# GEOMETRIC BROWNIAN MOTION AND ORNSTEIN-UHLENBECK PROCESS MODELING BANKS' DEPOSITS

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*We propose two stochastic models for banks' demand deposits, based on the geometric Brownian Motion and on the Ornstein-Uhlenbeck process. We formulate the problem in terms of optimal control in each model and we find a relation between the value function and the cost function. We calibrate data on banks' deposits to three models (Brownian motion, geometric Brownian motion and Ornstein-Uhlenbeck process). We compare the goodness-of-fit using the Kolmogorov-Smirnov test.*

**Keywords:** Geometric Brownian Motion, Ornstein-Uhlenbeck process, stochastic control, banks' deposits, calibration, Kolmogorov-Smirnov test
**MSC2010:** 91G 70, 91B 70.

## 1. Introduction

Stochastic control has a wide range of applicability, from agriculture, engineering, hidrology to computer science and economics. The 2008 financial turmoil showed the need for new mathematical and realistic models to prevent a future similar economic crisis. Mathematical finance offers mathematicians new problems of research, as well as a very useful intuition on the phenomenon.

In this paper we provide a mathematical model for the optimal strategy followed by one bank in the federal funds market. Such a strategy can be used by a profit-seeking financial institution in order to satisfy its reserve requirements and to maximize its profit in an idealized contemporaneous reserve requirement regime. The bank's task is to find an optimal amount to buy or to sell, at each time, while minimizing the cost of buying, selling and holding funds. The reserve requirements represent a certain percent of the demand deposits that the bank receives from its depositors. We consider that the bank meets its reserve requirements if the excess reserve process (i.e. the difference between the aggregate deposits and the required reserves) remains

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positive. If we model the excess reserve process as a stochastic diffusion process, the problem turns into a stochastic control problem, as in [5]. We extend a model based on a singular control problem for the Brownian motion from [6]. The theoretical model (based on the Brownian motion) was adapted for the bank's excess reserves in [3], and further developed in [1] and [2]. In comparison to [6], [3] and [2], we model the uncontrolled excess reserve process as a geometric Brownian motion and as an Ornstein-Uhlenbeck process and we prove that the problem formulation remains the same as in the Brownian motion case. However, a formula for the optimal policy can not be obtained by applying the Dynamic Programming Principle, since the latter is based on modeling the controlled process (such a model was approached in [7]).

We calibrate the model to the data on banks' deposits in order to check how realistic our model is. We compare the goodness-of-fit of the Brownian motion model from [3] and [2] with the proposed models in this paper. The Kolmogorov-Smirnov test shows that all three models give a satisfactory fit, meaning that the calibration to the data of almost 90% of the banks is not rejected by the test. We obtain that the best fit for banks' deposits is given by the geometric Brownian motion, followed by the Ornstein-Uhlenbeck process; the last fit is the Brownian motion.

The article is structured as follows: section 2 presents the general model for the strategy followed by one bank in the federal funds market. In section 3 we propose a model in which the uncontrolled excess reserve process is modeled by a geometric Brownian motion. Section 4 presents the analogous model based on the Ornstein-Uhlenbeck process. In section 5 we present the results on the calibration and on the goodness-of-fit tests.

## 2. **The general model**

We consider one bank that maintains an account with the Federal Reserve Bank. We assume that the bank can get funds from only two sources: depositors (through demand deposits, i.e. deposits that must be available if depositors need them) and the federal funds market (transactions in this market imply buying or selling funds, with the payment of the corresponding transaction costs). The bank tries to maximize its profits out of buying and selling overnight (federal) funds, while meeting the reserve requirements. The net deposit flow is an input in our system and it is modeled as a diffusion process.

We propose an idealized contemporaneous reserve requirement regime, with an infinite time horizon: the bank instantaneously modifies its reserve account to the required percentage of the value of the deposits. The excess reserve process (i.e. the difference between the deposits and the required reserves) can be used for transactions in the federal funds market. Therefore,

modeling the deposit flow is equivalent to modeling the excess reserve process. The optimal time and amount to buy or sell in the federal funds market represent the output of an optimal control problem.

The bank is characterized by the following processes:

(1) A demand deposit process $(D_t)_{t\geq 0}$.
(2) A required reserve process $(R_t)_{t\geq 0}$, where $R_t = qD_t$ and $0 < q < 1$.
(3) An excess reserve process $X_t = (1-q)D_t$.

