

NUMERICAL SIMULATION OF THE FLOW AROUND CIRCULAR CYLINDER WITH SLITS

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The flow around a cylinder with slits has been investigated numerically. The main purpose of this study is to show the effect of the slits on the vortex detachment behind a cylinder smooth, and stationary using the finite element approach to solve the fluid governing equations system at $Re = 200$. The variation of the spacing ratio (D/S) of the slit is strongly influenced both the various physical parameters and the vortex detachment behind a cylinder. We found that the drag coefficient varies between 1.43 and 1.24 in the case of the horizontal slit and between 1.45 and 1.39 in the case of the vertical slit. In the case of the double horizontal-vertical slit, the values vary between 1.47 and 1.38. We also found that the values of the Strouhal number vary between 0.201 and 0.268 according to the spacing ratio (D/S) of the slit. A comparison was made with experimental works for the Strouhal number. The results are very important, and the error found is less than 7%.

Keywords: Slit, Cylinder, Detachment, Vortex, Spacing ratio.

1. Introduction

The geometric shape of the cylinder strongly influences the wake behind a cylinder on the hydrodynamic forces. Several articles and scientific research papers based on the study of this physical phenomenon and the importance of the problem have been found [1-6].

These phenomena can be found in many applications: electronic components, electrical cables, buildings, heat exchangers, hydraulic networks, pipelines and turbine axles [3], [4], [6]. A group of articles discussing the characteristics of the flow around a cylinder at $Re = 2.67 \times 10^4$, are studied experimentally by Gao et al. [1] they found that the nature of the wake flow of the cylinder is strongly modified with the establishment of a slit. Various cluster

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arrangements in terms of the number of pipes (cylinders) treated by Zdravkovich [3] concluded the effect of pipe movement on vortex detachment. The change in vortex detachment in the synchronization range for a single oscillating pipe was shown to be excessive. Gonçalves et al. [7] presented a flow around circular cylinders with very low aspect ratio when carried out in a circulating water channel. The results showed a decrease in drag force coefficients with a decrease in aspect ratio (L/D). The two-dimensional and three-dimensional wake structures behind a cylinder have also been studied by Thompson et al. [8] investigations have shown there is a transfer of energy to mode B, that has a wavelength about a quarter of the length of mode A. The impact of the vertical arrangement of two cylinders on the flow with a high Reynolds number was studied via numerical modeling by Rahimi et al. [2] The results indicated that the flow pattern is irregular and unstable for denser arrangements, while the propagation of the vortices has no tendency in different spacings. The numerical results by Sohankar et al. [6] at Reynolds $Re = 70-150$ and a Prandtl number $Pr = 0.71$, with the shear rate $K = 0-0.2$ for flow around two square cylinders in tandem, found that the amplitude of lift increases linearly with K . For $Re = 150$ to 500 , Roshko [9], [10] observed an irregular regime with turbulent velocity fluctuations accompanying the periodic formation of vortices. He also showed that the wake width, drag coefficient and the distance between free streamlines for a cylinder are functions of basic pressure parameter only. Alam et al. [11] investigated the effects of the spacing ratio (g/D) for flow around four identical circular cylinders placed side by side. They found that the hydrodynamic forces increased exponentially with decreasing gap spacing ratio (g/D). On the other hand, Zhao et al. [12] observed that the lift force of a large cylinder and a small cylinder fluctuate at the same frequency, and the Strouhal number of the total fluctuating lift is always less than that of the equivalent cylinder. An experimental study of the interference between two circular cylinders treated by Assia et al. [13] observed the peak amplitude of an isolated cylinder case was reduced by 50% compared to a downstream cylinder. A computational algorithm developed by Meneghini et al. [14] of two cylinders arranged side-by-side observed the formation process and shed vortices from the second body due to the downstream cylinder. Also, Bearman and Wadcock [15] investigated the flow around two cylinders and they found that a repulsive force generated behind the two cylinders. In addition, Yangyang et al. [16] found the (L/D) spacing ratio influenced the vortex detachment and reattachment of the shear layer. Liu et al. [17] found the variation of the lift and drag coefficients depended on the elastic modulus and the Poisson's ratio. The latter influenced the frequency of vortex separation (vortex detachment).

The goal of the study is try to modify the wake and enhance the structural aspects of risers behind a cylinder smooth and stationary, for this, we varied the different slit spacing ratios (D/S) of a cylinder and its positions (horizontal,

vertical, and horizontal-vertical) on vortex detachment and hydrodynamic forces. For this, we varied the spacing ratio (D/S) between 32.0, 20.0, 12.5, and 8.0. Then we calculated the average drag coefficient, the average lift coefficient and the Strouhal number of each case and compared them.

