + \dots \right] \dots \quad (8)$$

Another general formula for representing the Legendre Polynomials is given in [2]:

$$P_j(x) = (-1)^j \cdot Q_j\left(\frac{x+1}{2}\right), x \in [-1, 1], \quad (9)$$

where

$$Q_j(u) = \sum_{k=0}^j (-1)^k \binom{j}{k} \binom{j+k}{k} u^k. \quad (10)$$

Legendre Polynomials are characterized by the following properties:

1) differential equation

$$(1-x^2)y'' - 2xy' + j(j+1)y = 0, y = P_j(x) \quad (11)$$

2) they can be calculated by Rodrigues formula

$$P_j(x) = \frac{(-1)^j}{2^j \cdot j!} \frac{d^j}{dx^j} (1-x^2)^j \quad (12)$$

3) they admit the integral representation

$$P_j(x) = \frac{1}{2\pi} \int_0^\pi \left(x + i\sqrt{1-x^2} \sin \varphi \right)^j d\varphi. \quad (13)$$

4) they can be obtained via a generator function

$$\frac{1}{\sqrt{1-2tx+t^2}} = \sum_{j=0}^{\infty} P_j(x) t^j. \quad (14)$$

5) some particular values are following

$$P_j(1) = 1; P_j(-1) = (-1)^j; P_{2j+1}(0) = 0; P_{2j}(0) = \frac{(-1)^j (2j)!}{2^{2j} (j!)^2}. \quad (15)$$

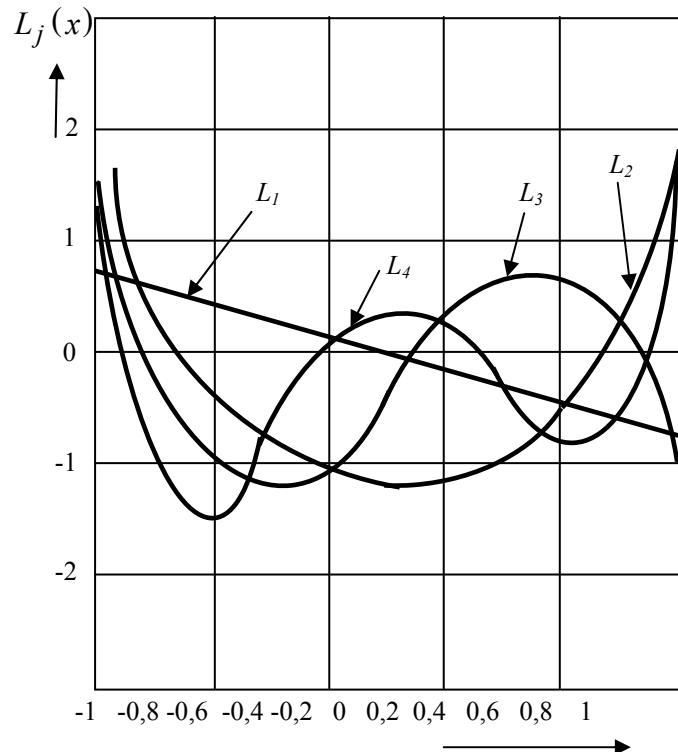


Fig.3. The polynomials $L_j(s)$ ($j = 1, 2, 3, 4$)

6) orthogonality and normalization

$$\int_{-1}^1 P_j(x) P_k(x) dx = \begin{cases} 0, & j \neq k \\ \frac{2}{2j+1}, & j = k \end{cases} \quad (16)$$

7) *Polynomials* roots

All of the roots of the polynomial $P_j(x)$ are real and belong to the interval $(-1, +1)$.