Let $(\Omega, F, P_x)$ be a probability space rich enough to allow the continuous excess reserves process $X$ and $P_x(X_0 = x) = 1$. We consider $\mathbf{F} = (F_t)$ to be the completion of the augmented filtration generated by $X$ (so that $(F_t)$ satisfies the usual conditions). Thus, the bank observes nothing except the sample path of $X$. We assume $X_0 = x \geq 0$.

A standard, one-dimensional, Brownian motion is a continuous, adapted process $B = (B_t, F_t, 0 \leq t \leq \infty)$, with the property that $B_t - B_s$ is independent of $F_s$ and is normally distributed with mean zero and variance $t - s$.

We consider that the excess reserve process is a diffusion process modeled by the following stochastic differential equation:

$$dX_t = \hat{\mu}(X_t)dt + \hat{\sigma}(X_t)dB_t, \tag{1}$$

where $\hat{\mu}, \hat{\sigma}$ are $C^2$ functions and $B$ is a standard Brownian motion on the previously defined filtered probability space $(\Omega, F, (F_t)_{t\geq 0}, P_x)$.

The case when $\hat{\mu}(.), \hat{\sigma}(.) > 0$ are constants (Brownian motion with drift) was discussed in [6], [3], and [2]. In this paper we will approach these cases:

(1) $\hat{\mu}(x) \equiv \mu x; \hat{\sigma}(x) \equiv \sigma x$, (geometric Brownian motion);
(2) $\hat{\mu}(x) \equiv \theta(\mu - x); \hat{\sigma}(x) \equiv \sigma > 0, \theta > 0$, (Ornstein Uhlenbeck).

We denote by $\lambda > 0$ the federal target rate, which is used as a discounting rate in our model. The bank can continuously modify its excess reserve account, by selling or buying funds. There are three types of transaction costs in our idealized market (similarly as in [6] and [3]): proportional transaction costs of buying ($\alpha$), or selling funds ($\beta$); and a continuous holding cost, accumulated at the rate $hX_t$.

We define:

$$r \equiv h/\lambda - \beta \qquad \text{and} \qquad c \equiv h/\lambda + \alpha. \tag{2}$$

Similarly as in [6], the ratio $h/\lambda$ can be interpreted as the cost of holding a unit of federal funds forever as its excess reserves. Then $r$ is the reward for selling a unit of federal funds: the holding cost that would have been paid is gained and the transaction cost for selling is lost. Similarly, the cost parameter $c$ is the cost of buying and holding a unit of federal funds.

**Definition 2.1.** *A policy is defined as a pair of processes $L$ and $U$ such that*

$$L, U \quad \text{are} \quad \boldsymbol{F}-\text{adapted, right-continuous, increasing and positive.} \tag{3}$$

The set of all policies that satisfy definition 2.1 are denoted by $\tilde{S}$. In our model for the federal funds market, $L_t$ and $U_t$ are the cumulative purchases and sales of funds that the bank undertakes up to time $t$, in order to satisfy the reserve requirements and to maximize its profit.

**Definition 2.2.** *A controlled process associated to the policy $(L, U)$ is a process $Z = X + L - U$.*

In our model for one bank in the federal funds market, $Z$ is the amount of excess funds in the bank's reserve account.

**Definition 2.3.** *The policy $(L, U)$ is said to be feasible if*

$$L_{0-} = U_{0-} = 0, \tag{4}$$

$$P_x \{Z_t \geq 0, \forall t\} = 1, \forall x \geq 0, \tag{5}$$

$$E_x \left[ \int_0^\infty e^{-\lambda t} dL \right] < \infty, \forall x \geq 0, \tag{6}$$

*and*

$$E_x \left[ \int_0^\infty e^{-\lambda t} dU \right] < \infty, \forall x \geq 0. \tag{7}$$

*We denote by $\tilde{S}(x)$ the set of all feasible policies associated with the continuous process $X$ that starts at $x$.*

Without loss of generality, as in [6], the last integrals are interpreted as

$$\int_0^\infty e^{-\lambda t} dL = L_0 + \int_{(0,\infty)} e^{-\lambda t} dL \tag{8}$$

and similarly for $U$. The definition allows a possible initial jump of the policy.