2. Problem statement and formulation

The computational domain and boundary conditions for a cylinder with horizontal and vertical slits are represented in Fig. 1. The domain (a rectangular area of $15D \times 5D$) with one inlet, three outlets and a cylinder with slots insides, the cylinder located in the middle of the domain is away from the inlet by $5D$. Shown in Fig. 1 is as follows: a diameter cylinder ($D = 2\text{cm}$) with horizontal and vertical slits, the slit spacing ratio (D/S) is changed between: $(D/S) = 32.0, 20.0, 12.5$, and 8.0 .

The main boundary condition at the inlet is the velocity, but we can manipulate it and change it through the following parameters: the Reynolds number $Re = 200$, density $\rho = 997.0 \text{ Kg/m}^3$, dynamic viscosity $\mu = 8.899 \times 10^{-4} \text{ kg/m}\cdot\text{s}$, and the pressure variation along the axis (x) $\partial p/\partial x = 0$. In outlet: free flow conditions are considered; namely $\partial u_1/\partial x_1 = 0$, $\partial u_2/\partial x_2 = 0$ and relative pressure $p_{\text{rel}} = 0$ ($p=1 \text{ atm}$). No slip boundary conditions are considered on the cylinder surfaces: $u_1 = 0$ and $u_2 = 0$. And in the boundary conditions of symmetrical are: $\partial u_1/\partial x_2 = 0$, $u_2 = 0$ and $\partial p/\partial x_2 = 0$.

It is assumed that the distribution of the velocity at the inlet is uniform and parallel, so the initial conditions are as follows: $u_1 = U_0$, and $u_2 = 0$.

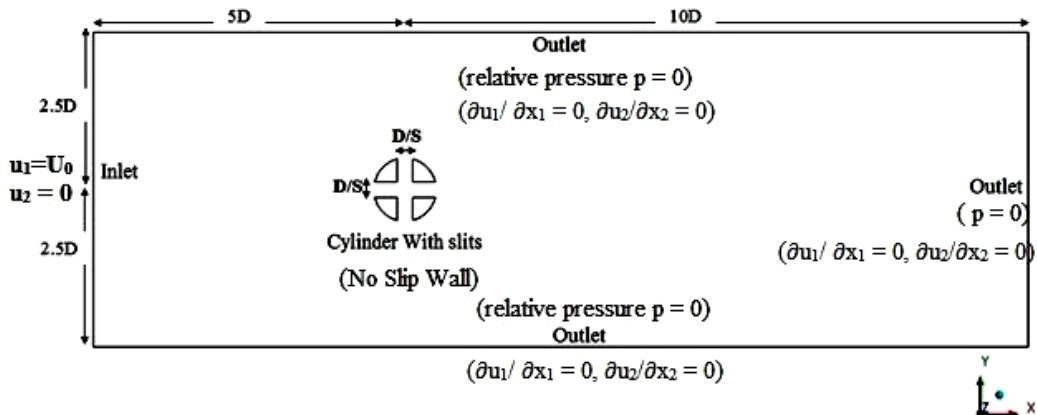


Fig. 1. Computational domain and boundary conditions for cylinder with slits

The second-order backward Euler transient scheme is selected for the transient CFD simulations with high-resolution advection scheme. The mesh of

22080 nodes \times 10800 elements is structured quadrilateral generated by ANSYS workbench is shown in Fig. 2.

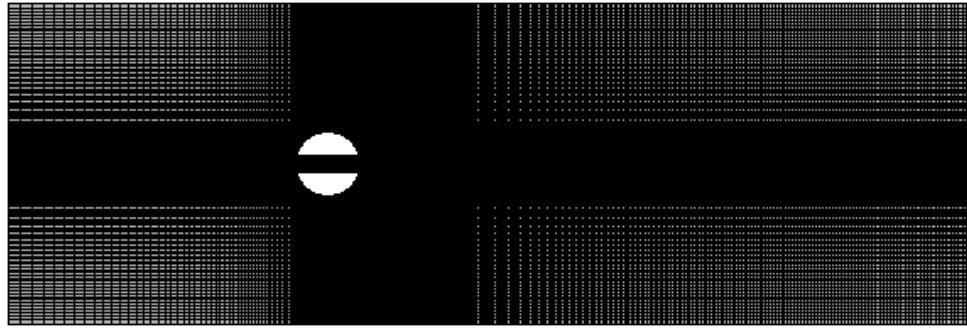


Fig. 2. Mesh of cylinder with horizontal slit for D/S = 12.5

The governing Navier-Stokes equations of continuity and momentum along the x axis and along the y axis for unsteady, laminar, viscous, incompressible, without heat transfer are written as follows [18]:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

$$\left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \left(\frac{\partial^2 u_i}{\partial x_j \partial x_j} \right) \quad (2)$$