8) recurrence relations:

$$\left. \begin{aligned} (1-x^2)P'_j(x) &= -(j+1)[P_{j+1}(x) - xP_j(x)]; \\ P_j(x) &= \frac{1}{j+1}[P'_{j+1}(x) - xP'_j(x)] = \frac{1}{2j+1}[P'_{j+1}(x) - P'_{j-1}(x)]; \\ (j+1)P_{j+1}(x) - (2j+1)xP_j(x) + jP_{j-1}(x) &= 0. \end{aligned} \right\} \quad (17)$$

The first orthonormalized Legendre Polynomials are represented in Fig.3. From relation (17) it results:

$$\begin{aligned} P_j(x) &= \frac{1}{j}[(2j-1)xP_{j-1}(x) - (j-1)P_{j-2}(x)], \\ (j = 2, \dots, n, P_0 = 1). \end{aligned} \quad (18)$$

Denoting:

$$P_j(x) = \sum_{k=0}^j S(k, j)x^k, \quad (19)$$

From (18) and (19) one deduced:

$$\left. \begin{aligned} S(0, 0) &= 1; \\ S(0, 1) &= 1; \\ S(0, j) &= -S(0, j-2) + \frac{S(0, j-2)}{j} \\ S(k, j) &= 2S(k-1, j-1) - S(k, j-2) - \frac{S(n-2, n-1)}{n}; \\ S(n-1, n) &= 2S(n-2, n-1) - \frac{S(n-2, n-1)}{n}; \\ S(n, n) &= 2S(n-1, n-1) - \frac{S(n-1, n-1)}{n}. \end{aligned} \right\} \quad (20)$$

To introduce a new orthogonal transform based on the Legendre Polynomials we firstly deduced a recurrence relation for orthonormalized Legendre Polynomials (Fig.3):

$$L_j(x) = \sqrt{\frac{2j+1}{2}}P_j(x). \quad (21)$$

From (18) and (21) it results:

$$\sqrt{\frac{2}{2j+1}}L_j(x) = \frac{1}{j}\left[(2j-1)x\sqrt{\frac{2}{2j-1}}L_{j-1}(x) - (j-1)\sqrt{\frac{2}{2j-3}}L_{j-2}(x)\right]. \quad (22)$$

Hence:

$$L_j(x) = \frac{(2j-1)x}{j} \sqrt{\frac{2j+1}{2j-1}} L_{j-1}(x) - \frac{j-1}{j} \sqrt{\frac{2j+1}{2j-1}} L_{j-2}(x), \quad (23)$$

$$\left(j = 2, \dots, n, L_0 = \frac{1}{\sqrt{2}} \right).$$

Denoting:

$$a_j = \frac{2j-1}{j} \sqrt{\frac{2j+1}{2j-1}}, \quad (24)$$

and

$$C_j = \frac{j-1}{j} \sqrt{\frac{2j+1}{2j-3}}, \quad (25)$$

One yields $L_j(x)$

$$L_j(x) = a_j x L_{j-1}(x) - C_j L_{j-2}(x), \quad (26)$$

$$(j = 1, \dots, N; L_{-1} = 0).$$

Relation (26) can be written in the following matrix form:

$$x \begin{pmatrix} L_0(x) \\ L_1(x) \\ L_2(x) \\ \vdots \\ L_{N-2}(x) \\ L_{N-1}(x) \end{pmatrix} = \begin{pmatrix} 0 & 1/a_1 & 0 & \cdots & 0 & 0 \\ c_2/a_2 & 0 & 1/a_1 & \cdots & 0 & 0 \\ 0 & c_3/a_3 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1/a_{N-1} \\ 0 & 0 & 0 & \cdots & c_N/a_N & 0 \end{pmatrix} \begin{pmatrix} L_0(x) \\ L_1(x) \\ L_2(x) \\ \vdots \\ L_{N-2}(x) \\ L_{N-1}(x) \end{pmatrix} +$$

$$+ (L_N(x)) / a_N \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}. \quad (27)$$

Relation (27) can also be written as

$$xL(x) = TL(x) + \left(\frac{1}{a_N} \right) L_N(x) E_N. \quad (28)$$

The significance of the matrices $L(x)$ and T in relation (28) results evidently from its correspondence with (27): E_N is the vector

$$E_N = \underbrace{(0, \dots, 0, 1)'}_{N-1}$$

Assume:

$$L(x_k) = 0, (k = 0, 1, \dots, N-1) \quad (29)$$

denoting:

$$L(x_k) = L^k, \quad (30)$$

Then relation (28) becomes:

$$x_k L^k = T L^k. \quad (31)$$

We deduced that x_k and L^k represent the characteristic roots and respectively the characteristic vectors of the matrix T . We shall prove that T is a symmetric matrix.