**Definition 2.4.** *The cost function associated to the feasible policy $(L, U)$ is*

$$k_{L,U}(x) \equiv E_x \left[ \int_0^\infty e^{-\lambda t} (hZ_t dt + \alpha dL + \beta dU) \right], \qquad x \geq 0. \tag{9}$$

**Definition 2.5.** *The control $(\hat{L}, \hat{U})$ is said to be optimal if $k_{\hat{L},\hat{U}}(x)$ is minimal among the cost functions $k_{L,U}(x)$ associated with feasible policies $(L, U)$, for each $x \geq 0$.*

The bank's reserve management and profit-making problem is therefore to find the optimal strategy $(\hat{L}, \hat{U})$. The problem can be approached more effectively using a value function. The problem of minimizing the cost can be translated to the task of maximizing a value function. We proceed by defining the value function. In the following sections we prove the equivalence of the problems when the underlying process is modeled as a geometric Brownian motion and as an Ornstein-Uhlenbeck process.

**Definition 2.6.** *Let $k_{L,U}(x)$ be the cost function for a feasible policy $(L, U)$ as in 2.4. We define the value function to be*

$$v_{L,U}(x) \equiv E_x \left\{ \int_0^\infty e^{-\lambda t}(rdU - cdL) \right\}, \qquad x \geq 0, \tag{10}$$

*where $r$ and $c$ are as in (2).*

Buying federal funds generates a cost of $c$ times the transaction amount, selling federal funds generates a reward of $r$ times the amount that was sold. The bank's task turns to be to maximize the expected value of rewards received minus costs incurred over an infinite horizon, subject to meeting reserve requirements.
We notice that $r \leq 0$ would imply that it is never optimal to sell, i.e. $U \equiv 0$. In addition, $r < c$ is a no-arbitrage condition, for otherwise the bank can make unlimited profits in a finite period of time. In order to exclude financial arbitrages and for the economical problem to make sense, we impose throughout this paper the following assumption:

$$0 < r < c < \infty. \tag{11}$$

Modeling the excess reserve process as a Brownian motion with drift was discussed in [6], [3], [2]. In section 3 and section 4 we focus on finding the relation between the cost function and the value function when the excess reserve process is modeled by a geometric Brownian motion and by an Ornstein-Uhlenbeck process, respectively. In this section we present the proposition for the Brownian motion case from [6], for completion.

### 2.1. **Brownian Motion case**

**Proposition 2.1.** *We consider that the excess reserve process is a diffusion process modeled by the following:*

$$dX_t = \mu dt + \sigma dB_t, \tag{12}$$

*where $\mu, \sigma$ are constants and $B$ is a standard Brownian motion as above. Then*

$$k_{L,U}(x) = hx/\lambda + h\mu/\lambda^2 - v_{L,U}(x), x \geq 0. \tag{13}$$

## 3. **Geometric Brownian motion case**

We assume that the excess reserve process follows a geometric Brownian motion: $X_t = X_0 e^{\mu t + \sigma B_t}$, where $(B_t)_t$ is a standard Brownian motion, $\mu \in \mathbb{R}$ and $\sigma > 0$. By taking logarithms we obtain:

$\log(X_t/X_0) = \log(X_t) - \log(X_0) = \mu t + \sigma B_t$.

Therefore, $\log(X_t) = \log(X_0) + \mu t + \sigma B_t$ is normal with mean $\mu t + \log(X_0)$, and variance $\sigma^2 t$. Consequently, for each $t$, $X_t$ has a lognormal distribution.

If $(X_t)$ follows a geometric Brownian motion then one can show, using Ito's Lemma, that $X$ must satisfy the following stochastic differential equation:

$$dX_t = \mu X_t dt + \sigma X_t dB_t. \tag{14}$$

By applying expectation in (14), we obtain that $EX_t = \mu \int_0^t (EX_s)ds + x$. By introducing the function $u(t) \equiv EX_t$, and solving the differential equation $u'(t) = \mu u(t), u(0) = x$, we deduce that $EX_t = e^{\mu t}x$, where $X_0 = x > 0$.

