

## JOINT FORCES IN DYNAMICS OF A SPATIAL PARALLEL MANIPULATOR

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*Articolul stabilește relații matriceale pentru analiza dinamică inversă a unui manipulator paralel spațial. Pornind de la o cinematică inversă cunoscută, dinamica mecanismului este rezolvată printr-un procedeu bazat pe principiul lucrului mecanic virtual. În final, se obțin expresii compacte și grafice de simulare pentru unele forțe interne din articulații.*

*Recursive matrix relations for the inverse dynamics analysis of a spatial parallel manipulator are established in this paper. Starting from a known inverse kinematics, the dynamics of the mechanism is solved using an approach based on the principle of virtual work. Finally, compact expressions and graphs of simulation for some internal joint forces are obtained.*

**Keywords:** Joint force; Kinematics; Dynamics; Parallel manipulator

### 1. Introduction

One of the research directions in recent decades is the construction of manipulators with parallel architecture. These mechanisms are closed-loop structures that consist of numerous separate serial chains acting in parallel and connecting the fixed base to the moving platform. Equipped with revolute or prismatic actuators, parallel manipulators have a robust construction and can move bodies of large dimensions with high velocities and accelerations. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [1]; Merlet [2]). Accuracy and precision in the direction of the tasks are essential since the positioning errors of the tool could end in costly damage. But, from the application point of view, the limited workspace and complicated singularities are two major drawbacks of these mechanical structures.

The prototype of the Delta parallel robot (Clavel [3]; Tsai [4]) and the Star parallel manipulator (Hervé and Sparacino [5]) are equipped with three motors, which train on the mobile platform in a three-degrees-of-freedom translational motion. Angeles [6], Wang and Gosselin [7] analysed the kinematics, dynamics and singularity loci of Agile Wrist spherical robot with three actuators.

The 3-*RPS* parallel mechanism, here analyzed, consists of a moving platform, a fixed base and three limbs of identical structure. Because the robot motion in the translation and orientation degrees of freedom are *interconnected*, this makes the

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investigations more complicated, in comparison to other manipulators with a different spatial structure.

The inverse and forward kinematics problem for this manipulator was analyzed by Shah and Lee [8], Song and Zhang [9], Fang and Huang [10]. An interesting solution of the inverse kinematics task for the manipulator with RPS joint construction is obtained by Sokolov and Xirouchakis [11].

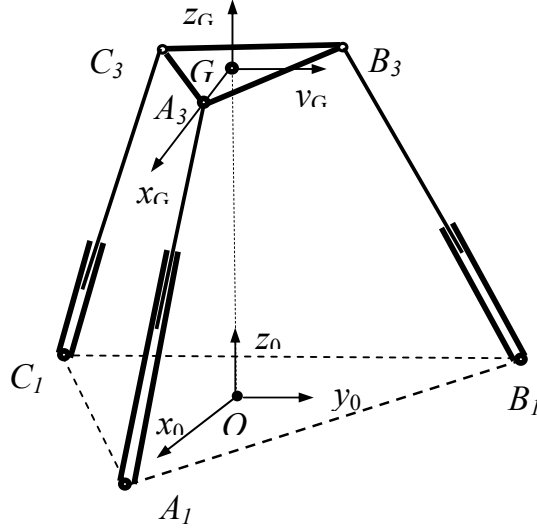


Fig. 1 The 3-RPS parallel mechanism

A recursive method is introduced in the present paper, to reduce significantly the number of equations and computation operations by using a set of matrices for the calculus of *internal joint forces* in the inverse dynamics of a 3-RPS spatial parallel mechanism.

## 2. Kinematics analysis

The special symmetrical 3-RPS architecture of parallel manipulators is already well known in the mechanism community. The manipulator consists of the base  $A_1B_1C_1$  and the upper platform  $A_3B_3C_3$  that are two equilateral triangles with  $L = l_0\sqrt{3}$  and  $l = r\sqrt{3}$  the lengths of the sides, respectively. The extensible legs connect the moving platform by spherical joints and the base by means of revolute joints, axe of which being parallel to the opposite edge. Each leg is made up of a cylinder and a piston connected together by a prismatic joint. Three actuated revolute joints of the legs drive the manipulator.

