

## MONTE-CARLO SIMULATION OF THE PIPELINE SYSTEM TO INVESTIGATE WATER TEMPERATURE'S EFFECTS

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*Apa geotermală este una dintre cele mai valoroase resurse termo-energetice. Fluctuația temperaturii și salinitatea ridicată a apei geotermale afectează parametrii sistemului de conducte, cum ar fi scăderea presiunii, cea ce necesită creșterea puterii pompei. Din punctul de vedere al inginerului de sistem, acești factori nedeterminanți ai parametrilor fizici ale fluidelor, sănătatea parametrii nesiguri. Acest studiu prezintă metodologia Monte-Carlo Simulation și posibilitatea de a o utiliza pentru a investiga influențele parametrilor fluidului în pierderile sistemului, într-un sistem de conducte simplu. Rezultatele obținute pot fi utilizate pentru investigarea parametrilor nesiguri ai sistemului de conducte geotermal, cum ar fi caracteristicile nedeterminante ale fluidelor.*

*The geothermal water is one of the most valuable thermo-energetic resources. The fluctuation of temperature and salinity of geothermal water has effects on pipeline system parameters such as its head loss; i.e. the required pump power. From the system engineering point of view these indeterminations of physical parameters of the fluid are parametric uncertainties. This paper shows the methodology of the Monte-Carlo Simulation and its possibility of use to investigate influences of fluid parameters on system losses by an easy pipeline system. The obtained consequents and experiences can be used to investigate parametric uncertainties of the geothermal pipeline system, such as fluid characteristics' indeterminations*

**Keywords:** Monte-Carlo Simulation, uncertainty analysis, pipeline system, geothermal system

### 1. Introduction

The geothermal water is very valuable thermo-energetic resource, especially used for heating, as well as a resource for hot water. The unsteadiness of temperature and salinity of geothermal water has effects to pipeline system parameters such as its head loss; i.e. the required pump power. From the system engineering and system modeling points of view these indeterminations of physical parameters of the fluid are parametric system (and model) uncertainties.

Pokorádi established that during mathematical modeling of the real technical system we can meet any type and rate model uncertainties [8]. Its

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reasons can be incognizance of modelers or data inaccuracy. So classification of uncertainties, with respect to its sources, can be distinguished between epistemic and aleatory ones. Epistemic uncertainty is due to the lack of the knowledge of quantities, processes of the system or the environment. The aleatory uncertainty is an inherent data variation associated with the investigated system or the environment; therefore it is named parametric uncertainty. Pokorádi mentioned that the Monte-Carlo Simulation (MCS) can be used for uncertainty analysis of a deterministic calculation because it yields a distribution describing the probability of alternative possible values about the nominal (designed) bias point.

The Monte-Carlo Simulation is one of the most well-known parametrical uncertainty investigation methods. There are several books and papers that state theory of the MCS and its applications. Rubinstein depicted detailed treatment of the theoretical backgrounds and the statistical aspects of these methods in his book [12]. Newman and Barkema applied the Monte-Carlo Simulation to investigate several statistical problems in physics [6].

The focus of Fang's paper was to propose a calculation method for evaluating thermal performance of the solar cavity receivers [1]. The Monte-Carlo method was employed to calculate radiation inside the receiver.

Vespignani et. al. used a global structured metapopulation model integrating mobility and transportation data worldwide [13]. The GLEaM (for GLobal Epidemic and Mobility) structured metapopulation model was used for the worldwide evolution of the pandemic and perform a maximum likelihood analysis of the parameters against the actual chronology of newly infected countries. The method was computationally intensive as it involved a Monte Carlo generation of the distribution of arrival time of the infection in each country based on the analysis of  $10^6$  worldwide simulations of the pandemic evolution with the GLEaM model.

In paper of Kusiak, Li and Zhang [4], a data-driven approach for steam load prediction was presented. Predicting building energy load is important in energy management. This load is often the result of steam heating and cooling of buildings.

Mavrotas, Florios, and Vlachou developed an energy planning framework combining Mathematical Programming and Monte-Carlo Simulation that can be properly used in buildings of the Services' sector (hospitals, hotels, sport centers, universities, etc.) taking into account the uncertainty in cost parameters that are expressed by probability distributions. Combining the systemic approach with Mathematical Programming and the uncertainty issue through stochastic approaches like Monte-Carlo Simulation is a challenging task. The basic innovation of the paper [5] relies on the way it represented the energy planning model and on the incorporation of the Monte-Carlo Simulation inside the optimization process.