**Proposition 3.1.** *If $(X_t)_{t\geq 0}$ follows 14, let $k_{L,U}(x)$ be the cost function for a feasible policy $(L, U)$ as in definition 2.4. Also let $v_{L,U}(x)$ be the associated value function as in definition 2.6. We consider $\lambda > \mu$. Then*

$$k_{L,U}(x) = hx/(\lambda - \mu) - v_{L,U}(x), \qquad x \geq 0. \tag{15}$$

*Proof.* Without loss of generality, we can assume that $U_0 = L_0 = 0$ (the other cases are similar, given (8)). Since $Z \equiv X + L - U$, we have:

$$hE_x\left(\int_0^\infty e^{-\lambda t}Z_t dt\right) = hE_x\left(\int_0^\infty e^{-\lambda t}X_t dt\right) + hE_x\left(\int_0^\infty e^{-\lambda t}(L_t - U_t)dt\right). \tag{16}$$

Applying Fubini's theorem ($X$ is positive), we obtain:

$$E_x\left(\int_0^\infty e^{-\lambda t}X_t dt\right) = \int_0^\infty e^{-\lambda t}E_x(X_t)dt.$$

Since $X$ is a $(\mu, \sigma)$ geometric Brownian motion, $E_x(X_t) = xe^{\mu t}$. Using the fact that $\lambda > \mu$ when computing the integral, we obtain:

$$\int_0^\infty e^{-\lambda t}E_x(X_t)dt = \int_0^\infty e^{-\lambda t}e^{\mu t}x dt = \frac{x}{\lambda - \mu}.$$

From the last two formulas we conclude that

$$E_x\left(\int_0^\infty e^{-\lambda t}X_t dt\right) = \frac{x}{\lambda - \mu}. \tag{17}$$

Next, we recall the Riemann- Stieltjes integration by parts theorem, which states that if two functions $f, g$ are $FV$ (of finite variation), then:

$$\int_0^t f dg = f(t)g(t) - f(0)g(0) - \int_0^t g(s)df(s).$$

Noticing that since $L$ is increasing, $L$ is $FV$ and applying the above-mentioned theorem, we obtain, for each fixed $T > 0$:

$$\int_0^T e^{-\lambda t}dL = e^{-\lambda T}L_T - e^{\lambda.0}L_0 + \lambda\int_0^T e^{-\lambda t}L_t dt. \tag{18}$$

Applying Fatou's lemma twice and using (18) and (6), we obtain

$$\begin{aligned} E_x[\liminf_{T\to\infty}(e^{-\lambda T}L_T + \lambda\int_0^T e^{-\lambda t}L_t dt)] &= E_x[\liminf_{T\to\infty} E_x\int_0^T e^{-\lambda t}dL] \leq \liminf_{T\to\infty} E_x\int_0^T e^{-\lambda t}dL \\ &\leq \limsup_{T\to\infty} E_x\int_0^T e^{-\lambda t}dL \leq E_x\limsup_{T\to\infty}\int_0^T e^{-\lambda t}dL \\ &= E_x\int_0^\infty e^{-\lambda t}dL < \infty. \end{aligned}$$

It follows that $e^{-\lambda t}L_t \to 0$ almost surely as $t \to \infty$. Indeed, if this were not true, then, since $e^{-\lambda t}L_t \geq 0$ on a set of non-zero measure, we would have $\int_0^\infty e^{-\lambda t}L_t dt = \infty$. We obtain therefore that
$E_x[\liminf_{T\to\infty}(e^{-\lambda T}L_T + \lambda \int_0^T e^{-\lambda t}L_t dt)]$ becomes unbounded, and we get to a contradiction.

Letting $T \to \infty$ in (18) and then taking $E_x$ on both sides, we obtain:

$$E_x\left(\int_0^\infty e^{-\lambda t}L_t dt\right) = \frac{1}{\lambda}E_x\left(\int_0^\infty e^{-\lambda t}dL\right). \tag{19}$$

We obtain a similar equation for $U$. Replacing it and (19), (17), (16) in the definition (9) for $k_{L,U}(x)$, we obtain:

$$k_{L,U}(x) = \frac{hx}{\lambda-\mu} + (\frac{h}{\lambda}+\alpha)E_x\left(\int_0^\infty e^{-\lambda t}dL_t\right) + (-\frac{h}{\lambda}+\beta)E_x\left(\int_0^\infty e^{-\lambda t}dU_t\right).$$

□

**Remark.** Analogously as in the Brownian motion case, the first term on the right-side of (15) does not depend on the particular policy $(L, U)$. Therefore, for fixed $x \in (0, \infty)$

$$\inf_{(L,U)\in\tilde{S}(x)} k_{L,U}(x) \Leftrightarrow \sup_{(L,U)\in\tilde{S}(x)} v_{L,U}(x),$$

where $\tilde{S}(x)$ is defined in 2.3.

## 4. **Ornstein-Uhlenbeck case**

We assume that the excess reserve process follows an Ornstein-Uhlenbeck process

$$dX_t = \theta(\mu - X_t)dt + \sigma dB_t,$$

where $\theta > 0$ is the mean reversion rate, $\mu$ is the mean, $\sigma > 0$ the volatility, and $(B_t)_{t\geq 0}$ a standard Brownian motion.

