

NEW MODELING FOR PROTECTION AGAINST FROST ON THE OVERHEAD POWER LINES

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În lucrare se studiază o metodă de eliminare a depunerilor de chiciură de pe conductoarele liniilor electrice aeriene. Metoda se bazează pe utilizarea unor materiale feromagnetice cu punct Curie scăzut incorporată în conductor sub forma unor fire, sau montate pe conductor sub forma unor manșoane. Se elaborează un model teoretic și un program de calcul care permite simularea numerică a fenomenelor electromagnetice și termice din conductoare, în prezența unei depunerii de chiciură.

This paper investigated a method to eliminate the deposition of frost on wires of overhead lines. The method is based on the use of ferromagnetic materials with low Curie temperature incorporated in the form of conductive wires, or mounted on the conductor in the form of sleeves. A theoretical model and a computer program that allows numerical simulation of electromagnetic and thermal phenomena in the presence of frost deposition on conductors were developed.

Keywords: overhead power lines, hoar-frost

1. Introduction

Accumulation of ice / hoar-frost on overhead line conductors (LEA) may cause serious damage in the power system, hence the need of reducing or even eliminating them. Ice / hoar-frost in the air is in dry or wet form, and it accumulates on line conductors under cold wind conditions.

A typical situation is under-loading LEA (steady current value below the rated current) with ambient temperatures below 0°C and in the presence of wind, the heat produced by Joule effect leads to a temperature on the conductor's surface lower than the hoar-frost formation temperature (about -5°C).

The idea is to provide additional heating of the HV conductor, to obtain a higher temperature than the hoar-frost formation temperature.

Additional heating is achieved by including ferromagnetic material in the structure of HV conductor. The self protection characteristic of the conductor is the its property to revert to higher temperatures, above hoar-frost formation temperatures after an environmental disturbance (low temperature, wind speed increase) which cooled the conductor under -5°C. Such materials are materials with low Curie temperature. In Fig. 1 the saturation induction-temperature

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characteristics for two types of ferromagnetic materials with Curie temperatures of 830°C or 60°C are presented.

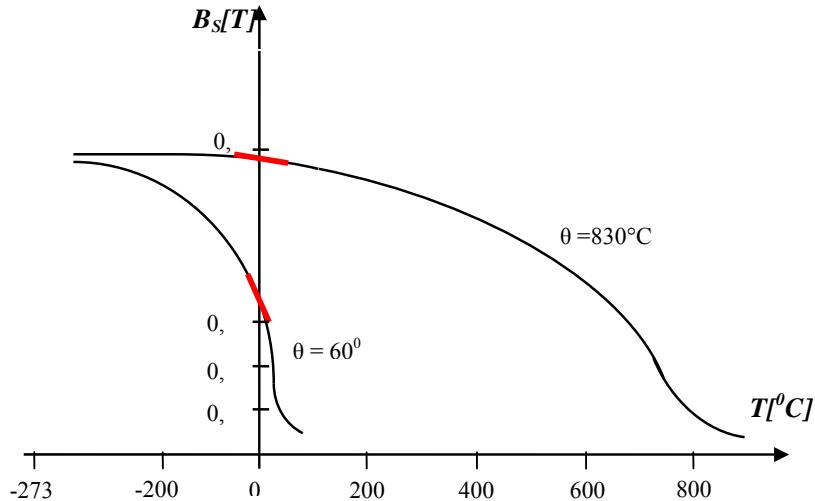


Fig. 1 Magnetic characteristic of the protective conductor material

With these materials, two models for the ice / hoar-frost self protected conductors (CAP) were considered, according to the arrangement of ferromagnetic material: the insertion of filaments (CAPIN) and the outer casing (CAPMA).

The paper proposes a mathematical model for calculating transient thermal regime of a conductor of an overhead line, on which a deposit of frost as a cylindrical sleeve was formed.

The heat source is represented by power losses in the conductor and the power developed by self-protection elements in the form of wire or sleeves.

2. Studied configurations

We consider the example studied as homogeneous conductor construction for the following reasons: firstly the greatest influence in the calculations has the outer conductor due to skin effect and secondly the new manufacturing technology eliminates conductor steel reinforcement inside and use superior alloy materials.