The modeling presented by Jones, Lacey and Walshe provided a synthesis of hydrological understanding of Toolibin Lake and a coarse approximation of the relative improvement in bird habitat expected in the short-term [2]. Lake Toolibin, an ephemeral lake in the agricultural zone of Western Australia, is under threat from secondary salinity due to land clearance throughout the catchment. The characterization of uncertainty associated with environmental variation and incertitude allows managers to make informed risk-weighted decisions.

Pokorádi, in his preceding publication [11], showed the methodology of a matrix-algebraic sensitivity analysis and its possibility of use by an easy pipeline system model to investigate effects of uncertainties of water and system parameters.

The aims of this paper are to show the methodology of the Monte-Carlo Simulation and its applicability to investigate influences of fluid parameters to system losses by an easy pipeline system model and discussions of simulation results. The (basically theoretical) obtained consequents and experiences can be used to investigate parametrical uncertainties of the geothermal pipeline system, such as fluid characteristic's indeterminations.

The outline of the paper is as follows: Section 2 shows the Monte-Carlo Simulation. Section 3 presents an easy case study by a pipeline system model, and interprets the result of the simulation. Section 4 summaries the paper, outlines the prospective scientific work of the authors.

## 2. The Monte-Carlo Simulation

One of the most well-known probabilistic parametric uncertainty investigation methods is the Monte-Carlo Simulation. The „classical” MCS is used as an uncertainty analysis of a deterministic calculation because it yields a distribution describing the probability of alternative possible values about the nominal (designed) point.

The idea of the Monte-Carlo calculation is much older than the computer. The name “Monte-Carlo” is relatively recent — it was coined by Nicolas Metropolis in 1949 — but under the older name of “statistical sampling” the method has a history stretching back well into the last century, when numerical calculations were performed by hand using pencil and paper and perhaps a slide-rule. An early example of what was effectively a Monte-Carlo calculation of the motion and collision of the molecules in a gas was described by William Thomson (Lord Kelvin) in 1901 [6]. Thomson's calculations were aimed at demonstrating the truth of the equipartition theorem for the internal energy of a classical system. The exponential growth in computer power since those early days is by now a familiar story to us all, and with this increase — in computational resources Monte-Carlo techniques have looked deeper and deeper

into the subject of statistical physics. The Monte-Carlo simulations have also become more accurate as a result of the invention of a new algorithm.

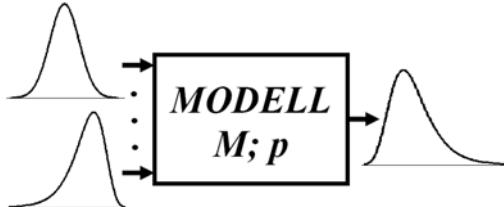


Fig. 1. The Monte-Carlo Simulation (source: [8])

The body of this simulation is that values of uncertain input variables are chosen randomly based on taken probability distributions. Using aforementioned input data determined above as excitation values solving the mathematical model to get an output of system (see Fig. 1.). The system behavior can be characterized by statistical analysis.

The biggest advantage of the Monte-Carlo Simulation is that it does not require complex and complicated analytical model investigations. Its disadvantage is that the mathematical model of investigated system should be solved scores of times to get acceptable population for statistical analysis which should require prolonged computing time.

The main step of Monte-Carlo Simulation is the random variables generation. Basically three generation methods are used:

- Inverse Transform Method;
- Composition Method;
- Acceptance-Rejection Method.

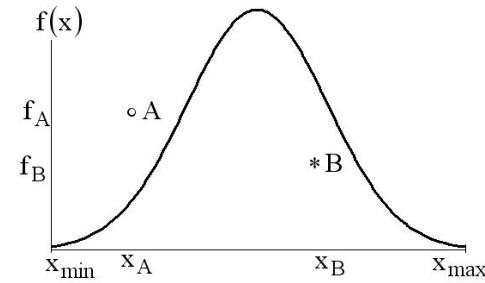


Fig. 2. Illustration of Acceptance-Rejection Method

During our investigation the Acceptance-Rejection (Hit and Miss) Method was used. This method is due to von Neumann and consists of sampling a random variate from an appropriate distribution and subjecting it to a test to determine whether or not it will be acceptable for use.