In order to compute the mean of this process, we apply the Ito's Lemma for the function $f(x,t) = xe^{\theta t}$.

We obtain that $df(X_t, t) = \theta X_t e^{\theta t}dt + e^{\theta t}dX_t = \theta\mu e^{\theta t}dt + \sigma e^{\theta t}dB_t$.

Therefore,

$X_t e^{\theta t} = X_0 + \theta\mu \int_0^t e^{\theta s}ds + \sigma \int_0^t e^{\theta s}dB_s$,

$X_t = X_0 e^{-\theta t} + \mu(1 - e^{-\theta t}) + \sigma \int_0^t e^{\theta(s-t)}dB_s$,

$E_x X_t = xe^{-\theta t} + \mu(1 - e^{-\theta t})$.

**Proposition 4.1.** *Let $k_{L,U}(x)$ be the cost function for a feasible policy $(L, U)$ as in 2.4. Also let $v_{L,U}(x)$ be the value function defined as in 2.6. Then*

$$k_{L,U}(x) = h[\frac{x-\mu}{\lambda+\theta} + \frac{\mu}{\lambda}] - v_{L,U}(x), x \geq 0. \tag{20}$$

*Proof.* Similarly as in the geometric Brownian motion case, we have:

$$hE_x\left(\int_0^\infty e^{-\lambda t}Z_t dt\right) = hE_x\left(\int_0^\infty e^{-\lambda t}X_t dt\right) + hE_x\left(\int_0^\infty e^{-\lambda t}(L_t - U_t)dt\right).$$

Further on, since $E_x X_t = xe^{-\theta t} + \mu(1 - e^{-\theta t})$, we obtain:

$$E_x\left(\int_0^\infty e^{-\lambda t}X_t dt\right) = \int_0^\infty e^{-\lambda t}E_x(X_t)dt = \int_0^\infty e^{-\lambda t}(xe^{-\theta t} + \mu(1 - e^{-\theta t}))dt.$$

By applying Fubini's theorem and using the fact that $\lambda > 0, \theta > 0$ when computing the integral, we obtain:

$$E_x\left(\int_0^\infty e^{-\lambda t}X_t dt\right) = \frac{x-\mu}{\lambda+\theta} + \frac{\mu}{\lambda}. \tag{21}$$

Further on, the proof is identical to the proof in proposition 3.1. Finally, we obtain:

$$k_{L,U}(x) = h[\frac{x-\mu}{\lambda+\theta} + \frac{\mu}{h}] + (\frac{h}{\lambda} + \alpha)E_x\left(\int_0^\infty e^{-\lambda t}dL_t\right) + (-\frac{h}{\lambda} + \beta)E_x\left(\int_0^\infty e^{-\lambda t}dU_t\right).$$

□

**Remark.** Analogously as in the geometric Brownian motion case, the first two terms on the right-side of (20) do not depend on the particular policy $(L, U)$. Therefore,

$$\inf_{(L,U)\in\tilde{S}(x)} k_{L,U}(x) \Leftrightarrow \sup_{(L,U)\in\tilde{S}(x)} v_{L,U}(x),$$

where $\tilde{S}(x)$ is defined in 2.3.

## 5. Application to the data on banks' deposits

A very important aspect in mathematical finance is whether the proposed models are realistic. In this section we calibrate the models to real data. We assume that banks' deposits evolve as a Brownian motion, geometric Brownian motion and as an Ornstein-Uhlenbeck process, respectively. We calibrate all these three models to our data. The question is which model fits the demand deposits best? We test the goodness-of-fit and we present the results on the Kolmogorov-Smirnov test.

### 5.1. Data description

Our data is obtained from WRDS (Wharton Research Data Services)[1].

For each commercial bank in the United States there is a record of $RCON2210$ (demand deposit amounts- net of withdrawals) on the last business day of each quarter between March 1991 and December 2000. The total number of banks is 1221. We included in our data set only the banks which had recorded non-zero total asset size and demand deposit amount, for all 40

[1] $http://wrds-web.wharton.upenn.edu/wrds/ds/bank/banks/balance.cfm$

quarters between 1991 and 2000. The data excludes savings banks, savings and loan associations, credit unions, investment banks, mutual funds, and credit card banks.

