

OPTIMAL POWER FLOW BASED ON PARTICLE SWARM OPTIMIZATION

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Optimal Power Flow (OPF) is one of the most important requirements in all developed power system. It is an optimization problem try to make a re-distribution of the active and reactive power caused a minimizing of an Objective Function with respect of a set of technical and economical constraints. This process involves adjustment and use a set of control variables. Usually, control variables are the generator voltage magnitude, transformer tap changing, injection shunt capacitance and generator active power at PU bus, while the state variables are the active power at the slack bus, the load voltage and the generator reactive power. Particle Swarm Optimization (PSO) is used in this paper to solve the OPF problems according to the two Objective Functions: active power losses and Fuel Cost. The proposed algorithm is applied and tested for IEEE 30 bus and shows better results when compared to other previous work.

Keywords: Optimal Power Flow, minimization of active power losses and Fuel Cost, Particle Swarm Optimization

1. Introduction

The OPF that was initially developed by Carpentier in 1962 and used to find the minimum generation cost of generator units in case of normal operation conditions holding classical power flow results within operation limits [1].

Generally two mainly types of optimization techniques have been used to solve the problem of OPF. The first one is known the classical or conventional optimization techniques. Different classical optimization techniques have been applied in solving the OPF problems such as Gradient base, Linear programming, Non linear program, Quadratic programming, etc. However all of these methods suffer from main problems such as: They may not be able to provide an optimal solution and usually getting stuck at a local optimal; all these methods are based on assumption of linearity, continuity and differentiability of Objective Function which is not actually allowed in a practical system. Also these methods depend on the assumption of convex system of the Objective Function while OPF problem is

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an optimization problem with a highly non linear, non convex system, non-smooth Objective Function [2, 3].

In order to overcome the limitations of the classical (conventional) optimization techniques, the second type of optimization techniques which are known the Artificial Intelligence or Heuristic Optimization techniques has been used, such as Genetic Algorithm, Differential Evolution and Particle Swarm Optimization, etc. This type of optimization technique inspired from the natural phenomena or the social behavior of humans or animals [4, 5].

2. The mathematical model of Optimal Power Flow

Mathematically, the OPF problem can be formulated as follows [6, 7]

$$\begin{cases} \text{minimum } f(x_1, x_2) \\ g(x_1, x_2) = 0 \\ h(x_1, x_2) \leq 0 \end{cases} \quad (1)$$

where f is the Objective Function to be minimized; g is a function corresponding to the equality constraints that represent typical load flow equations; h is a function corresponding to the system operating constraints.

x_1 is the vector of dependent variables (state vector) consisting of active generating power at slack bus P_{G1} ; Load-bus voltage U_L and generator reactive power outputs Q_G .

x_2 is the vector of independent variables (control variables) consisting of generator real power output P_G at PU bus; Generator voltage U_G ; Shunt VAr compensation Q_C and transformer tap setting T_i .

The equality constraints represent typical load flow equations. i.e. active and reactive power balance at each node given by the next equations:

$$P_i - U_i \sum_{j=1}^{NB} U_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \quad (2)$$

$$i = 1, 2, \dots, NB-1 \text{ and } P_i = P_{Gi} - P_{Li}$$

$$Q_i - U_i \sum_{j=1}^{NB} U_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \quad (3)$$

$$i = 1, 2, \dots, NL \text{ and } Q_i = Q_{Gi} - Q_{Li}$$

where P_i is the active power injected into network at bus i ; Q_i is the reactive power injected into network at bus i ; P_{Gi} is the active power generation at bus i ; Q_{Gi} is the reactive power generation at bus i ; P_{Li} is the load active power at bus i ; Q_{Li} is the load reactive power at bus i ; NB is the total number of buses; G_{ij} is the conductance of the branch $i j$; B_{ij} is the susceptance of the branch $i j$; $NB-1$ the total number of buses excluding slack bus; NL is the number of load buses [8, 9].