Firstly the  $f(x)$  probability density function and interval of the generated parameter should be determined (see Fig. 2.). They can be defined by statistical

investigation of the real measured data or they can be assumed by preliminary experiences.

Then two independent random values  $x$  from  $[x_{min}, x_{max}]$  and  $y_x$  from  $(0; 1)$  intervals are generated, and test to see whether or not the inequality

$$y_x < f(x) \quad (1)$$

holds:

- if the inequality holds, then accept  $x$  as a variate generated from  $f(x)$  (see B point in Figure 2.);
- if the inequality is violated, reject the pair  $x, y_x$  (see A point in Figure 2.) and try again.

The Acceptance-Rejection Method, which is simple to implement and can generate random numbers according to any distribution, whether it is integrable or not. The method has some drawbacks, however, which make it inferior to the transformation method in case integrable functions.

### 3. Application of Monte-Carlo Simulation (Case Study)

In this case study the illustrative system consists of only one lineal pipe and only one pipe fitting. Therefore, the pressure loss and head loss of a simple pipeline system and its two structural elements (lineal pipe and pipe fitting) will be investigated.

Table 1.

Technical System Parameters	
Diameter:	$d = 20 \text{ mm};$
Pipeline length:	$l = 4.2 \text{ m};$
Loss coefficient of pipe fitting:	$\zeta = 2.1;$
Minimal water temperature by measured data:	$t_{min} = 38 \text{ }^{\circ}\text{C};$
Maximum water temperature by measured data:	$t_{max} = 51 \text{ }^{\circ}\text{C};$
Average flow velocity:	$c = 0.1 \text{ m/s}$

The Table 1. shows technical parameters of the investigated (illustrative) system. For preliminary results of the model, the minimal and maximal temperature of water were determined by data measured in a simple solar collector hot water system during 317 days. The average flow velocity was determined by measuring the flow on a usual day using of the system.

#### 3.1. The Applied System of Equations

For Monte-Carlo Simulation, firstly the system's model should be depicted. It is important to mention that the following equations will compose the system of equations of the simulation, although some of them are elementary ones. The model of a water system's loss depends on the water temperature is

composed of following equations:

$$\rho = At^2 + Bt + C \quad , \quad (2)$$

where (by Table 2. and Figure 1.):

$$A = -0.0033 \text{ [kg/m}^3\text{C}^2\text{]};$$

$$B = -0.1193 \text{ [kg/m}^3\text{C]};$$

$$C = 1002.2 \text{ [kg/m}^3\text{].}$$

Table 2.

Water Density Depend on Temperature (source: [3])

	Water Temperature [°C]							
	30	35	40	45	50	55	60	70
Density [kg/m <sup>3</sup> ]	995.7	994.1	992.2	990.2	988.1	985.7	983.2	977.8

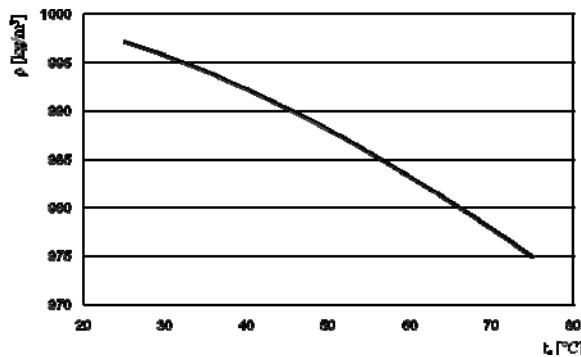


Fig. 3. Water Density Depend on Temperature

$$\mu(t) = \frac{\mu_0}{1 + Dt + Et^2} \quad , \quad (3)$$

where:

$\mu_0$  dynamic viscosity of  $t = 0^\circ\text{C}$  water:  $\mu_0 = 1.792 \text{ [mPas]}$ ;

$D = 0.0337 \text{ [1/}^\circ\text{C]}$ ;

$E = 0.00022 \text{ [1/}^\circ\text{C}^2]$ .