### 5.2. Calibration

We model and calibrate the demand deposit process as a Brownian motion with drift, a geometric Brownian motion and as an Ornstein-Uhlenbeck process. Further on, we present the formulas that we use. For a thorough discussion on the estimators we refer to [4].

Let $(D_t)_t$ be the demand deposit process for an arbitrary bank. We denote by $(B_t)_{t\geq 0}$ a standard Brownian motion. Our data consists of $m = 40$ time-records of demand deposit amounts for each of the 1221 banks:

$$(D_t^i)_{i=1,N;t\in[0=t_1<t_2<...<t_m=T]}.$$

The demand deposits are recorded for each bank on the last business day of all quarters between March 1991 and December 2000. Therefore, further on, we consider that demand deposits are recorded at $m$ equidistant moments of time, i.e. $t_j - t_{j-1} = \delta$, for all $j = 2, m$. Consequently, $t_m - t_1 = (m-1)\delta$.

5.2.1. *Demand deposits as Brownian motions with drifts.* Let $(D_t)_{t\geq 0}$ follow a Brownian motion (BM) with drift $\mu$ and volatility $\sigma > 0$, i.e.:

$$D_t = \mu t + \sigma B_t.$$

A unbiased estimate for the drift is $\hat{\mu} = \frac{D_{t_m} - D_{t_1}}{t_m - t_1}$.

A good candidate for the volatility estimator is:
$\hat{\sigma}^2 = \sum_{j=2}^{m} \frac{(D_{t_j} - D_{t_{j-1}})^2}{t_m - t_1} - \frac{1}{m-1}\frac{(D_{t_m} - D_{t_1})^2}{t_m - t_1}$.

5.2.2. *Demand deposits calibrated as geometric Brownian motions.* Let $(D_t)_t$ follow a geometric Brownian motion (GB) with drift $\mu$ and volatility $\sigma > 0$, i.e. $D_t = D_0 e^{\mu t + \sigma B_t}$, where $D_0 > 0$.

Using the fact that $\log(D)$ is a normal random variable and applying Ito's Lemma one can show that $(D_t)_{t>0}$ must satisfy the following equation:

$$dD_t = \mu D_t dt + \sigma D_t dB_t.$$

Since demand deposits are recorded at $m$ equidistant moments of time, a unbiased drift estimator is: $\hat{\mu} = \frac{log(D_{t_m}) - log(D_{t_1})}{t_m - t_1}$.

A good volatility estimator is
$\hat{\sigma}^2 = \sum_{j=2}^{m} \frac{(log(D_{t_j}) - log(D_{t_{j-1}}))^2}{t_m - t_1} - \frac{1}{m-1}\frac{(log(D_{t_m}) - log(D_{t_1}))^2}{t_m - t_1}$.

5.2.3. *Demand deposits calibrated as Ornstein- Uhlenbeck processes.* We assume that the demand deposit process $(D_t)_{t\geq 0}$ follows an Ornstein-Uhlenbeck process (OU), i.e.

$$dD_t = \theta(\mu - D_t)dt + \sigma dB_t,$$

where $\theta > 0$ is the mean reversion rate, $\mu$ is the mean, and $\sigma > 0$ is the volatility.

Using the fact that $ED_t = D_0 e^{-\theta t} + \mu(1 - e^{-\theta t})$ and $Var(D_t) = \frac{\sigma^2}{2\theta}$ one can find the parameter estimators using a least square regression:

$$D_{t_j} = D_{t_{j-1}} e^{-\theta\delta} + \mu(1 - e^{-\theta\delta}) + \sigma\sqrt{\frac{1 - e^{-2\theta\delta}}{2\theta}} N(0,1) = aD_{t_{j-1}} + b + cN(0,1).$$

The model parameters are given by:
$\theta = -\frac{\log a}{\delta}, \mu = \frac{b}{1-a}, \sigma = c\sqrt{\frac{-2\log a}{\delta(1-a^2)}}$.

### 5.3. Results on the Kolmogorov-Smirnov test

We have 1221 banks and 40 data points for each bank. We obtained that the Kolmogorov Smirnov test did not reject 89% of the banks when calibrated to Brownian motion. More, the test did not reject the hypothesis of the Ornstein- Uhlenbeck model for 92% of the banks. Finally, the best fit among the three models is the geometric Brownian motion: the Kolmogorov- Smirnov test did not reject 94% of the banks. Overall, the Kolmogorov-Smirnov test did not show a significant discrepancy between the models and the data on the banks' deposits.

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