For the purpose of analysis, a Cartesian frame  $Ox_0y_0z_0(T_0)$  is attached to the fixed base with its origin located at triangle centre  $O$ , the  $z_0$  axis perpendicular to the base and the  $x_0$  axis pointing towards  $A_1$  from  $O$ . Another mobile reference frame  $Gx_Gy_Gz_G$  is attached to the moving platform (Fig. 1). The moving platform is initially located at a *central configuration*, where the platform's mass centre  $G$  is fixed at  $OG = h$  elevation. Six variables, namely three coordinates  $x_0^G, y_0^G, z_0^G$  of the mass centre and three classical Euler angles  $\alpha_1, \alpha_2, \alpha_3$  can describe the position and orientation of the moving platform with respect to the fixed reference frame. However, the mechanism is only a 3-DOF device, therefore only three of them are independent. Here,  $z_0^G, \alpha_1$  and  $\alpha_2$  are chosen as the independent variables and  $x_0^G, y_0^G, \alpha_3$  are parameters of *parasitic* motions. The three parasitic motions from the six commonly known motions of the moving platform are permanently dependent on three independent variables.

Since all rotations take place successively about the moving coordinate axes, the general rotation matrix  $R_{30} = R_{32}R_{21}R_{10}$  of the moving platform is obtained by multiplying three known transformation matrices

$$R_{10} = a_1\theta_1, \quad R_{21} = a_2\theta_1\theta_2, \quad R_{32} = a_3\theta_2\theta_1\theta_2$$

$$\theta_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \theta_2 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad a_k = \text{rot}(z, \alpha_k), \quad (k=1, 2, 3). \quad (1)$$

First active leg  $A$ , for example, consists of a fixed revolute joint, a moving cylinder of length  $l_1$ , mass  $m_1$  and tensor of inertia  $\hat{J}_1$ , which has a rotation about  $z_1^A$  axis with the angle  $\varphi_{10}^A$ . A prismatic joint is as well as a piston with the length  $l_2$ , the mass  $m_2$  and the tensor of inertia  $\hat{J}_2$  linked at the  $A_2x_2^Ay_2^Az_2^A$  frame, having a relative translation motion with the displacement  $\lambda_{21}^A$ . Finally, a spherical joint  $A_3$  is introduced to a planar moving platform, which is schematised as an equilateral triangle with edge  $l$ , mass  $m_p$  and tensor of inertia  $\hat{J}_p$  (Fig. 2).

At the central configuration, we also consider that all legs are initially extended at equal length  $\lambda_0 + l_2 = h / \cos\beta = (l_0 - r) / \sin\beta$  and that the angles of orientation of fixed pivots are given by

$$\alpha_A = 0, \alpha_B = 2\pi/3, \alpha_C = -2\pi/3. \quad (2)$$

Pursuing the first leg  $A$  in the  $OA_1A_2A_3$  way, for example, we obtain the following matrices of transformation [12]:

$$a_{10} = a_{10}^{\varphi} a_{\beta} \theta_3 a_{\alpha}^A, \quad a_{21} = \theta_3^T, \quad a_{20} = a_{21} a_{10}, \quad (3)$$

with

$$\theta_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \quad a_{10}^{\varphi} = \text{rot}(z, \varphi_{10}^A), \quad a_{\alpha}^A = \text{rot}(z, \alpha_A), \quad a_{\beta} = \text{rot}(z, \beta). \quad (4)$$

We consider that the position of the 3-RPS mechanism is completely given in the inverse geometric problem through the independent variable  $z_0^G$  and the angles of rotation  $\alpha_1, \alpha_2$ .