Where u_i is the velocity component in the x and y directions (in this study $x_1 = x$ and $x_2 = y$), p is the pressure, t is the time, and Re is the Reynolds number in this flow is defined as $Re = \frac{\rho U_0 D}{\mu}$, where U_0 is the inlet velocity, D is the diameter of the cylinder, and μ is the dynamic viscosity of the fluid.

The drag coefficients C_D and lift C_L given by the following expressions [19]:

$$C_D(t) = \frac{F_D(t)}{\frac{1}{2} \rho D U_0^2} = - \int_0^{2\pi} p(t) \cos \theta d\theta - \int_0^{2\pi} \frac{1}{Re} \left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \right) \sin \theta d\theta \quad (3)$$

$$C_L(t) = \frac{F_L(t)}{\frac{1}{2} \rho D U_0^2} = - \int_0^{2\pi} p(t) \sin \theta d\theta + \int_0^{2\pi} \frac{1}{Re} \left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \right) \cos \theta d\theta \quad (4)$$

Where and are the streamwise and transverse forces, u_1 and u_2 are the velocity component in the x_1 and x_2 directions, respectively, U_0 is the inlet velocity, D is the diameter of the cylinder, ρ is the density of the fluid, θ is an angle in the counterclockwise direction.

The mean hydrodynamic drag and lift coefficients are calculated by:

$$\bar{C}_D = \frac{1}{\Delta T} \int_{t_1}^{t_2} \frac{2F_D(t)}{\rho D U_0^2} dt \quad (5)$$

$$\bar{C}_L = \frac{1}{\Delta T} \int_{t_1}^{t_2} \frac{2F_L(t)}{\rho D U_0^2} dt \quad (6)$$

Where $\Delta T = t_2 - t_1$ is the time difference between the instant t_1 and t_2 .

The Strouhal number (St) is given by:

$$St = \frac{f_t \times D}{U_0} \quad (7)$$

Where D is the diameter of the cylinder, U_0 is the inlet velocity and f_t is the vortex-shedding frequency of the lift coefficient calculated by Fast Fourier Transform (FFT) method.

3. Results and Discussion

3.1. Validation of Numerical Model

First, we validated our result of the flow around a cylinder without a slit at $Re = 100$ and 200 with experimental results from Norberg [20] and Roshko [9] and other numerical simulations from Thompson et al. [8]. After a series of simulations, it was found that the error between this present work and the experimental results do not exceed 7% and are shown in Table. 1 and Table. 2.

We studied the independence of the flow mesh around a cylinder without a slit at $Re = 100$ and after five (5) series of simulation, we found that the mesh of case 5 at 22200 elements gave sufficiently precise results. We therefore chose this mesh.

Table 1

Influence of the refinement of the mesh of cylinder without slits at Re =100

| Present study | | | | | Experiment (St) | |
|---------------|-------|----------|-----------|-------|-----------------|------------|
| Mesh | Nodes | Elements | C_{Dav} | St | Norberg [20] | Roshko [9] |
| Case 1 | 5540 | 2648 | 1.656 | 0.179 | 0.168 | 0.16-0.17 |
| Case 2 | 9520 | 4600 | 1.627 | | | |
| Case 3 | 11698 | 5672 | 1.609 | | | |
| Case 4 | 22080 | 10800 | 1.597 | | | |
| Case 5 | 45100 | 22200 | 1.591 | | | |

Table 2

Values of C_{Lav} , C_{Dav} and St of cylinder without slits at Re =200

| Present study | | Experiments | | | | | | Other simulation | |
|---------------|-----------|-----------------|-----------|-------------------|-----------|------------|-----------|---------------------|-----------|
| St | C_{Dav} | Williamson [21] | | Braza et al. [22] | | Roshko [9] | | Thompson et al. [8] | |
| | | St | C_{Dav} | St | C_{Dav} | St | C_{Dav} | St | C_{Dav} |
| 0.208 | 1.45 | 0.196 | - | 0.200 | 1.35 | 0.17-0.19 | - | 0.21 | - |

Table 3 summarizes the variation of the different physical parameters (C_{Lav} , C_{Dav} , and St) and the different slit positions with the variation of the spacing ratio (D/S). The table shows that the variation of the position and the ratio spacing (D/S) plays a very important on the variation of these parameters.