From (24), (25) and (27) yields

$$\left. \begin{array}{l} \frac{c_j}{a_j} = \frac{j-1}{\sqrt{(2j-1)(2j-3)}} = \frac{1}{a_{j-1}}; \\ \frac{c_2}{a_2} = \frac{1}{\sqrt{3}}; \\ \frac{c_N}{a_N} = \frac{1}{a_{N-1}} = \frac{N-1}{\sqrt{(2N-3)(2N-1)}}. \end{array} \right\} \quad (32)$$

Therefore, T is a symmetric matrix. We obtain:

$$T = \begin{pmatrix} 0 & \frac{1}{\sqrt{3}} & 0 & \dots & \dots & 0 \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{15}} & \dots & \dots & 0 \\ 0 & \frac{2}{\sqrt{15}} & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \frac{N-1}{\sqrt{(2N-3)(2N-1)}} & 0 \end{pmatrix} \quad (33)$$

The matrix T is a real symmetric matrix having distinct characteristic roots given by

$$\begin{cases} P_N(x_k) = 0, (k = 0, 1, \dots, N-1); \\ x_k \in (-1, 1), 1 > x_0 > x_1 > \dots > x_{N-1} > -1. \end{cases} \quad (34)$$

But the eigenvectors of a symmetric matrix that correspond to distinct eigenvalues are orthogonal among themselves; hence

$$\Psi = (L^0 / \|L^0\|, L^1 / \|L^1\|, \dots, L^{N-1} / \|L^{N-1}\|), \quad (35)$$

where:

$$L_k = (L_0(x_k), L_1(x_k), \dots, L_{N-1}(x_k)), (k = 0, 1, \dots, N-1), \quad (36)$$

represents an orthogonal matrix. From (35) and (36) we obtain

$$\Psi = \begin{pmatrix} \frac{L_0(x_0)}{N_0} & \frac{L_0(x_1)}{N_1} & \dots & \frac{L_0(x_{N-1})}{N_{N-1}} \\ \frac{L_1(x_0)}{N_0} & \frac{L_1(x_1)}{N_1} & \dots & \frac{L_1(x_{N-1})}{N_{N-1}} \\ \vdots & \vdots & \dots & \vdots \\ \frac{L_{N-1}(x_0)}{N_0} & \frac{L_{N-1}(x_1)}{N_1} & \dots & \frac{L_{N-1}(x_{N-1})}{N_{N-1}} \end{pmatrix} \quad (37)$$

where:

$$N_k = \|L_k\| = \sqrt{\sum_{i=0}^{N-1} L_i^2(x_k)} = \sqrt{\sum_{i=0}^{N-1} \left(\sqrt{\frac{2i+1}{2}} P_i(x_k) \right)^2}, \quad (38)$$

$$(k = 0, 1, \dots, N-1; L_N(x_k) = 0).$$

4. Asymptotical properties of the Discrete Legendre Transform

We call the new transform, characterized by the matrix Ψ , the Discrete Legendre Transform (*DLT*).

Theorem. *DLT* is asymptotically equivalent to the Karhunen Loeve Transform for a first order Markov random process.