It is considered a portion of any length from a conductor on the surface which is a deposit of frost. Outside temperature is Θ_E , and the convection coefficient between the nozzle surface and environment is α_k .

The great length of the line allows the adoption of a one-dimensional geometric pattern, in which the physical values depends only of the spatial radial coordinate (r) and time.

The frost is represented by a homogeneous medium, whose thermo-physical properties are like water properties under the two states of aggregation encountered: liquid and solid.

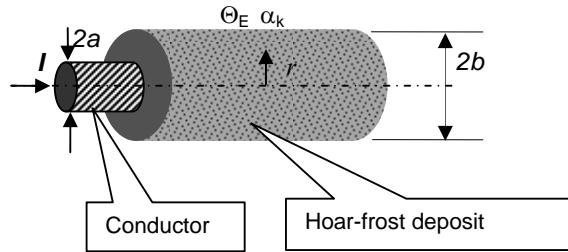


Fig. 2. Geometric configuration studied

The state equation of the deposit presents three areas:

- At temperatures below the melting temperature (0°C), the characteristic is practically linear, with a slope equal to the specific heat of ice (or frost, if applicable).
- A vertical portion corresponding to phase transformation (melting or solidification), corresponding to the associated latent heat.
- At temperatures above the melting temperature (0°C), the characteristic is practically a straight line, with a slope equal to the specific heat of water.

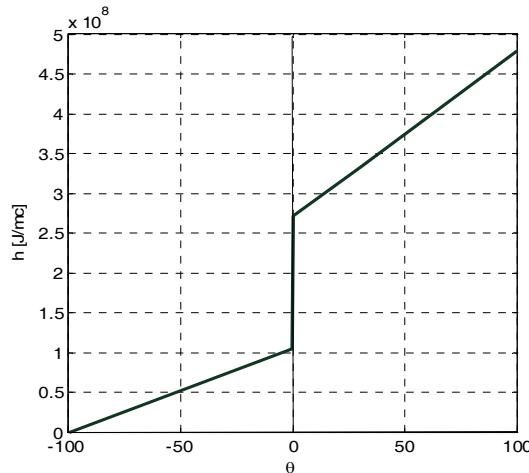


Fig.3. Equation of state of water

It is difficult to estimate appropriate material properties of the hoar-frost. The hoar-frost looks like a porous medium with a density that can be significantly less than that of ice (between 90 and 300 kg/m^3), which may considerably affect the development of the phenomenon.

We suppose that the cylindrical symmetry is maintained throughout the studied process.

The problem consists in the study of the transient thermal regime represented by the conductor (metal), self-protection sleeve (in various constructive solutions) and frost coating, in contact with ambient temperature and with conditions of heat transmission.

The main difficulty consists in variations of the material properties with the temperature. The most difficult to model is the highly nonlinear character of the thermo-physical properties of water (or frost).

3. Heat conduction equation

The thermal regime is described by the equation of Stefan:

$$\int_{D_\Sigma} \frac{\partial h}{\partial t} dv = \int_{D_\Sigma} p_J dv + \oint_{\Sigma} (\lambda \operatorname{grad} \theta) \overline{dA} \quad (1)$$

for a domain D_Σ bounded by a closed surface Σ . Moving bodies are neglected.

On separation surfaces, crossing relations are satisfied:

- boundary between two solids, designated by indices 1 and 2:

$$-\lambda_1 \frac{d\theta_1}{dn} \Big|_1 = -\lambda_2 \frac{d\theta_2}{dn} \Big|_2 \quad (2)$$

- contact surface of a solid with surface temperature θ_s , and a fluid with temperature θ_f (heat convection):

$$-\lambda_S \operatorname{grad} \theta_S = \alpha_k (\theta_S - \theta_f) \quad \forall M \in D_\Sigma, t > 0 \quad (3)$$