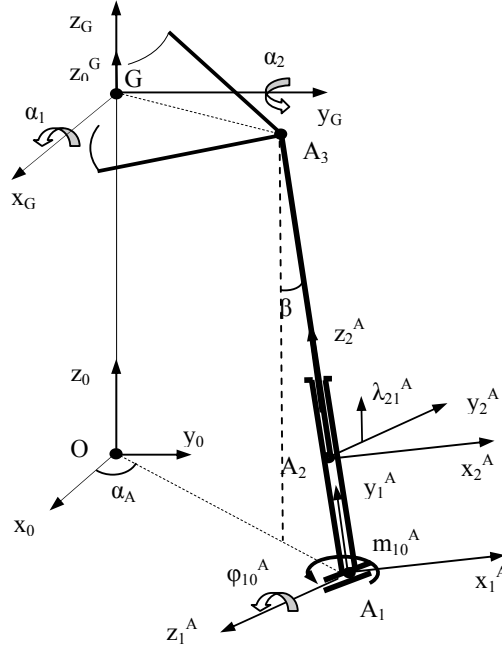


Fig. 2 Kinematical scheme of first leg  $A$  of the mechanism

Six independent variables  $\varphi_{10}^A, \lambda_{21}^A, \varphi_{10}^B, \lambda_{21}^B, \varphi_{10}^C, \lambda_{21}^C$  and the parameters  $\alpha_3, x_0^G, y_0^G$  can be determined by following vector-loop equations

$$\vec{r}_{10}^i + \sum_{k=1}^2 p_{k0}^T \vec{r}_{k+1,k}^i - R_{30}^T \vec{r}_{13}^i = \vec{r}_0^G \quad (p = a, b, c) \quad (i = A, B, C), \quad (5)$$

with the notations

$$\begin{aligned}\vec{r}_{10}^i &= l_0 a_\alpha^{iT} \vec{u}_1, \vec{r}_{21}^i = (\lambda_0 + \lambda_{21}^i) \vec{u}_2, \vec{r}_{32}^i = l_2 \vec{u}_3, \vec{r}_G^i = r a_\alpha^{iT} \vec{u}_1 \\ \vec{u}_1 &= [1 \ 0 \ 0]^T, \vec{u}_2 = [0 \ 1 \ 0]^T, \vec{u}_3 = [0 \ 0 \ 1]^T.\end{aligned}\quad (6)$$

Now, we compute in terms of the angular velocities  $\dot{\alpha}_1, \dot{\alpha}_2$  and the vertical velocity  $\dot{z}_0^G$  the angular velocities  $\dot{\alpha}_3, \omega_{10}^A, \omega_{10}^B, \omega_{10}^C$  and the linear velocities  $\dot{x}_0^G, \dot{y}_0^G, v_{21}^A, v_{21}^B, v_{21}^C$ , starting from following *matrix conditions of connectivity* [13]

$$\omega_{10}^A \vec{u}_j^T a_{10}^T \vec{u}_3 \{\vec{r}_{21}^A + l_2 a_{21}^T \vec{u}_3\} + v_{21}^A \vec{u}_j^T a_{10}^T \vec{u}_2 = \vec{u}_j^T \dot{\vec{r}}_0^G + \vec{u}_j^T R_{30}^T \tilde{\omega}_p \vec{r}_G^{A_3}, (j = 1, 2, 3) \quad (7)$$

$$\tilde{\omega}_p = \dot{\alpha}_1 R_{32} R_{21} \tilde{u}_3 R_{21}^T R_{32}^T + \dot{\alpha}_2 R_{32} \tilde{u}_3 R_{32}^T + \dot{\alpha}_3 \tilde{u}_3.$$

If other two kinematical chains *B* and *C* of the robot are pursued, analogous relations can be easily obtained.