Proof. If relation (31) is satisfied, then for any real constants a, b we deduce that the following relation is also true:

$$(aE_N + bT)L^k = (a + bx_k)L^k. \quad (39)$$

Denote:

$$a + bx_k = \omega_k, \quad (40)$$

choose:

$$\begin{cases} a = (1 + \rho^2) / (1 - \rho^2); \\ b = (-2\rho) / (1 - \rho^2), \end{cases} \quad (41)$$

$(0 \leq \rho < 1)$, and denote

$$\tilde{T} = a\tilde{E}_N + bT, (\tilde{E}_N \text{ being the unit matrix}) \quad (42)$$

one yields

$$\tilde{T}L^k = \omega_k L^k. \quad (43)$$

From relation (43) we deduce that L^k represent the eigenvectors of the matrix \tilde{T} where:

$$\tilde{T} = \frac{1}{1 - \rho^2} \begin{pmatrix} 1 + \rho^2 & -\frac{2\rho}{\sqrt{3}} & \dots & 0 \\ -\frac{2\rho}{\sqrt{3}} & 1 + \rho^2 & \dots & 0 \\ 0 & -\frac{2\rho}{\sqrt{3}} & \dots & 0 \\ \vdots & \vdots & \dots & -2(N-1)\rho / \sqrt{(2N-3)(2N-1)} \\ 0 & 0 & \dots & (1 + \rho^2) \end{pmatrix}. \quad (44)$$

Since the column vectors $\Psi^{(i)}$ and $\Psi^{(j)}$ are orthonormal, where:

$$\Psi(\Psi^{(0)}, \Psi^{(1)}, \dots, \Psi^{(N-1)}), \quad (45)$$

It implies:

$$\Psi' \tilde{T} \Psi = \text{diag}(\omega_0, \omega_1, \dots, \omega_{N-1}). \quad (46)$$

By considering a zero-mean random process:

$$X = \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{pmatrix} \quad (47)$$

We transform X into Y so that:

$$Y = \Psi' X \quad (48)$$

Assuming X is a first order Markov process, then its covariance matrix has the form [3]:

$$\Sigma_X = (\rho^{|i-j|}) = \begin{pmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{N-1} \\ \rho & 1 & \rho & \cdots & \rho^{N-2} \\ \vdots & \vdots & \vdots & & \vdots \\ \rho^{N-1} & \rho^{N-2} & \rho^{N-3} & \cdots & 1 \end{pmatrix}. \quad (49)$$

According to a known rule [3] from (48) and (49) we can obtain the covariance matrix of Y :

$$\Sigma_Y = \Psi' \Sigma_X \Psi \quad (50)$$

and its inverse:

$$(\Sigma_Y)^{-1} = \Psi^{-1} (\Sigma_X)^{-1} (\Psi')^{-1}, \quad (51)$$

or:

$$(\Sigma_Y)^{-1} = \Psi' \Sigma_X^{-1} \Psi. \quad (52)$$

But for a zero mean order random process [7]:

$$(\Sigma_X)^{-1} = \frac{1}{1-\rho^2} \begin{pmatrix} 1 & -\rho & \cdots & 0 & 0 \\ -\rho & 1+\rho^2 & \cdots & \vdots & \vdots \\ 0 & -\rho & \cdots & 0 & 0 \\ \vdots & \vdots & & 1+\rho^2 & -\rho \\ 0 & 0 & \cdots & -\rho & 1 \end{pmatrix} \quad (53)$$

We observe that:

$$\lim_{N \rightarrow \infty} \left(-1 + \frac{2(N-1)}{\sqrt{(2N-3)(2N-1)}} \right) = 0, \quad (54)$$

and the set corresponding to the two extreme diagonals in $(\Sigma_X)^{-1}$

$$a_N = \left\{ \left(-1 + \frac{2}{\sqrt{3}} \right) \rho, \dots, \left(-1 + \frac{2(N-1)}{\sqrt{(2N-3)(2N-1)}} \right) \rho \right\} \quad (55)$$

has the elements satisfying the relation:

$$\left| \left(-1 + \frac{2}{\sqrt{3}} \right) \rho \right| > \left| \left(-1 + \frac{4}{\sqrt{3 \cdot 5}} \right) \rho \right| > \dots > \left| \left(-1 + \frac{2(N-1)}{\sqrt{(2N-3)(2N-1)}} \right) \rho \right|. \quad (56)$$