4. Dimensionless forms

It is convenient to express relations in dimensionless form, obtained by comparing physical quantities to conveniently chosen reference values. Thus, choosing a reference length l_0 , a reference temperature θ_0 , a reference enthalpy $h_0 = h(\theta_0)$, a reference thermal conductivity $\lambda_0 = \lambda(\theta_0)$, a reference time $t_0 = (h_0 l_0^2) / (\lambda_0 \theta_0)$ and a reference power $p_0 = \lambda_0 \theta_0 / l_0^2$, dimensionless quantities are defined as follows:

$$X = \frac{x}{l_0}, \quad T = \frac{t}{t_0}, \quad \Theta = \frac{\theta}{\theta_0}, \quad H = \frac{h}{h_0}, \quad \Lambda = \frac{\lambda}{\lambda_0}, \quad P = \frac{p}{p_0}, \quad \operatorname{Grad} = l_0 \nabla, \quad dV = \frac{dv}{l_0^3}, \quad \overline{dA} = \frac{\overline{da}}{l_0^2}$$

Equation (1) becomes, in dimensionless form:

$$\int_{D_\Sigma} \frac{\partial H}{\partial T} dV = \int_{D_\Sigma} P dV + \oint_{\Sigma} (\Lambda \operatorname{Grad} \Theta) \overline{dA} \quad (5)$$

and the convection condition (3) is:

$$-\Lambda_S \frac{d\Theta_S}{dX_n} = Bi (\Theta_S - \Theta_f) \quad Bi = \frac{\alpha_k l_0}{\lambda_f} \quad (\text{Biot number}) \quad (6)$$

5. Finite volumes method

In traditional approaches, thermal problem is treated from the local form of Stefan equation.

This approach presents considerable difficulties, especially for nonlinear problems in materials whose properties depend on temperature.

For this reason, finite volume method is adopted, the field is discretized into sufficiently small disjoint elements, for each of which the Stefan equation being applied, taking into account the heat exchange between contiguous elements.

In evaluating the surface integrals which express the heat exchange between elements, the conditions of passage through areas between them are used [1]. In the case of outer surfaces, a specific form of conditions for transition is used.

Thus, a system of differential equations that equal numbers of mesh elements is obtained. Unknowns are temperatures attached to each mesh element. It's reasonable that these temperatures to be considered like average temperatures of the elements.

In principle, for a mesh of N elements, this system is of the form:

$$[\Delta V] \frac{d}{dT} [H] = [\Delta V] [P] - [G] [\Theta] \quad (7)$$

where $[\Theta] = [\Theta_1, \dots, \Theta_N]^T$ are the temperatures of elements (in relative units), $[H] = [H_1, \dots, H_N]^T$ are the specific enthalpy, $[P] = [P_1, \dots, P_N]^T$ are the specific power dissipated and $[\Delta V]$ is a square matrix whose diagonal elements are meshing elements volumes. The coefficients matrix $[G]$ is a symmetrical square matrix with N rows and columns. By appropriate numbering of the N elements, the coefficient matrix will present a shape of "band", with favorable implications for numerical calculations.

Further, the time is discretized, with constant or variable time step, and the time derivatives are replaced by finite difference expressions. For the simplest case (but also most stable) of the "2 Steps" methods (e.g. Euler method) results:

$$\frac{d}{dT} [H] \approx \frac{1}{\Delta T} ([H^*] - [H]) \quad (8)$$

where H are the "current" values and H^* are the future (unknown) values.

For numerical stability reasons, the implicit method is adopted, by taking the right member of equation (7) for the next time step (i.e. with values still unknown). In this way, making a step in time requires solving a system of equations, what apparently considerably complicates the calculations. Instead, this method is stable, which it is desirable and will be considered further.

$$\frac{[\Delta V]}{dT}([H^*] - [H]) = [\Delta V][P^*] - [G][\Theta^*] \quad (9)$$

6. Treatment of nonlinearities

The main difficulty in solving system (9) consists in the presence of nonlinearities of the materials with temperature dependent properties, such as those suffering phase transformations.

In such cases, current practice consists in developing iterative methods such as Newton-Raphson, involving numerous numerically problems.

An advantageous alternative consists in using a method proposed by Hăntilă [2], for electromagnetic field problems, but which can be applied in other situations.