Table 3

Values of C_{Lav} , C_{Dav} and St of cylinder with different slits at Re =200

| Ratios (D/S) | Horizontal slit | | | Vertical slit | | | Horizontal and Vertical slits | | |
|--------------|-----------------|-----------|-------|---------------|-----------|-------|-------------------------------|-----------|-------|
| | C_{Lav} | C_{Dav} | St | C_{Lav} | C_{Dav} | St | C_{Lav} | C_{Dav} | St |
| D/S=32.0 | 0.00123 | 1.434 | 0.201 | -0.00421 | 1.455 | 0.208 | 0.00349 | 1.479 | 0.208 |
| D/S=20.0 | - 0.00057 | 1.417 | 0.208 | 0.00146 | 1.466 | 0.216 | 0.00294 | 1.535 | 0.201 |
| D/S=12.5 | 0.00019 | 1.347 | 0.216 | -0.00208 | 1.423 | 0.208 | -0.00161 | 1.570 | 0.201 |
| D/S=8.0 | - 0.00024 | 1.245 | 0.201 | 0.00297 | 1.391 | 0.224 | -0.00047 | 1.388 | 0.268 |

3.2 Effect of the slit

a) Behavior of drag with horizontal slit

The variation of the drag coefficient as a function of the spacing ratio (D/S) is shown in Fig. 3. The drag coefficient curves are periodic and varied, so the amplitude and period depend on the spacing ratio (D/S) of the slit. When the spacing ratio (D/S) of the slit increases, the period is increased but the amplitude is decreased. This is especially true in the case of (D/S) = 8.0 compared with the case (D/S) = 32.0. Therefore, the horizontal slit strongly weakens the appearance of vortex detachment and influences the wake structure.

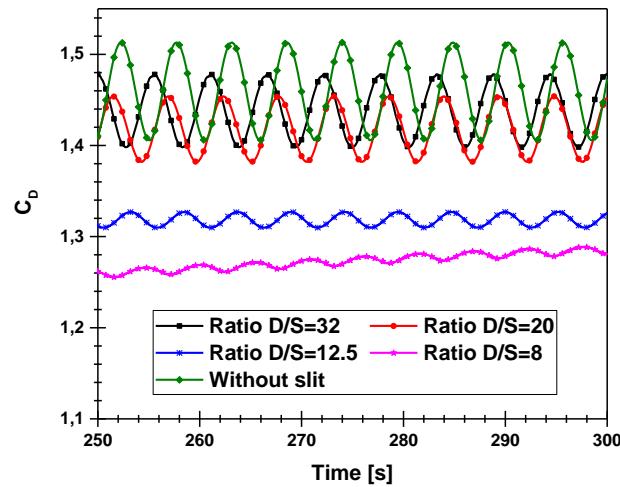
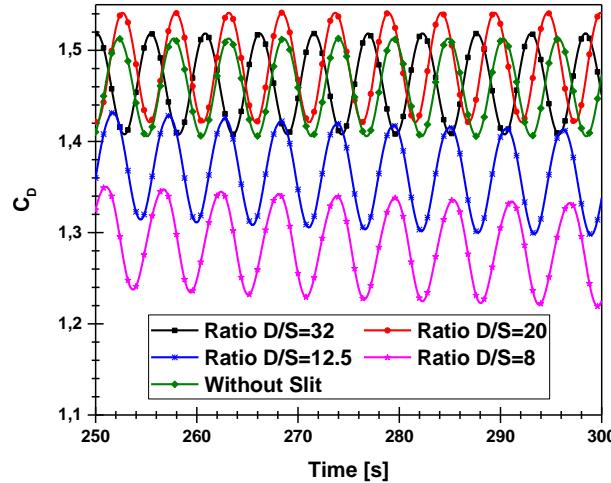