With a good approximation we can neglect the elements from (55), starting with the second

$$a_N = \left\{ \left(-1 + \frac{2}{\sqrt{3}} \right) \rho, 0, \dots, 0 \right\}, \quad (57)$$

where the first neglected element is:

$$\left(-1 + \frac{4}{\sqrt{3 \cdot 5}} \right) \rho \approx 0.03 \rho. \quad (58)$$

Since, usually $\rho \approx (0.9 \div 0.95)$, it is clear that with a good approximation:

$$(\hat{\Sigma}_X)^{-1} = \tilde{T} + \frac{1}{1 - \rho^2} \begin{pmatrix} -\rho^2 & \left(-1 + \frac{2}{\sqrt{3}} \right) \rho & \dots & 0 & 0 \\ \left(-1 + \frac{2}{\sqrt{3}} \right) \rho & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & -\rho^2 \end{pmatrix} \quad (59)$$

If we compare our transform with the Symmetric Cosine Transform (SCT) of Kitajima [7], deduce ours is better because the residuum:

$$\left(\left(-1 + \frac{2}{\sqrt{3}} \right) \rho \approx 0.15 \rho \right)_{DLT} < \left(\left(-1 + \sqrt{2} \right) \rho \approx 0.41 \rho \right)_{SCT} \quad (60)$$

Denoting:

$$(\Sigma_X)^{-1} - \tilde{T} = \Phi, \quad (61)$$

$$\text{one yields from (61):} \quad (\Sigma_X)^{-1} = \Psi' (T + \Phi) \Psi. \quad (62)$$

and from (46):

$$(\Sigma_X)^{-1} = \text{diag}(\omega_0, \omega_1, \dots, \omega_{N-1}) + \Psi' \Phi \Psi. \quad (63)$$

One can observe that the elements Φ_{ij} of the matrix Ψ have the form:

$$\Phi_{ij} = \frac{L_i(x_j)}{\sqrt{\sum_{i=0}^{N-1} L_i^2(x_j)}} = \frac{\sqrt{\frac{i+1}{2}} P_i(x_j)}{\sqrt{\sum_{i=0}^{N-1} \left(\frac{2i+1}{2} \right) P_i^2(x_j)}}, \quad (64)$$

so that:

$$\lim_{N \rightarrow \infty} \Psi = \lim_{N \rightarrow \infty} \Psi' = 0, \quad (65)$$

Therefore:

$$\lim_{N \rightarrow \infty} (\Sigma_Y)^{-1} = \text{diag}(\omega_0, \omega_1, \dots, \omega_{N-1}), \quad (66)$$

and hence:

$$\lim_{N \rightarrow \infty} \Sigma_Y = \text{diag}(\omega_0, \omega_1, \dots, \omega_{N-1}), \quad (67)$$

and the asymptotical equivalence between the Discrete Legendre Transform and the Karhunen – Loève Transform is proved.

5. Evaluating the performances of the DLT for picture data compression

We wanted to evaluate the performances obtained by using the Discrete Legendre Transform for picture data compression, comparing them with those obtained using other already known transforms.

We used a 64×64 pixel and 32 amplitude level image (Lincoln) given in the Appendix of the book of Gonzales and Wintz [1]. This picture corresponds to a data matrix:

$$MX = \begin{pmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,64} \\ \vdots & \vdots & & \vdots \\ x_{64,1} & x_{64,2} & \cdots & x_{64,64} \end{pmatrix} \quad (68)$$

We made a zonal coding such that the picture corresponding to the data matrix MX is divided into 512 vectors having each of them 8 pixels (8 vectors on each row).