Thus, the caloric equation of state can be represented by the relation:

$$h(\theta) = c_m \theta + q(\theta) \quad (10)$$

respectively, in relative units:

$$H(\Theta) = C_m \Theta + Q(\Theta), \quad C_m = c_m \theta_0 / h_0; \quad Q(\Theta) = q(\theta) / h_0 \quad (11)$$

The coefficient c_m is a kind of an "average" specific heat, and $q(\theta)$ is a "difference" term. For the discretized field and using the relative values, this leads to a representation of the type:

$$[H(\Theta)] = [C_m][\Theta] + [Q(\Theta)] \quad (12)$$

where $[C_m]$ is a diagonal matrix of N^*N terms representing "average" specific heat of mesh elements. They may differ, but most often they can be chosen equally. In this case the matrix $[C_m]$ is reduced to a scalar.

System (7) becomes:

$$[G^*][\Theta^*] = \frac{[\Delta V]}{\Delta T} [H] + [\Delta V][P^*] - \frac{[\Delta V]}{\Delta T} [Q(\Theta^*)]; \quad [G^*] = [G] + \frac{[\Delta V]}{\Delta T} [C_m] \quad (13)$$

It appears that the coefficients of matrix $[G^*]$ differ from the original system only by diagonal terms. Therefore, the algorithm of solving the problem of steady state can be easily adapted to solve the problem of transient regime.

The main disadvantage of this algorithm comes from passing through nonlinear characteristic given by $[Q^*] = [Q(\Theta)]$. When passing through the phase transformation, the algorithm becomes unstable, because small changes in temperature leads to great variations of enthalpy, and hence of the "difference" term Q .

To avoid this, the algorithm can be modified by choosing as unknowns the enthalpies of the elements, instead of temperatures. This is possible because the relationship $h(\theta)$ is bijective.

Thus, thermal equation of state can be represented by the relation:

$$\theta(h) = k_m h + q(h); \quad [k_m] = \text{grad } m^3/J \quad (14)$$

respectively, in relative units:

$$\Theta(H) = K_m H + Q(H); \quad K_m = k_m h_0 / \theta_0; \quad Q(H) = q(h) \theta_0 \quad (15)$$

In discretized field and using relative values, it is a representation of the type:

$$[\Theta(H)] = [K_m] [H] + [Q(H)] \quad (16)$$

in which $[K_m]$ is a diagonal matrix of $N \times N$ terms. As in the previous case, different values of k_m , or a common value for all elements mesh may be taken.

System (9) becomes:

$$[G^*] [H^*] = \frac{[\Delta V]}{\Delta T} [H] + [\Delta V] [P^*] - [G^*] [Q(H^*)]; \quad [G^*] = [G] [K] + \frac{[\Delta V]}{\Delta T} \quad (17)$$

It should be noted that, if different values of the coefficient k_m for the mesh elements are adopted, the coefficients matrix $[G^*]$ is no more symmetric. Hence would be advantageous, in terms of computation economy, to adopt a single k_m value (in which case the matrix $[K]$ is replaced by the scalar K_m). In this case, system coefficients matrix $[G^*]$ differs from the original in that it is multiplied by the scalar, and diagonal terms change.

Iterative algorithm can be represented by the following pseudo-code:

```

 $[\Theta] \leftarrow \text{initial conditions}$ 
 $[H] = [H(\Theta)]$ 
// Time steps:
first factorization of matrix  $G^*$ 
For  $T = \Delta T$  to  $T_{\text{final}}$  step  $\Delta T$  {
  if step  $\Delta T$  is changed, refactoring the matrix  $G^*$ 
  // Iterations:
  Repeat {
     $[Q^*] = [Q(H)]$  ( $\leq$ =linear characteristic)
     $[H^*] \leftarrow \text{system solution (9) with factorized matrix}$ 
     $\text{Err} = || [H^*] - [H] ||$ 
     $[H] = [H^*]$ 
  while  $\text{Err} > \text{admissible error}$ 
   $[\Theta] = [\Theta(H)]$ 
}

```

The main advantage of this algorithm comes from passing through nonlinear characteristic by $[\Theta^*] = [\Theta(H)]$. When passing through the phase transformation, even large changes in enthalpy leads to small variations of temperature, and of the "difference" term $[Q(H)]$. This makes the algorithm stable.

It should be noted that, by this representation, the matrix $[G^*]$ contains only linear terms. Therefore, to solve the system of equations by factorization, it is

enough that factorization of system coefficients matrix, respectively the most intensive time phase (number of operations is $O(N^3)$) to be made only once, as long as the time step or thermal conductivities of the material does not change.