The system operating constraints include:

- The inequality constraints on control variables limits are described as

$$\begin{aligned}
 U_{Gi}^{min} &\leq U_{Gi} \leq U_{Gi}^{max} & i=1,2,\dots,NG \\
 P_{Gi}^{min} &\leq P_{Gi} \leq P_{Gi}^{max} & i=1,2,\dots,NG \quad \text{for PU bus} \\
 T_i^{min} &\leq T_i \leq T_i^{max} & i=1,2,\dots,NT \\
 Q_{Ci}^{min} &\leq Q_{Ci} \leq Q_{Ci}^{max} & i=1,2,\dots,NC
 \end{aligned}$$

where U_{Gi}^{min} , U_{Gi}^{max} are the lower and upper voltage limit of generator i ; T_i^{min} , T_i^{max} are the lower and upper tap changing limit of the transformer i ; Q_{Ci}^{min} , Q_{Ci}^{max} are the lower and upper limit reactive power compensator of shunt injection capacitor i ; P_{Gi}^{min} , P_{Gi}^{max} are the generator lower and upper active power limit at PU bus i ; NG , NT and NC are the number of generators, number of the regulating transformers and number of VAr compensators, respectively.

- The inequality constraints on state variables limits are given by

$$\begin{aligned}
 U_{Li}^{min} &\leq U_{Li} \leq U_{Li}^{max} & i=1,2,\dots,NL \\
 Q_{Gi}^{min} &\leq Q_{Gi} \leq Q_{Gi}^{max} & i=1,2,\dots,NG \\
 P_{Gs}^{min} &\leq P_{Gs} \leq P_{Gs}^{max}
 \end{aligned}$$

where U_{Li}^{min} , U_{Li}^{max} are the lower and upper voltage limit of load-bus i ; Q_{Gi}^{min} , Q_{Gi}^{max} are the lower and upper reactive power limit of generator i and P_{Gs}^{min} , P_{Gs}^{max} the lower and upper active power limit of slack generator.

The following Objective Functions are considered in this paper:

- **The active power losses of the system (MW)**

The total real power losses (F) is given by

$$F = P_{loss} = \sum_{k=1}^{NE} G_{ij} (U_i^2 + U_j^2 - 2U_i U_j \cos\theta_{ij}) \quad (4)$$

where P_{loss} is the network active power losses; U_i & U_j are the voltage magnitude at buses i & j respectively; G_{ij} is the mutual conductance between bus i and j ; θ_{ij} is the voltage angle difference between bus i and j ; NE is the number of branches in the system [10].

- **The Fuel Cost (\$/h)**

The total system cost is modelled as the sum of the cost function of each generator as shown in (5). The generator Fuel Cost curves are modelled with smooth quadratic functions and measured by the unit \$/h as below:

$$F = Cost = \sum_{i=1}^{NGT} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad (\$/h) \quad (5)$$

where NGT is the number of thermal units, P_{Gi} is the active power generation at unit i and a_i, b_i, c_i are the Cost Coefficients of the i th generator [11].

It should be noted that the control variables are self constrained and the state variables are constrained by adding them to the objective function. Therefore the new objective function is generalized as

$$\min f = F + \lambda_v \sum_{i=1}^{NL} \Delta U_{Li}^2 + \lambda_Q \sum_{i=1}^{NG} \Delta Q_{Gi}^2 + \lambda_s \Delta P_s^2 \quad (6)$$

$$\Delta U_{Li} = \begin{cases} U_{Li} - U_{Li}^{max} & (U_{Li} > U_{Li}^{max}) \\ 0 & (U_{Li}^{min} < U_{Li} < U_{Li}^{max}) \\ U_{Li}^{min} - U_{Li} & (U_{Li} < U_{Li}^{min}) \end{cases} \quad (7)$$

$$\Delta Q_{Gi} = \begin{cases} Q_{Gi} - Q_{Gi}^{max} & (Q_{Gi} > Q_{Gi}^{max}) \\ 0 & (Q_{Gi}^{min} < Q_{Gi} < Q_{Gi}^{max}) \\ Q_{Gi}^{min} - Q_{Gi} & (Q_{Gi} < Q_{Gi}^{min}) \end{cases} \quad (8)$$

$$\Delta P_s = \begin{cases} P_s - P_s^{max} & (P_s > P_s^{max}) \\ 0 & (P_s^{min} < P_s < P_s^{max}) \\ P_s^{min} - P_s & (P_s < P_s^{min}) \end{cases} \quad (9)$$

where λ_v , λ_Q and λ_s are penalty factor of load-bus, generator reactive power and slack active power violation respectively. ΔU_{Li} is the violation voltage of load-bus i ; ΔQ_{Gi} is the reactive power violation of generator i ; ΔP_s is the active power violation of slack bus and F is the Objective Function (active power losses or Fuel Cost) [12]