The kinematic viscosity:

$$\nu = \frac{\mu}{\rho} \quad . \quad (4)$$

The Reynolds-number:

$$Re = \frac{cd}{\nu} \quad . \quad (5)$$

Because of the simulated Reynolds numbers are in the  $2320 < Re < 8 \cdot 10^4$  interval (see Table 3.) and pipe is smooth, only the following equation was used to

determine pipe loss coefficients

$$\lambda_b = \frac{0,316}{\sqrt[4]{\text{Re}}} . \quad (6)$$

Losses of the pipe:

$$h'_{cs} = \frac{c^2}{2g} \frac{l}{d} \lambda , \quad (7)$$

Because the water temperature's effects to loss pressures and head losses are investigated, these parameters should be determined separately (for example to deduce the 4. conclusion!).

$$\Delta p_{cs} = \frac{\rho}{2} c^2 \frac{l}{d} \lambda . \quad (8)$$

Losses of the pipe fitting:

$$h'_{sz} = \frac{c^2}{2g} \xi , \quad (9)$$

$$\Delta p_{sz} = \frac{\rho}{2} c^2 \xi . \quad (10)$$

Losses of the system:

$$h' = h'_{cs} + h'_{sz} , \quad (11)$$

$$\Delta p = \Delta p_{cs} + \Delta p_{sz} . \quad (12)$$

Table 3. shows the preliminary results of model, that is equations (2) – (12) in cases of the minimum and the maximum temperatures.

Table 3.

**Preliminary Results of System of Equations**

Parameter	Results		Unit
	“minimum”	“maximum”	
t	38	51	[°C]
ρ	992.9014	987.5324	kg/m <sup>3</sup>
μ	0.00068969	0.00054453	Ns/m <sup>2</sup>
v	6.94618 10 <sup>-7</sup>	5.51403 10 <sup>-7</sup>	m <sup>2</sup> /s
Re	2879	3627	[–]
λ	0.043139	0.040719	[–]
h' <sub>cs</sub>	0.004617	0.004358	[m]
Δp <sub>cs</sub>	44.974	42.22188	[Pa]
h' <sub>sz</sub>	0.00107	0.00107	[m]
Δp <sub>sz</sub>	10.42546	10.36909	[Pa]
h'	0.005688	0.005429	[m]
Δp	55.39946	52.59097	[Pa]

### 3.2. The Simulation and its Results

During simulation, the system of equations (2) — (12) will be solved using water temperature generated randomly. For the simulation the water temperature data were measured by the Ltd. Merkapt in Debrecen, from 03<sup>rd</sup> of January to 13<sup>th</sup> of December 2010. The histogram of these 17.934 measured data can be seen in Figure 3. (Regarding paper limitations measurement data attachment is omitted.) The MiniTab<sup>®</sup> Release 14 software was used for statistical analysis. The Table 4. shows the results of analysis.

Table 4.

Result of Statistical Analysis of Water Temperature

Unit	Mean	Deviation	Minimum Value	Median	Maximum Value
°C	44.638	1.6	37.2	44.7	50.2

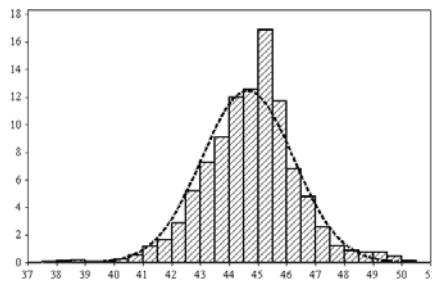


Fig. 4. Histogram of Measured Temperatures

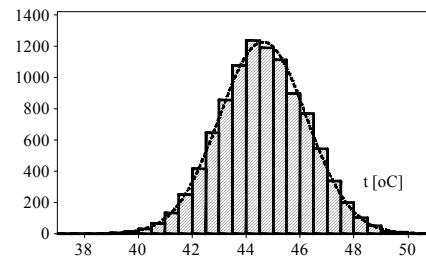


Fig. 5. Histogram of Generated Temperatures

During simulation to generate actual value of water temperature the acceptance-rejection method shown in Chapter 2. was used by density function.

$$f(t) = \frac{1}{\sqrt{3.2\pi}} e^{-\frac{(t-44.638)^2}{5.12}} \quad (13)$$

Number of model generations  $n$  was 10 000. The histogram of these generated temperatures can be seen in Figure 5.

Table 5.