7. Numerical model

The studied case is a one-dimensional geometric model, with cylindrical symmetry.

Meshing elements will be N_r coaxial layers, whose thicknesses are arbitrary. The first element is the conductor, the second is the protection element (sleeve or wire), and following N_r-2 are layers of frost.

Let:

a = outer radius of conductor (including protection)

b = outer radius of frost deposition

thickness of the protective sleeve

and:

: radius of conductor ; : average radius of nozzle

$d_i = (b-a)/(N_r-2)$: thickness for the frost layer

$r_3 = a+d_3/2$: average radius of layers

for $i = 4, N_r$: $r_i = r_{i-1} + (d_{i-1}+d_i)/2$

Stefan equation for layer (i , $i = 1, \dots, N_r$)

$$\frac{dh_i}{dt} \Delta v_i = p_i + [g_{i,i+1}(\theta_{i+1} - \theta_i) + g_{i,i-1}(\theta_{i-1} - \theta_i)] \quad i = 1, \dots, N_r \quad (18)$$

where:

$$\Delta v_i = \begin{cases} 2\pi r_i d_i & i = 2, \dots, N_r \\ \pi r_1^2 & i = 1 \quad (\text{conductor}) \end{cases} \quad (19)$$

are element volumes, p_1 is the line loss in the conductor, p_2 is the power dissipated in the line protection elements (in W/m), $p_i = 0$ for $i = 3, \dots, N_r$, and g coefficients models the heat transfer between neighboring elements.

These factors are evaluated in [1] for a two-dimensional problem and a rectangular mesh network. In the present case, it becomes:

$$\begin{aligned} g_{i,i-1} &= g_E = 2\pi(r_i - \frac{d_i}{2})\gamma(\frac{\lambda_i}{d_i}, \frac{\lambda_{i-1}}{d_{i-1}}); \quad i = 2, N_r \\ g_{i,i+1} &= g_I = 2\pi(r_i + \frac{d_i}{2})\gamma(\frac{\lambda_i}{d_i}, \frac{\lambda_{i+1}}{d_{i+1}}); \quad i = 1, N_r - 1 \end{aligned} \quad (20)$$

For simplicity, is denoted by g_E and g_I coefficients that links a median element (i) of its neighbors from outside ($i+1$), and inside ($i-1$).

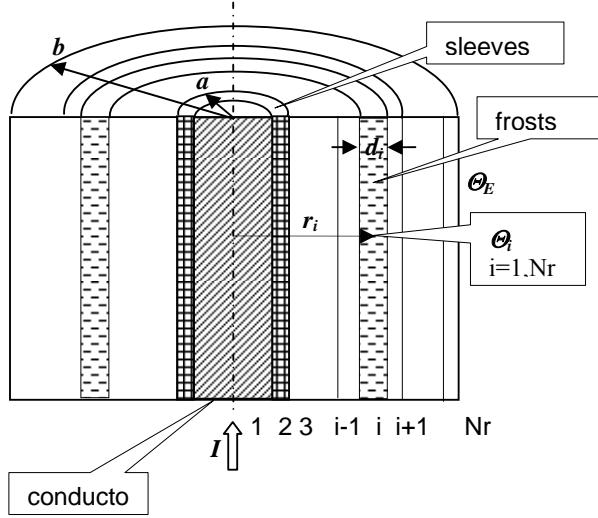


Fig.4 Configuration studied

Function γ has the expression:

$$\gamma(x_1, x_2) = \frac{x_1 x_2}{x_1 + x_2} \quad (21)$$

On the outer surface ($i=N_r$), the thermal convection equation leads to a particular expression of that coefficient, in function of the number of Biot associated to the outer surface:

$$g_E = 2\pi(r_{N_r} + \frac{d_{N_r}}{2})\gamma(\frac{\lambda_{N_r}}{d_{N_r}}, Bi) \quad (22)$$

Under dimensionless form, shall be adopted following reference sizes:

- Reference temperature: θ_0 (for numerical modeling was adopted $\theta_0=1$)
- Reference length: $L_0 = a$
- Reference enthalpy $H_0 = h(\theta_0)$, where h is specific enthalpy of water at reference temperature.
- Conductivity reference: $\Lambda_0 = \lambda(\theta_0)$, where λ is the thermal conductivity of water at reference temperature.
- Reference time: $T_0 = \frac{H_0 L_0^2}{\Lambda_0 \theta_0}$
- Lineic power reference: $P_0 = \Lambda_0 \theta_0$