Fig. 3. Behavior of C_D with horizontal slit at $Re = 200$

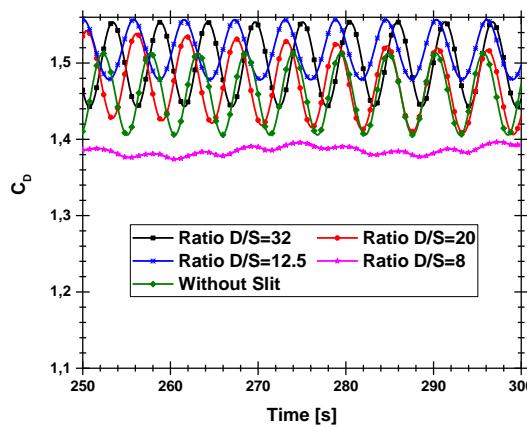
b) Behavior of drag with vertical slit

Fig. 4 shows the behavior of the drag coefficient of a cylinder with a vertical slit. In the case of the vertical slit, the curves of drag coefficients are periodic with a constant period and amplitude, and a phase shift between the curves. An increase in the value of the drag coefficient was observed in the case (D/S) = 20.0, then a remarkable decrease in the cases (D/S) = 12.5 and (D/S) = 8.0. Therefore, the fluid resistance, or the friction with the cylinder walls, are very small as the spacing ratio (D/S) increases. This decrease is accompanied by easy vortex detachment.

Fig. 4. Behavior of C_D with vertical slit at $Re = 200$

c) Behavior of drag with horizontal and vertical slit

Fig. 5 shows the behavior of the hydrodynamic drag coefficient of a cylinder with double slit (one vertical slit and the other horizontal). It was found that the curves are periodic with a varied period and amplitude. In the case of the spacing ratio $(D/S) = 12.5$, the amplitude reduced with an increase in the drag force, and in the case of the ratio $(D/S) = 8.0$, the amplitude reduced accompanied by a decrease in the drag force. But in the cases of ratio $(D/S) = 32.0$ and $(D/S) = 20.0$, the period and the amplitude are almost constant with a phase shift and a decrease in the drag force. It means that the period and the amplitude relate to the resistance and the interference of the fluid.

Fig. 5. Behavior of C_D with horizontal and vertical slits at $Re = 200$

d) Behavior of lift with different slit Horizontal

The lift coefficient of a cylinder with a horizontal slit is shown in Fig. 6. The curves are periodic with different amplitudes and periods. It has been noticed that this slit has a strong influence on the lift force, especially in the case where the spacing ratio (D/S) = 8.0. The lift force tends to zero and there is a great repulsive pressure force between the two cylinder parts. However, in the case where the spacing ratio (D/S) = 32.0, the attractive force is dominant, and therefore there is a great interference in the wake. The intensity of this force depends on the spacing ratio (D/S).

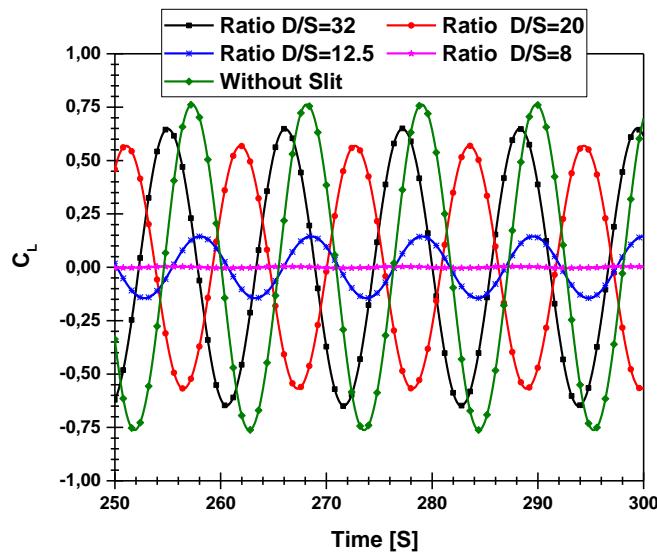
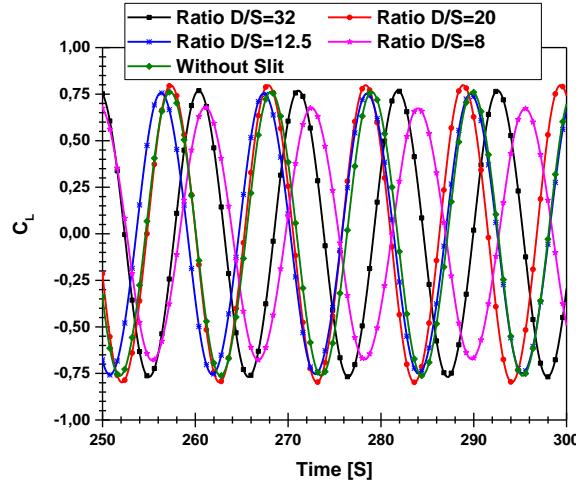