Statistical Data of the Simulation Results

Parameter	Unit	Mean	Deviation	Minimum Value	Maximum Value
$t$	°C	44.624	1.619	38.166	50.796
$\rho$	kg/m <sup>3</sup>	990.3	0.67	987.63	992.84
$\mu$	Pa·s	0.00061	$1.79 \cdot 10^{-5}$	0.000546	0.000687

$v$	$\text{m}^2/\text{s}$	$6.1549 \cdot 10^{-7}$	$1.767 \cdot 10^{-8}$	$5.5327 \cdot 10^{-7}$	$6.924 \cdot 10^{-7}$
$\text{Re}$	-	3252.1	93.3	2888.4	3614.8
$\lambda$	-	0.041851	0.0003	0.040753	0.043105
$\Delta p_{cs}$	Pa	43.517	0.342	42.262	44.936
$h'_{cs}$	m	0.004479	$3.21 \cdot 10^{-5}$	0.004362	0.004614
$\Delta p_{sz}$	Pa	10.398	0.00704	10.37	10.425
$h'_{sz}$	m	0.00107	0	0.00107	0.00107
$\Delta p$	Pa	53.915	0.349	52.632	55.361
$h'$	m	0.00555	$3.21 \cdot 10^{-5}$	0.005432	0.005684

The block diagram of the simulation can be seen in Figure 6. The Table 5. and Figures 7. — 16. show the histograms of simulation results — the result obtained from 10 000 system model program running, that is applying equations (2) — (12).

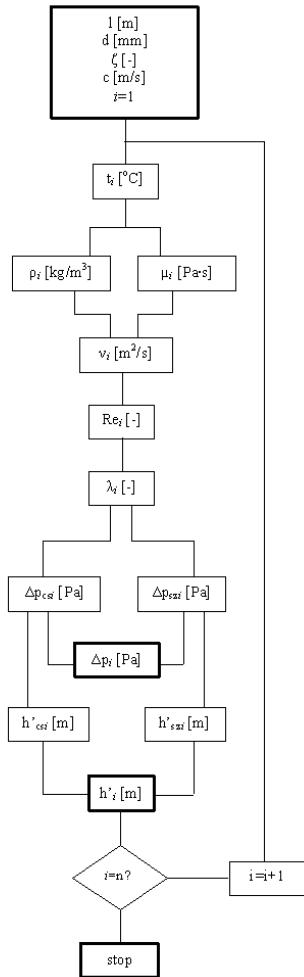


Fig. 6. Block Diagram of the Monte-Carlo Simulation

### 3.3. Discussions of Simulation's Results

From simulation's results the following conclusions can be deduced:

1. The Monte-Carlo Simulation can be used to analyze the effects arising from the characteristics of geothermal water.

Subsequent conclusions justify the Monte-Carlo Simulation method for evaluating the effects of the salinity of the water can be improved by supplementation of the applied model. (Unfortunately we do not currently have any information which will enable this model supplementation.)

2. In case of relatively high water temperature and low-speed flow, the Reynolds-number is higher than 2320.

It can be stated that flow is not stable laminar according to simulation results. This fact must be taken into account during the determination of pipe loss coefficients by Reynolds-numbers.

3. The pressure loss of the pipe fitting has marginal sensitivity on the water temperature.

As the Table 5. shows the relative deviation (ratio of the absolute values of the deviation and the mean value) of the pipeline pressure loss is  $7,86 \cdot 10^{-3}$ , and relative deviation of pipe fitting's pressure loss is  $6,77 \cdot 10^{-4}$ . The Figure 17. also confirms that shows the loss pressures of system, pipe and pipe fitting depending on the water temperature. It can be seen that loss pressure of pipe fitting curve is basically linear one on the same pressure scale.