Therefore, dimensionless values which will operate as follows:

- Relative Coordinates: $R_i = r_i/L_0$, $D_i = d_i/L_0$, $i = 1, N_r$
- Relative temperature: $\Theta = \theta / \theta_0$;
- Relative line powers: $P_i = p_i/P_0$

- Reference enthalpy: $H = h/H_0$;
- Biot's number: $Bi = \alpha_k L_0 / \lambda$

Differential equations assume the following dimensionless form:

$$\frac{dH_i}{dT} \Delta V_i = P_i + [G_{i,i+1}(\Theta_{i+1} - \Theta_i) + G_{i,i-1}(\Theta_{i-1} - \Theta_i)] \quad i = 1, \dots, N_r \quad (23)$$

where:

$$G_{i,i-1} = G_E = 2\pi(R_i - \frac{D_i}{2})\gamma(\frac{\Lambda_i}{D_i}, \frac{\Lambda_{i-1}}{D_{i-1}}); \quad i = 2, N_r \quad (24)$$

$$G_{i,i+1} = G_I = 2\pi(R_i + \frac{D_i}{2})\gamma(\frac{\Lambda_i}{D_i}, \frac{\Lambda_{i+1}}{D_{i+1}}); \quad i = 1, N_r - 1$$

$$\Delta V_i = \begin{cases} 2\pi R_i D_i & i = 2, \dots, N_r \\ \pi R_1^2 & i = 1 \quad (\text{conductor}) \end{cases} \quad (25)$$

G is the tri-diagonal and symmetric matrix of coefficients. Because homogeneous coefficients k_m were adopted, the matrix G^* is also tri-diagonal and symmetric.

The power dissipated in the protection element is calculated according to its temperature (function of the magnetic flux density) and other relevant data.

Two types of protection elements (CAPIN respectively CAPMA) were took into consideration.

In order to study a variety of "scenarios", the program allows setting of time varying conditions, namely:

- external temperature,
- intensity of electric current in conductor,
- wind speed, leading to convection coefficients, by criterial relations: $Nu = 0.037 Re^{0.8} Pr^{0.43}$, to finally obtain $\alpha_k = 6.126 v^{0.8}$.

8. Numerical example

Consider the following numerical example:

- Protective sleeve:
 - Model: CAPIN
 - outer radius of layer: $a = 12$ mm
 - filament diameters: $d_f = 3.90$ mm
 - number of filaments: $N = 4$
- Conductor:
 - Aluminum
 - Current: $I_c = 400$ A
 - Line losses: $P_c = P(1) = 27$ [W/m]
- Frost layer:

- inner radius: $a = 12 \text{ mm}$
- outer radius $b = 50 \text{ mm}$
- initial temperature: $\theta_{\text{ini}} = -5 \text{ }^{\circ}\text{C}$
- equation of state approximated by a polygonal line (table 1):

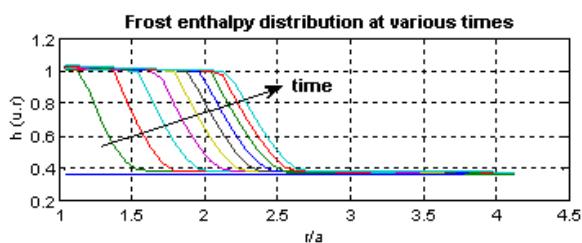
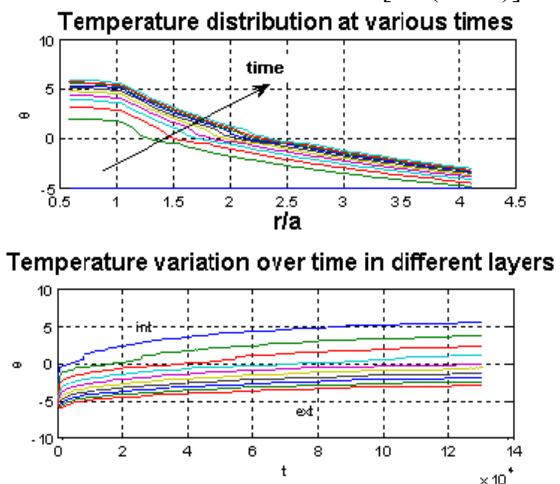
Table 1