Fig. 6. Behavior C_L with horizontal slit at $Re = 200$

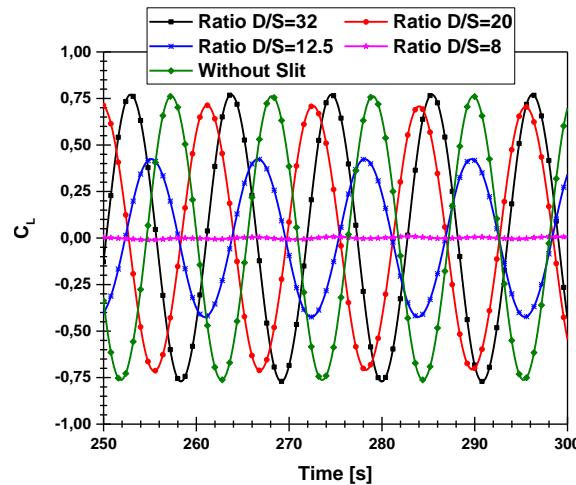
e) Behaviors lift with different slit vertical

The lift coefficient of a cylinder with a vertical slit represented in Fig. 7, shows that the curves are periodic with a variation of period and amplitude, thus the variation of the behavior lift due to the attractive pressure force created between the two cylinder parts depends on the nature of the spacing ratio (D/S).

Fig. 7. Behavior of C_L with vertical slit at $Re = 200$

f) Behavior of lift with different slit horizontal and vertical

Fig. 8 shows the behavior of the lift coefficient of the flow around a cylinder with a horizontal slit and another vertical (double slits). The curves are periodic with different periods and amplitudes and there is degradation or decrease lift force, that is inversely proportional to the spacing ratio (D/S) of the slit. We noticed that in the case where the ratio spacing (D/S) = 8.0, the amplitude is very low or very small, so the vortex detachment is also very low compared to the other cases. The force applied to the cylinder was then dispersed or divided by another repulsive force due to water passing through the slits.

Fig. 8. Behavior of C_L with horizontal and vertical slits at $Re = 200$

g) Drag comparison with and without slit

According to Fig. 9, which represents the average drag coefficients of different slits as a function of the spacing ratio (D/S), it was observed in the case of the horizontal slit that a remarkable decrease in drag coefficient occurs when the slit widens because the fluid pressure force increase. However, in the case of the vertical slit, there is an increase in average drag coefficient between $(D/S) = 32.0$ and 8.0 due to the decrease in the pressure force of the fluid, then a decrease. In the case of the horizontal slit and other vertical slit, an increase in drag coefficients average between $(D/S) = 32.0$ and 12.5 , then the forces applied around cylinder are dominant so there is a great inference in the wake, then a decrease to the value 1.38. That means there is a force created by the slits influenced on these forces applied therefore on the fluid resistance.

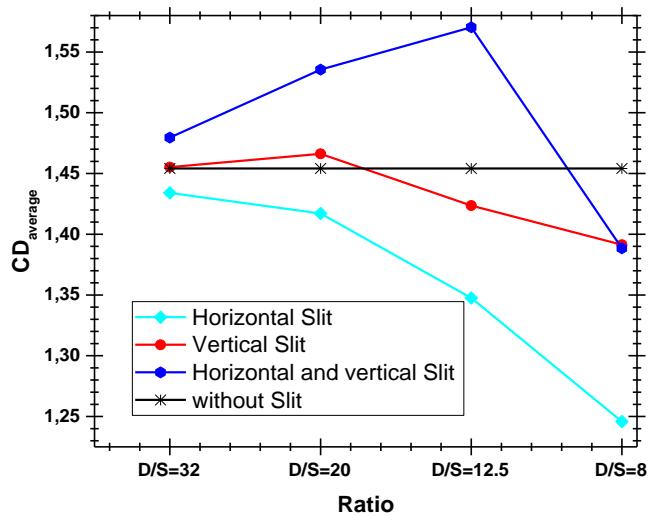


Fig. 9. Drag coefficient as a function of the ratio (D/S) at $Re=200$

h) Lift comparison with and without slit

Fig. 10 represents the average lift coefficient of different types of slits as a function of spacing ratio (D/S). It shows a large variation in lift coefficient especially in the case of the horizontal-vertical slit, where the average lift coefficient is reduced between the spacing ratios $(D/S) = 32.0$ and $(D/S) = 12.5$ because the pressure force of the fluid increases. However, in the case of the horizontal slit there is less variation than the other types of cases, and by comparing this with the cylinder without slit, there is a significant variation. The values of this parameter vary with opposite signs and do not have the same value. This implies that the passage of fluid through the slits creates a reverse force, the latter strongly influenced on the values of the lift coefficient. The intensity of this force depends on the spacing ratio (D/S) and the fluid velocity.