Concerning the first leg *A*, the characteristic *virtual velocities* are expressed as functions of the position of the mechanism by the kinematical constraint equations (7), where we add the contributions of some virtual translations during three successive fictitious displacements of distal spherical joint  $A_3$  in the directions  $y_1^A, z_1^A$  and  $x_1^A$ , respectively, as follows:

$$\begin{aligned}\omega_{10}^{Av} \vec{u}_j^T a_{10}^T \vec{u}_3 \{\vec{r}_{21}^A + l_2 a_{21}^T \vec{u}_3\} + v_{21}^{Av} \vec{u}_j^T a_{10}^T \vec{u}_2 + v_{32}^{Av} \vec{u}_j^T a_{10}^T \vec{u}_2 - v_{43}^{Av} \vec{u}_j^T a_{10}^T \vec{u}_3 + v_{54}^{Av} \vec{u}_j^T a_{10}^T \vec{u}_1 = \\ = \vec{u}_j^T \tilde{v}_0^{Gv} + \vec{u}_j^T R_{30}^T \tilde{\omega}_p \vec{r}_G^{A_3}, (j = 1, 2, 3).\end{aligned}\quad (8)$$

Let us assume that the robot has successively some *virtual motions* determined by following three sets of velocities concerning the first leg *A*, for example:

$$\begin{aligned}\omega_{10a}^{iv} &= 0, v_{32a}^{Av} = 1, v_{32a}^{Bv} = 0, v_{32a}^{Cv} = 0, v_{43a}^{iv} = 0, v_{54a}^{iv} = 0 \\ \omega_{10a}^{iv} &= 0, v_{32a}^{iv} = 0, v_{43a}^{Av} = 1, v_{43a}^{Bv} = 0, v_{43a}^{Cv} = 0, v_{54a}^{iv} = 0 \\ \omega_{10a}^{iv} &= 0, v_{32a}^{iv} = 0, v_{43a}^{iv} = 0, v_{54a}^{Av} = 1, v_{54a}^{Bv} = 0, v_{54a}^{Cv} = 0 \quad (i = A, B, C).\end{aligned}\quad (9)$$

These virtual velocities are required into the computation of the virtual work of all the forces applied to the component elements of the mechanism.

As for the angular accelerations  $\ddot{\alpha}_3, \varepsilon_{10}^A, \varepsilon_{10}^B, \varepsilon_{10}^C$  and the linear accelerations  $\ddot{x}_0^G, \ddot{y}_0^G, \gamma_{21}^A, \gamma_{21}^B, \gamma_{21}^C$ , the derivatives with respect to time of the equations (7) give other following conditions of connectivity [14]

$$\begin{aligned}\varepsilon_{10}^A \vec{u}_j^T a_{10}^T \vec{u}_3 \{\vec{r}_{21}^A + a_{21}^T \vec{r}_{32}^A\} + \gamma_{21}^A \vec{u}_j^T a_{10}^T \vec{u}_2 = \vec{u}_j^T \ddot{\vec{r}}_0^G + \vec{u}_j^T R_{30}^T \{\tilde{\omega}_p \tilde{\omega}_p + \dot{\tilde{\omega}}_p\} \vec{r}_G^{A_3} - \\ - \omega_{10}^A \omega_{10}^A \vec{u}_j^T a_{10}^T \vec{u}_3 \{\vec{r}_{21}^A + a_{21}^T \vec{r}_{32}^A\} - 2 \omega_{10}^A v_{21}^A \vec{u}_j^T a_{10}^T \vec{u}_2 \quad (j = 1, 2, 3).\end{aligned}\quad (10)$$

The matrix relations (7) and (10) will be further used for the computation of the wrench of the inertia forces for every rigid component of the mechanism.

### 3. Dynamics modelling

The dynamics of parallel mechanisms is complicated by existence of multiple closed-loop chains. In the context of the real-time control, neglecting the friction forces and considering the gravitational effects, an important objective of the dynamics is first to determine the input torques or forces which must be exerted by the actuators in order to produce a given trajectory of the end-effector, but also to calculate all *internal joint forces or torques*.