θ	$^{\circ}\text{C}$	-100	-0,5	0,5	30	100
H	J/mc	0	208950000	541950000	665555000	958855000

The transition phase at 0°C was split at -0.5°C and 0.5°C , for reasons imposed by the interpolation algorithm used in MATLAB program. Between these values, the characteristic was approximated by linear interpolation, the only able to ensure the absence of parasitic oscillations. The average coefficient k_m is equal to the initial slope of the characteristic $\theta(h)$.

- Environment:

- Ambient temperature: $T_e = -10^{\circ}\text{C}$
- Convection coefficient. $\alpha_k = 20 \text{ [W/(m }^{\circ}\text{C)]}$



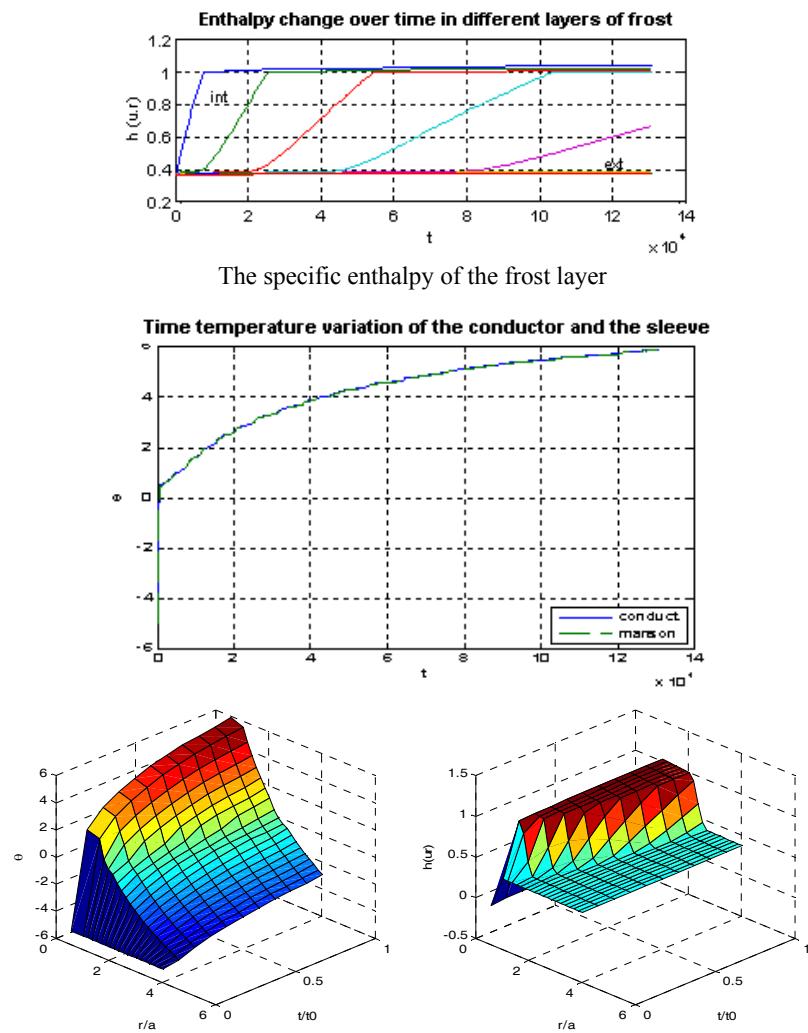
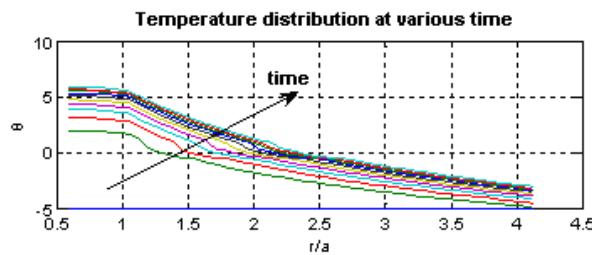