Assume the eight pixel vector MV has a covariance matrix $SIGN$ (denoted by Σ_X in relation (49)):

$$SIGN = \Sigma_X = (\rho^{|i-j|}) = \begin{pmatrix} 1 & \rho & \cdots & \rho^7 \\ \rho & 1 & \cdots & \rho^6 \\ \vdots & \vdots & & \vdots \\ \rho^7 & \rho^6 & \cdots & 1 \end{pmatrix} \quad (69)$$

and consider a transformation:

$$RY = FI \cdot MV, \quad (70)$$

where FI is a transformation matrix, MV is the transformed vector.

According to (50) and (69) we obtain

$$SY = FI \cdot SIGN \cdot FIT \quad (71)$$

where FIT signifies the transpose of FI . If we extract the elements of the principal diagonal of SY

$$D = (d_1, \dots, d_8), \quad (72)$$

and recorder the above elements of D into the row vector:

$$\Delta = (d_{i_1}, \dots, d_{i_8}), \quad (73)$$

such that:

$$|d_{i_1}| \geq \dots \geq |d_{i_8}|. \quad (74)$$

(The row vectors D and Δ have the same elements but in a different order). Denote:

$$IP = (i_8, \dots, i_1) \quad (75)$$

To each of the 256 vectors MV we apply firstly relation (70) and secondly consider the element of RY having the indices $\{i_8, i_7, i_6, i_5\}$ (see (75)) to be zero.

We denoted by RYN the truncated vectors obtained in this way. We applied then the Inverse Discrete Legendre Transform to the vectors RYN , thus obtaining the reconstituted picture vectors called

$$VN = FIT \cdot RYN. \quad (76)$$

The reconstructed vectors are put in a new 64×64 picture data matrix denoted by RXN .

$$RXN = \begin{pmatrix} \hat{x}_{1,1} & \hat{x}_{1,2} & \cdots & \hat{x}_{1,64} \\ \vdots & \vdots & & \vdots \\ \hat{x}_{64,1} & \hat{x}_{64,2} & \cdots & \hat{x}_{64,64} \end{pmatrix} \quad (77)$$

The 64×64 error matrix, $REPS$, may be computed as a difference:

$$REPS = |MX - RXN| = \begin{pmatrix} \varepsilon_{1,1} & \varepsilon_{1,2} & \cdots & \varepsilon_{1,64} \\ \vdots & \vdots & & \vdots \\ \varepsilon_{64,1} & \varepsilon_{64,2} & \cdots & \varepsilon_{64,64} \end{pmatrix}, \quad (78)$$

where:

$$\varepsilon_{i,j} = |x_{i,j} - \hat{x}_{i,j}|, i, j = 1, \dots, 64.$$

To evaluate the performances of a picture coding algorithm, we used the following total parameters:

- (1) The error norm

$$N_1 = \frac{1}{4096} \sum_i \sum_j \varepsilon_{i,j}; \quad (79)$$

(2) The signal to noise ratio

$$N_2 = 10 \lg \left(\sum_i \sum_j x_{i,j}^2 \right) / \left(\sum_i \sum_j \varepsilon_{i,j}^2 \right); \quad (80)$$

(3) The error histogram:

$$H(k) = \begin{cases} \left\{ \begin{array}{l} \text{the number of errors } \varepsilon_{ij} / k \leq \varepsilon_{i,j} \leq k+1, \text{ for } k = 1, 2, 3, 4, 5 \end{array} \right\} \\ \left\{ \begin{array}{l} \text{the number of errors } \varepsilon_{ij} / \varepsilon_{ij} \geq 6 \end{array} \right\}. \end{cases} \quad (81)$$

6. Conclusions

Within this paper we introduce a new orthogonal transform called the Discrete Legendre Transform (*DTL*), starting from the Legendre Polynomials.

We proved that the performances obtained using on (*DTL*) for picture coding are very close to those corresponding to the best suboptimal orthogonal transform known (the Discrete Cosine Transform in variants of Ahmed and respectively Kitajima).

R E F E R E N C E S

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