Upon to now, several methods have been applied to formulate the dynamics of parallel mechanisms, which could provide the same results concerning these actuating torques or forces. First method applied to formulate the dynamics modelling is using the Newton-Euler procedure [15], the second one applies the Lagrange's equations and multipliers formalism [16] and the third approach is based on the principle of virtual work [17], [18].

Knowing the position and kinematics state of each link as well as the external forces acting on the 3-*RPS* parallel mechanism, in the present paper we apply the principle of virtual work for the inverse dynamic problem in order to establish some definitive recursive matrix relations for calculus of the internal joint forces.

Spatial evolution of the moving platform is controlled by three electric motors that generate three couples of moments  $\vec{m}_{10}^A = m_{10}^A \vec{u}_3$ ,  $\vec{m}_{10}^B = m_{10}^B \vec{u}_3$ ,  $\vec{m}_{10}^C = m_{10}^C \vec{u}_3$ . The force of inertia  $\vec{f}_{k0}^{inA} = -m_k^A [\ddot{\gamma}_{k0}^A + (\ddot{\omega}_{k0}^A \tilde{\omega}_{k0}^A + \tilde{\varepsilon}_{k0}^A) \tilde{r}_k^{CA}]$  and the resulting moment of inertia forces  $\vec{m}_{k0}^{inA} = -[m_k^A \tilde{r}_k^{CA} \ddot{\gamma}_{k0}^A + \hat{J}_k^A \tilde{\varepsilon}_{k0}^A + \tilde{\omega}_{k0}^A \hat{J}_k^A \tilde{\omega}_{k0}^A]$  of an arbitrary rigid body  $T_k^A$ , for example, are determined with respect to the centre of joint  $A_k$ . On the other hand, the wrench of two vectors  $\vec{f}_k^{*A}$  and  $\vec{m}_k^{*A}$  evaluates the influence of the action of the weight  $m_k^A \vec{g}$  and of other external and internal forces applied to the same element  $T_k^A$  of the manipulator.

Two significant recursive relations generate the vectors

$$\vec{F}_k^A = \vec{F}_{k0}^A + a_{k+1,k}^T \vec{F}_{k+1}^A, \quad \vec{M}_k^A = \vec{M}_{k0}^A + a_{k+1,k}^T \vec{M}_{k+1}^A + \tilde{r}_{k+1,k}^A a_{k+1,k}^T \vec{F}_{k+1}^A, \quad (11)$$

with the notations  $\vec{F}_{k0}^A = -\vec{f}_{k0}^{inA} - \vec{f}_k^{*A}$ ,  $\vec{M}_{k0}^A = -\vec{m}_{k0}^{inA} - \vec{m}_k^{*A}$ .

The fundamental principle of the virtual work states that a mechanism is under dynamic equilibrium if and only if the virtual work developed by all external, internal and inertia forces vanish during any general virtual displacement, which

is compatible with the constraints imposed on the mechanism. Starting from *the fundamental equations of the parallel robots dynamics* [19], [20], following compact matrix relations results

$$f_{k,k-1}^A = \bar{u}_3^T \{v_{21a}^{Av} \bar{F}_2^A + v_{21a}^{Bv} \bar{F}_2^B + v_{21a}^{Cv} \bar{F}_2^C + v_{10a}^{Gv} \bar{F}_1^G + v_{21a}^{Gv} \bar{F}_2^G + v_{32a}^{Gv} \bar{F}_3^G + \omega_{10a}^{Gv} \bar{M}_1^G + \omega_{21a}^{Gv} \bar{M}_2^G + \omega_{32a}^{Gv} \bar{M}_3^G\} \quad (k=3, 4, 5) \quad (12)$$

for the *internal joint forces*  $f_{32}^A, f_{43}^A, f_{54}^A$  acting along three orthogonal axes, where the virtual velocities  $v_{21a}^{Av}, v_{21a}^{Bv}, v_{21a}^{Cv}, v_{10a}^{Gv}, v_{21a}^{Gv}, v_{32a}^{Gv}, \omega_{10a}^{Gv}, \omega_{21a}^{Gv}, \omega_{32a}^{Gv}$  are determined starting from the conditions (8) and (9).