3. Particle Swarm Optimization (PSO)

PSO is one of the modern heuristic optimization techniques developed by Kennedy and Eberhart in 1995. It is a population technique based on the evolutionary computation inspired from the social behaviors of bird flocking or fish schooling for searching the food [4, 7]. The word *particle* means as a fish in schooling or bird in a flock. The aim of PSO technique is to find the optimal solution using a population of particles, where each particle represents a candidate solution to the problem. These particles constitute a swarm. The particles fly over the search space with a random velocity looking for the optimal solution (minimum path for the food) and each particles change its position according to its own experience (own intelligence), and the experience of neighboring particles (intelligence of the swarm). The experience of each particle is a memory that

keeps the path of the particle in the best position according to the previous best position. The best position of each particle is called the individual best position or local best position (p_{best}), while the best value over all the individual best position (p_{best}) of the particles in the swarm is called the global best position (g_{best}). The particles always update its position and velocity towards their p_{best} and g_{best} positions at each time step [13]

The basic elements of the PSO technique

The basic elements of the PSO can be illustrated as follows [7, 13]:

*Particle x_{id}

It is a candidate solution of the control variables where $i = 1, 2, \dots, n$ & $d = 1, 2, \dots, D$; n is the number of control variables; D is the number of candidates (particles) of each control variables. Assume the vector of the control variables are $[X_1, X_2, X_3, X_4, \dots, X_n]$, then:

- The set of particles of 1st control variables X_1 are $\{x_{11}, x_{12}, x_{13}, \dots, x_{1D}\}$;
- The set of particles of 2nd control variables X_2 are $\{x_{21}, x_{22}, x_{23}, \dots, x_{2D}\}$ and so on for the n th control variables;
- The set of particles of n th control variables X_n are $\{x_{n1}, x_{n2}, x_{n3}, \dots, x_{nD}\}$.

Each particle represents a position in the search space solution.

*Population

The vector of the control variables $[x_{11}, x_{21}, x_{31}, \dots, x_{n1}]^k$ is one of the populations in the swarm at iteration k . Swarm may be defined as the total number of the populations in the whole search spacing.

*Particle velocity v_{id}^k

Particle velocity is the velocity of particles movement in the swarm population at iteration k .

Individual best position (x_{id}^ or $p_{best_{id}}$)

The best position that related with the best fitness value for each particle is called the individual best position (local best position).

*Global best position (x_i^{**} or g_{best_i})

Global best is the best position among all of the individual best positions achieved so far, where $g_{best_i}^k$ represent the best position over the individual best position (global position) for the i -th control variable at iteration k .

*Velocity updating

The id -th particle velocity is updated according to the following equation:

$$v_{id}^{k+1} = w v_{id}^k + c_1 \times rand_1 \times (pbest_{id}^k - x_{id}^k) + c_2 \times rand_2 \times (gbest_i^k - x_{id}^k) \quad (10)$$

where v_{id}^k is the velocity of particle at iteration k ; v_{id}^{k+1} is the current velocity of particle x_{id} at iteration $k+1$; x_{id}^k is the particle position at iteration k ; w is the inertia weight; c_1 & c_2 are a randomly choosing number; $rand_1$ & $rand_2$ are a uniformly distributed random number between $[0,1]$; k is the iteration number.

If a particle violates the velocity limits, the algorithm set its velocity equal to the violated limit.

*Inertia weight

The weight factor must be chosen in a way to make a faster convergence, it is sensible to make a balance of local and global search and choose a large value of the weight factor for the initial iterations and gradually reduce weight factor in successive iterations as in equation (11)

$$w = w_{max} - k * (w_{max} - w_{min}) / w_{min} \quad (11)$$

where $w_{max} = 0.9$; $w_{min} = 0.4$; k is currently iteration number; $iTmax$ is the maximum iteration number.

*Position update

The current position can be update using (12)

$$x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \quad (12)$$

If a particle violates the position limits, the algorithm set its position to the violated limit.

*Individual best position updating ($pbest_{i,d}^{k+1}$)

In the first, we will calculate the Objective Function $f(x_{id}^{k+1})$ for the new position x_{id}^{k+1} (updated position)

(1) Secondly, we will compare $f(x_{id}^{k+1})$ with $f(pbest_{id}^k)$ as follow:

If $f(x_{id}^{k+1}) < f(pbest_{id}^k)$ then

$$f_{new}^{k+1} = f(x_{id}^{k+1}) \text{ and } pbest_{id}^{k+1} = x_{id}^{k+1} \quad (13)$$