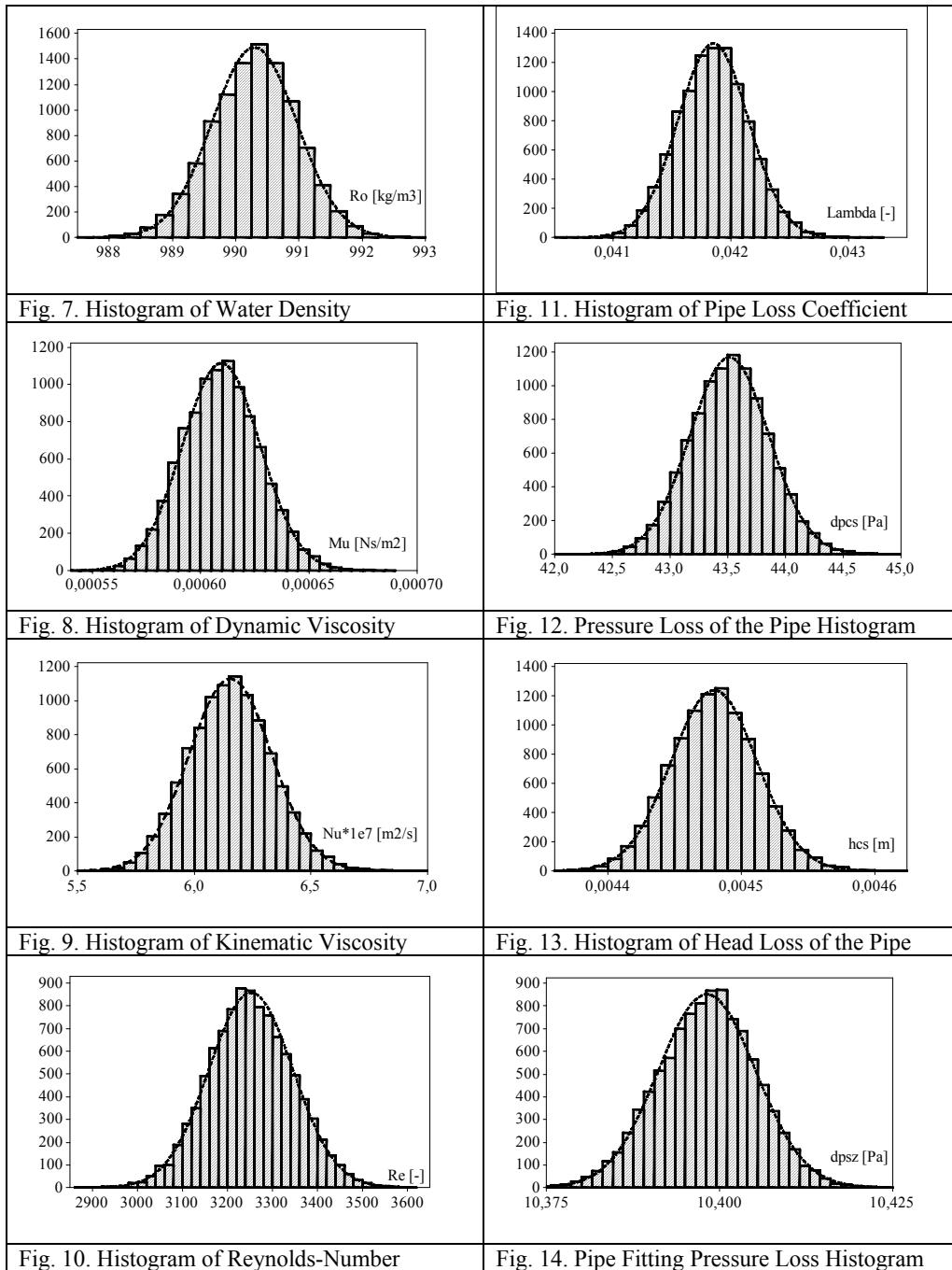
4. The water temperature has no influence on the head loss of the pipe fitting. Table 5. also shows that the head losses of the pipe fittings are 0,00107 meters in cases of all the 10 000 generations. This conclusion has been draw by Pokorádi, as a result of matrix algebraic sensitivity analysis of the pipeline system. See Figures 4. – 7. of the reference [11].

5. During investigation of a complex pipeline system, the equivalent pipe length cannot be used.

Because of the water temperature has no influence on the head loss of the pipe fitting; the using of its equivalent pipe length leads to incorrect results. The water salinity has an effect on the head loss and pressure loss of the system as well; therefore during investigation of its effect similar conclusion can be deduced.

6. Deviations of system's and pipe's head losses are equal.

This is an outcome from Conclusion 4.