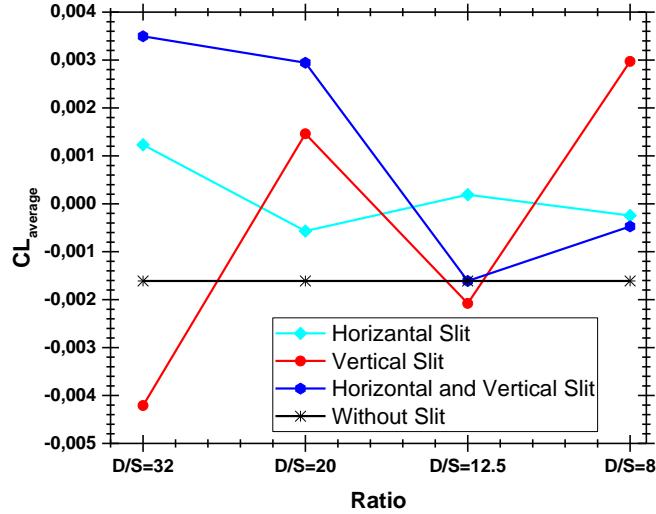


Fig. 10. Lift coefficient as a function of the ratio (D/S) at Re=200

i) Strouhal number

Fig. 11 shows the variation of Strouhal number (St) as a function of the spacing ratio (D/S) with different types of slits. This parameter represents the intensity of vortex detachment behind a cylinder. According to Fig. 11, the Strouhal number varies with the variation of the spacing ratio (D/S). It shows an increase in the case of the horizontal slit until $(D/S) = 12.5$, then a decrease and an alternative variation of this parameter in the case of the vertical slit. On the other hand, in the case of the horizontal slit and the other vertical, this parameter decreases slightly at the beginning then increases enormously. So, the spacing ratio (D/S) of the slit strongly influences this parameter, implying that the velocity of flow behind a cylinder and the forces applied to the latter have changed, meaning that it influences the detachment intensity and the wake structure especially in the case of horizontal and vertical slits.

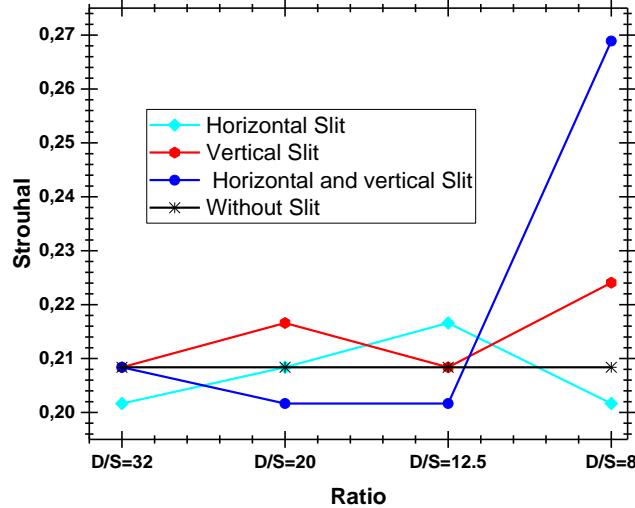
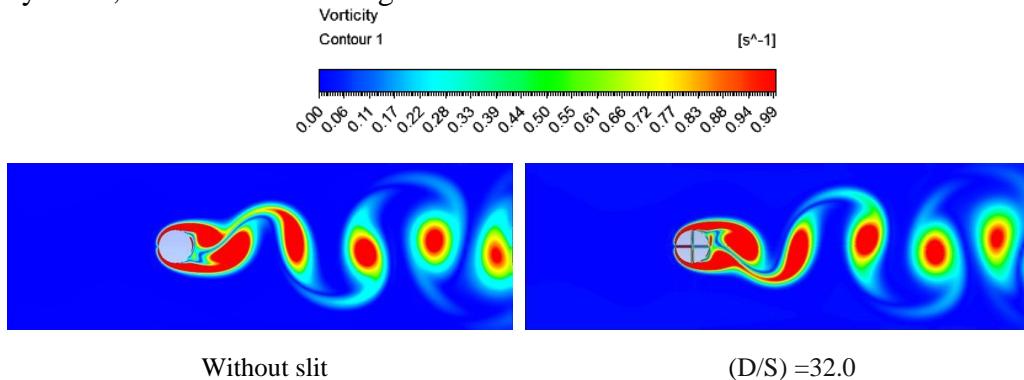