If $f(x_{id}^{k+1}) \geq f(pbest_{id}^k)$ then

$$f_{new}^{k+1} = f(pbest_{id}^k) \text{ and } pbest_{id}^{k+1} = pbest_{id}^k \quad (14)$$

*Updating the global best position ($gbest_i^{k+1}$)

(1) Find the Objective Function for the new global best position ($gbest_i^{k+1}$)
 (2) Compare $f(gbest_i^{k+1})$ with $f(gbest_i^k)$ as follow

If $f(gbest_i^{k+1}) < f(gbest_i^k)$ then

$$f(gbest_i^{k+1}) = f(gbest_i^{k+1}) \text{ and } gbest_i^{k+1} = gbest_i^{k+1} \quad (15)$$

If $f(gbest_i^{k+1}) \geq f(gbest_i^k)$ then

$$f(gbest_i^{k+1}) = f(gbest_i^k) \text{ and } gbest_i^{k+1} = gbest_i^k \quad (16)$$

***Stopping criteria**

The search will stop if the number of iterations reaches the maximum.

The flow chart of PSO for OPF is shown in the Fig. 1

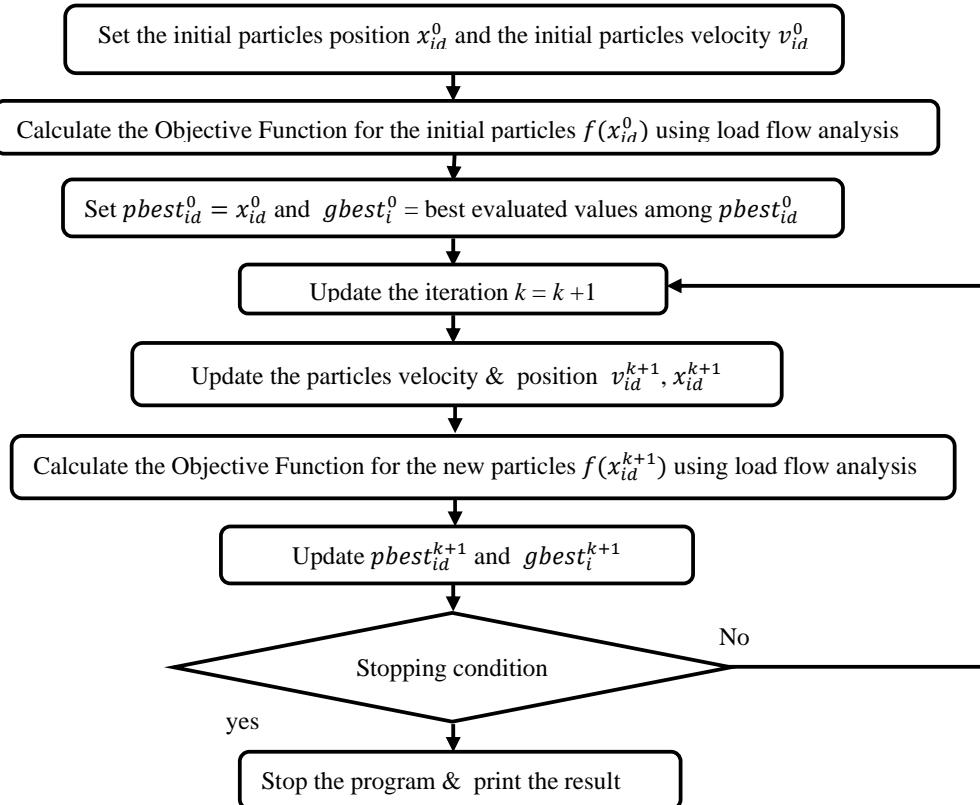


Fig. 1 Flow chart of the Particle Swarm Optimization in Optimal Power Flow

4. The application of OPF based on PSO on IEEE 30 buses

The IEEE 30 bus represent a middle case between the small systems like IEEE 6, 14 bus and large systems like 57 and 118 bus. Many practical systems especially the extra high voltage systems are similar to the IEEE 30 bus. Also this system contains the four types of the control variables: generator active power, generator voltage, transformer tap changing and shunt injection capacitance. For these reasons many authors prefer this system in their application. In the system IEEE 30 bus as in Fig. 2 [6, 14], the bus 1 is the slack bus. Also this system contains 24 control variables as follow:

- 6 generators voltage magnitude ($U_{G1}, U_{G2}, U_{G5}, U_{G8}, U_{G11}, U_{G13}$);
- 4 Transformers tap changing ($T_{4-12}, T_{6-9}, T_{6-10}, T_{28-27}$);
- 9 VAr compensators according to the shunt injection capacitances ($Q_{C10}, Q_{C12}, Q_{C15}, Q_{C17}, Q_{C20}, Q_{C21}, Q_{C23}, Q_{C24}, Q_{C25}$);
- 5 generators active power at PU bus ($P_{G2}, P_{G5}, P_{G8}, P_{G11}, P_{G13}$)

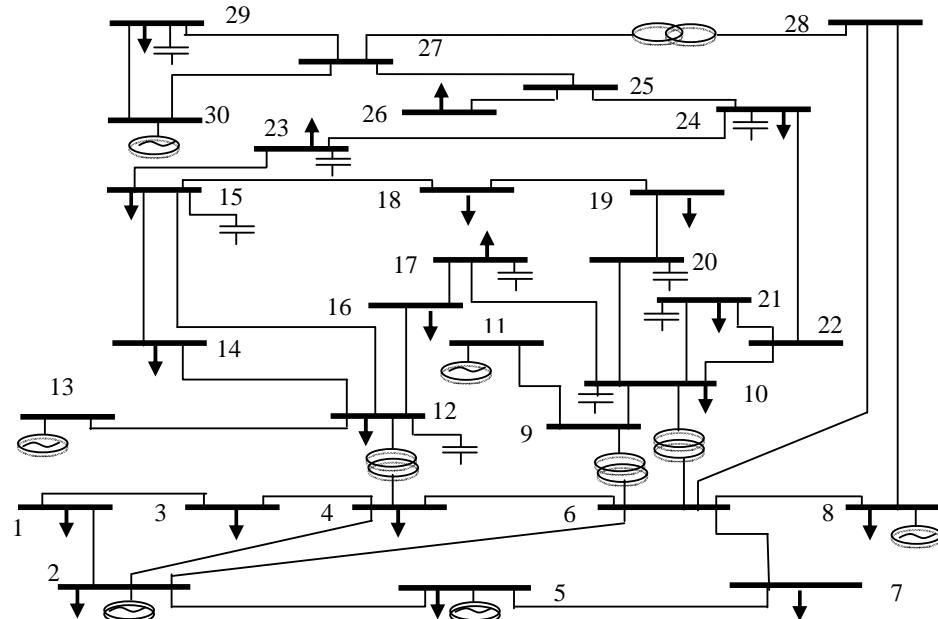


Fig. 2 IEEE 30 bus system

Different candidate number ($D = 4, 11$ and 20) are used for the OPF based on PSO of the IEEE 30 bus with two Objective Function (active power losses and Fuel Cost) treated with each one separately as shown in Table 1.

According to Table 1, PSO reduce the Objective Function active power losses from the initial state 5.8419 MW to the optimal state 3.0329 (the reduction

in the active power losses is 2.809 MW). Also PSO reduce the Objective Function Fuel Cost from the initial state 902.05 \$/h to the optimal state 801.66 \$/h (the reduction in the Fuel Cost is 100.39 \$/h (Economic Dispatch)). Therefore applying the Economic Dispatch help us to save money as follow:
 $100.39 \times 24 \times 30 \times 12 = 867\ 369$ \$/year.

Table 1
Two Objective Function (active power losses and Fuel Cost) based on Particle Swarm Optimization of the IEEE 30 bus with different candidates number D

	Initial	Number of Candidate		
		D = 4	D = 11	D = 20
Active power losses (MW) (Objective Function)	5.8419	3.3691	3.0329	3.0663
Reduction (%)		42.23	48.08	47.51
Fuel Cost (\$/h) Objective Function	902.05	801.68	801.66	803.327
Reduction (%)		11.126	11.129	10.944

Tables 2 and 3 show the comparisons between different optimization techniques and the proposed algorithm PSO for the IEEE 30 bus based on the two Objective Functions active power losses and Fuel Cost respectively.