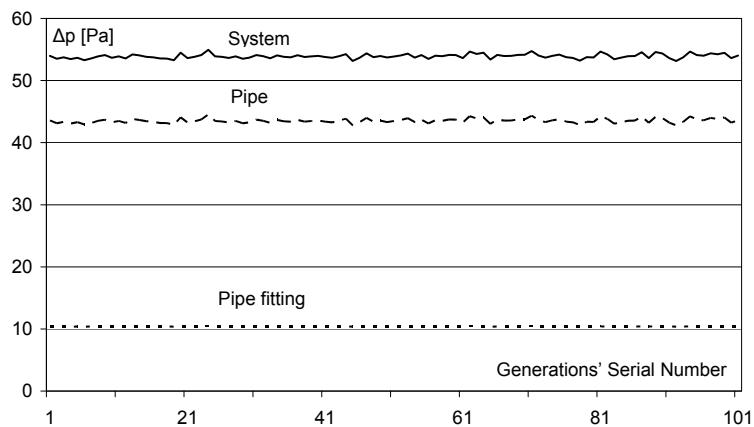
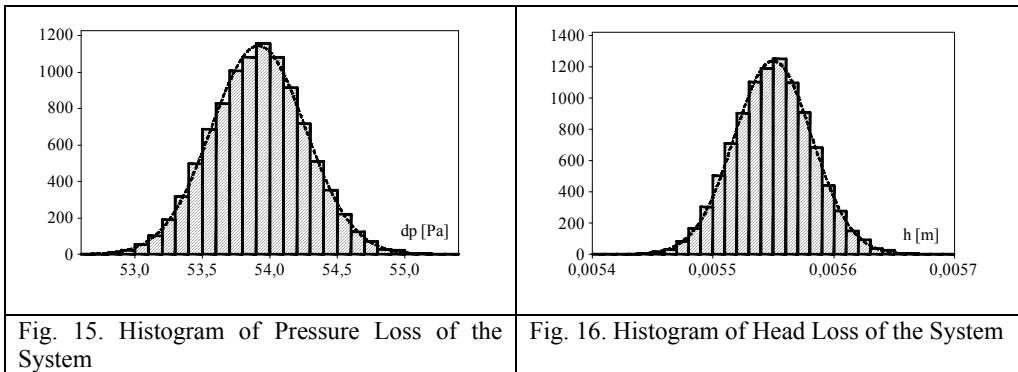


Fig. 17. Pressure Losses by Generations' Serial Number

#### 4. Closing Remarks, Future Works

The authors of this paper showed the methodology of the Monte-Carlo Simulation and its possibility of use to investigate influences of fluid parameters on system losses by a simple pipeline system model and discussions about simulation results. This work has proved that The Monte-Carlo Simulation can be used to analyze the effects arising from the characteristics of geothermal water. The obtained consequents and experiences can be used to investigate parametrical uncertainties of the geothermal pipeline system, such as fluid characteristic's indeterminations. The methodology of the Monte-Carlo Simulation, which based uncertainty analysis, has been shown by a short case study of an easy pipeline system.

During prospective scientific research related to this field of applied mathematics and technical system modeling the authors would like to complete the following tasks:

- the Monte-Carlo Simulation of multiple pipeline system and pipe-network;
- data collection for depicting correctly the influence of the salinity to system losses and required pump power;
- to investigate interconnection between temperature and exploited volume of geothermal water and its influence on system losses.

## 5. List of Symbols

$\mu$	—	dynamic viscosity $\left[ \frac{\text{Ns}}{\text{m}^2} \right]$ ;
$\rho$	—	fluid density $\left[ \frac{\text{kg}}{\text{m}^3} \right]$ ;
$\nu$	—	kinematic viscosity $\left[ \frac{\text{m}^2}{\text{s}} \right]$ ;
$c$	—	average flow velocity $\left[ \frac{\text{m}}{\text{s}} \right]$ ;
$d$	—	internal diameter [ $\text{m}$ ];
$l$	—	tube length [ $\text{m}$ ];
$\text{Re}$	—	Reynolds-number [-];
$t$	—	temperature [ $^{\circ}\text{C}$ ];
$\lambda$	—	pipe loss coefficient [-];
$h'_{cs}$	—	head loss of pipe [ $\text{m}$ ].
$\Delta p_{cs}$	—	the pressure loss of the pipe [ $\text{Pa}$ ];
$h'_{sz}$	—	the head loss of the pipe fitting [ $\text{m}$ ];
$\Delta p_{sz}$	—	the pressure loss of the pipe fitting [ $\text{Pa}$ ];
$\xi$	—	pipe fitting loss coefficient [-];
$x$	—	general parameter [-];
$y$	—	temporary parameter [-];
$n$	—	number of model excitations [-].

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