Fig. 11. Strouhal number as a function of the ratio (D/S) at $Re=200$

j) Wake structure behind the cylinder

It was observed, in the case of a cylinder with one horizontal slit and the other vertical with the spacing ratio (D/S) = 32.0, that there is a slight variation of the wake structure. On the other hand, in the case of spacing ratio (D/S) = 20.0 and (D/S) = 12.5 the number of vortices generated behind the cylinder is enormous, i.e., the flow of water passing through the slit is low and conversely, the flow of water that passes around two sides of a cylinder is enormous. In this case, the vertical slit accelerates the fluid, and creates a great interference in the wake structure generated. However, in the case of spacing ratio (D/S) = 8.0, the flow of water passing through the slit is enormous and the latter creates a great repulsive force that has a strong influence on vortex detachment behind the cylinder, as can be seen in Fig. 12.



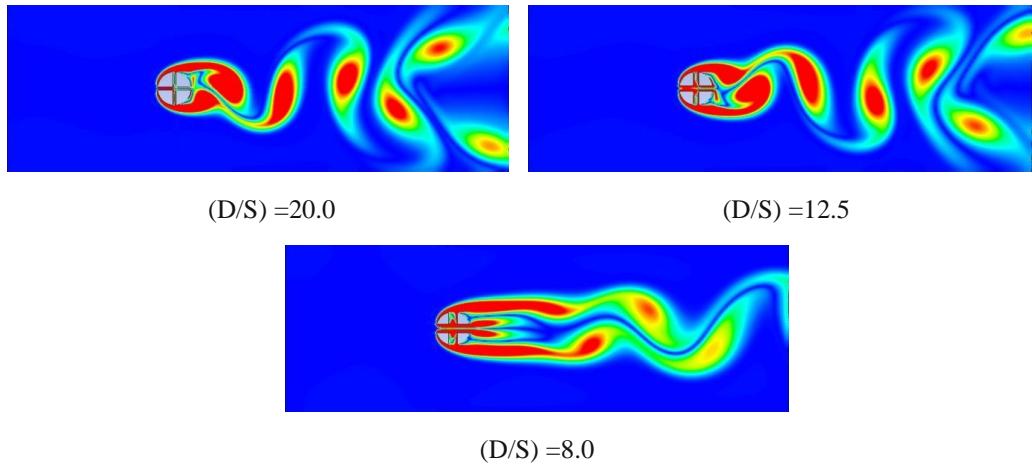


Fig. 12. Vorticity contours of different spacing ratios of horizontal and vertical slits at $Re=200$

6. Conclusions

In this paper, the effect of different slits on the vortex detachment and wake structure of the flow around a cylinder with different spacing ratios (D/S) at Reynolds number $Re=200$ is treated numerically.

Horizontal, vertical, and horizontal-vertical slits were made on the cylinder, with spacing ratios (D/S) equal to 32.0, 20.0, 12.5 and 8.0. The different physical parameters calculated were the average hydrodynamic coefficient of drag, the average hydrodynamic coefficient of lift, and the Strouhal number.

It was found that the slits strongly influenced the wake structure and the vortices generated behind the cylinder were due to the interference of the fluid. This means that the influence on the resistive forces of the fluid (attractive force and repulsive force) implies that the intensity of these forces depends on the spacing ratio (D/S) and its position (the flow of fluid passing through it). Also, there is a great difference between the average hydrodynamic coefficient of drag and average hydrodynamic of lift coefficient.

A uniform reduction in the drag coefficient was observed in the case of the horizontal slit, but in the other cases, because the fluid resistance is low or the friction is low, an increase was seen between the values of the spacing ratios (D/S) = 32.0 and (D/S) = 20.0. For the case of the vertical slit, due to fluid resistance or the friction forces, and in the case of horizontal-vertical slits, a large increase occurred between the values of the spacing ratios (D/S) = 32.0 and (D/S) = 12.5 due to high fluid resistance or high friction. Finally, for the coefficient of lift there is a significantly alternating variation because the fluid velocity is varied in the case of horizontal and vertical slits. But in the case of the horizontal-vertical slit, this coefficient is reduced between the spacing ratios (D/S) = 32.0 and (D/S)

= 12.5 because the fluid pressure forces are important. Ultimately, the number of Strouhal has an important relation with the vortex detachment. This parameter was influenced by the position and the nature of the slit, especially where the spacing ratio (D/S) = 8.0 for the case of the horizontal-vertical slit.

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