Table 2
Comparison between different optimization techniques and the proposed algorithm PSO according to the active power losses of IEEE 30 bus

References	Optimization techniques	Active power losses (MW)
[15]	Standard Genetic Algorithm	5.011
[15]	Particle Swarm Optimization	5.092
[15]	conventional Interior Point Method	5.101
[16]	Differential Evolution	4.720
[17]	Differential Evolution (DE)	4.760
[18]	Real coded Genetic Algorithm	4.501
Proposed algorithm	Particle Swarm Optimization	3.0329

Table 3
Comparison between different optimization techniques and the proposed algorithm PSO according to the Objective Function Fuel Cost for IEEE 30 bus

References	Optimization techniques	Fuel Cost (\$/h)
[1]	Genetic Algorithm	884.8
[1]	Particle Swarm Optimization	880.05
[14]	Gradient	804.85
[19]	Differential Evolution	803.05
[19]	Imperialistic Competitive Algorithm	802.881
[20]	Improved PSO	802.63
[20]	Evolutionary Programming	802.62
Proposed algorithm	Particle Swarm Optimization	801.66

Figs. 3 and 4 show the variation of the Objective Function active power losses and Fuel Cost respectively with respect to the number of iteration, based on PSO at different candidate number.

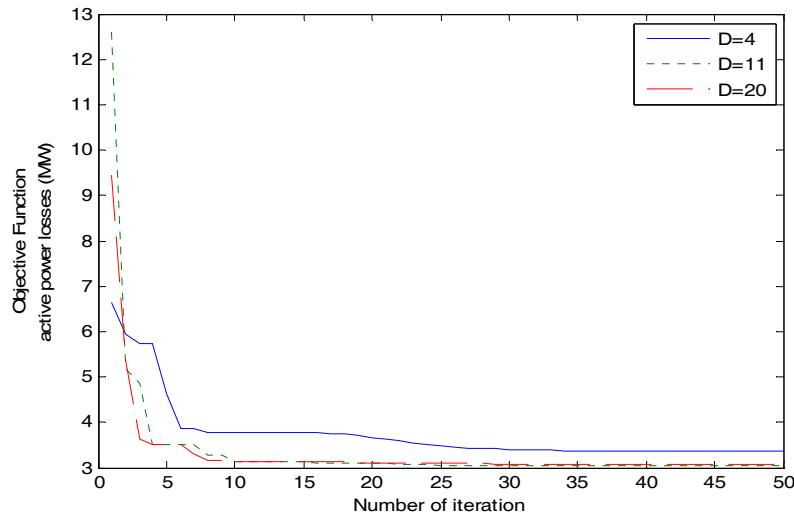


Fig. 3 The active power losses based on PSO at different candidates number D for IEEE 30 bus

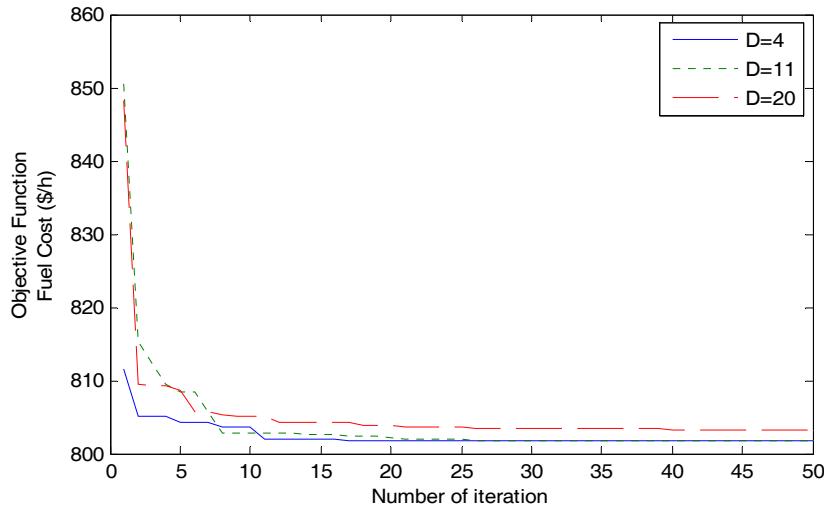


Fig. 4 The Fuel Cost based on PSO at different candidates number D for the IEEE 30 bus

5. Conclusions

The Optimal Power Flow is one of the important and necessary issue that make the power system more security and economically. OPF is an optimization process which search through the control variables to minimize the Objective Function and satisfying the constraints imposed. The proposed algorithm PSO has been applied for IEEE 30 bus with four type of control variables and different candidates numbers ($D = 4, 11$ and 20). The control variables are the generator voltage magnitude, transformer tap changing, shunt injection capacitance and generator active power at PU bus. The state variables are the active power at slack bus, load bus voltage and generator reactive power. Two Objective Functions are used for OPF problem, the active power losses and the Fuel Cost deals with each one separately. The proposed algorithm PSO provides better results at candidate number $D = 11$ when compare with other optimization techniques. All the programs need for OPF problem are written by the authors on Matlab software.

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