Fig. 7. Conductor and sleeve temperature

Influence of the protection element:



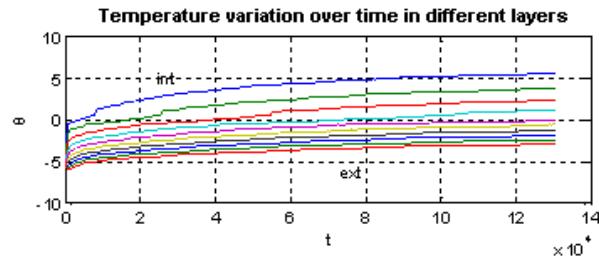


Fig. 8 With thermal protection.

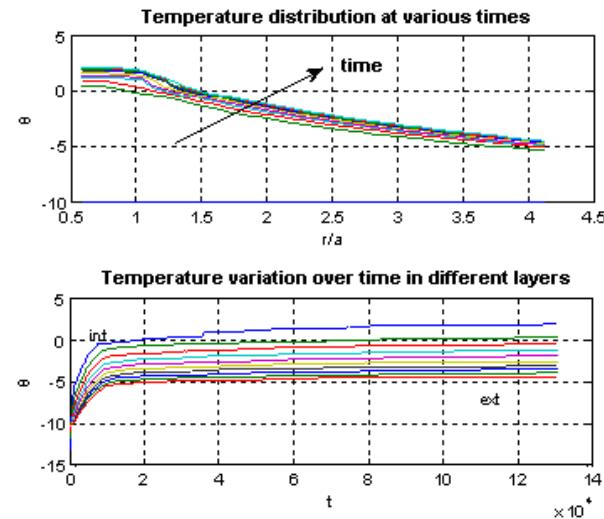


Fig. 9 In the absence of thermal protection (zero auxiliary power)

The presence of protective factors leads to a higher temperature of conductor than in its absence, thus facilitating the frost melting.

It appears (fig.10) that the iteration method adopted provides an accurate representation of highly nonlinear behavior of frost (including phase change).

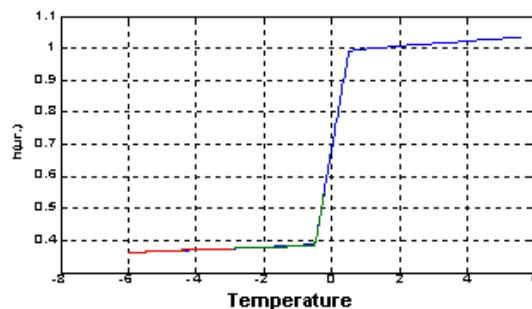


Fig.10. Equation of state of frost (actual calculations)

9. Conclusions

This paper brings the following original contributions:

1. Modeling by similitude of physical system and experimental model of the self protection conductor, to obtain its electro-thermal parameters.
2. A theoretical model and a computer program for the two types of protection of conductors in different operating conditions: temperature, wind and current intensity in conductor.
3. Experimental investigations with different materials to highlight and analyze the practical phenomena and processes occurring on the protected conductors.

Experimental investigations were carried out at INCDIE ICPE-CA, in order to confirm the adopted calculation model.

R E F E R E N C E S

- [1] *Fluerașu Corina, Fluerașu Cezar* - Lessons of electro-heat, Printech, Bucharest, 2008
- [2] *F.I. Hănilă*, Mathematical models of the relation between B and H for non-linear media. Rev.Roum.Sci.Techn. Electrotechn. et Energ., 19, 3, P.429-448, Bucharest, 1974
- [3] (Conduction through solid), , 1956
- [4] (Construction and operation of electricity networks in regions of intense frost)Moscova 1947
- [5] (Systems and electrical networks), , 1947
- [6] (Contribution to the establishment prescriptions for calculating of overhead electrical transmission) 1927
- [7] Procédé **vol. II**
- [8] *G. Iacobescu, I. Electrical networks*
- [9] *Fluerașu Cezar, Fluerașu Corina*, Calcul des régimes transitoires thermiques dans des matériaux avec propriétés dépendantes de la température, Rev.Roum.Sci.Techn.- Electrotechn. et Energ., **vol. 53**, 3, p.269-278, Bucharest, 2008.