Further, we suppose that during three seconds following analytical functions can describe the general absolute motion of the moving platform

$$\frac{\alpha_1}{\alpha_1^*} = \frac{\alpha_2}{\alpha_2^*} = \frac{h - z_0^G}{z_0^{G*}} = 1 - \cos \frac{\pi}{3} t. \quad (13)$$

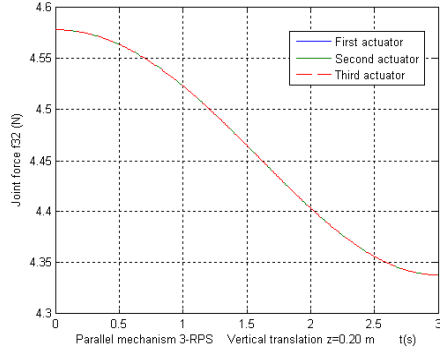


Fig. 3 Internal joint forces  $f_{32}^i$  from three legs

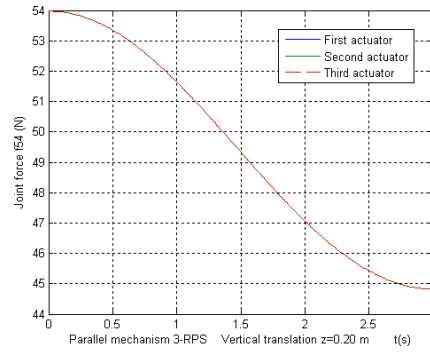


Fig. 4 Internal joint forces  $f_{54}^i$  from three legs

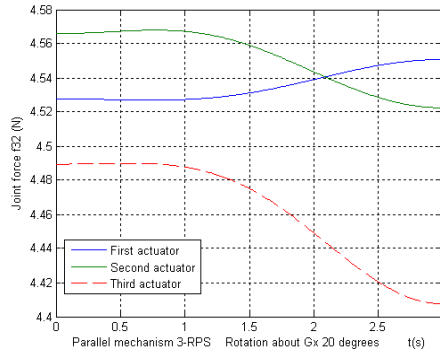


Fig. 5 Internal joint forces  $f_{32}^i$  from three legs

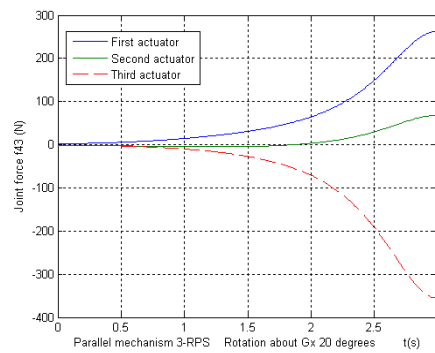


Fig. 6 Internal joint forces  $f_{43}^i$  from three legs

As application let us consider a manipulator which has following architectural and mechanical characteristics

$$z_0^{G*} = 0.1 \text{ m}, \alpha_1^* = \alpha_2^* = \frac{\pi}{18}, l_0 = 1 \text{ m}, r = 0.5 \text{ m}, l = r\sqrt{3}, h = 1.2 \text{ m}$$

$$\lambda_0 = 0.35 \text{ m}, l_1 = 0.7 \text{ m}, m_1 = m_2 = 0.5 \text{ kg}, m_p = 5 \text{ kg}, \Delta t = 3 \text{ s}.$$

Using the MATLAB software, a computer program was developed to solve the inverse dynamics of the spatial 3-*RPS* parallel manipulator. To illustrate the algorithm, it is assumed that the platform starts at rest from a central configuration and moves or rotates successively about two orthogonal directions.

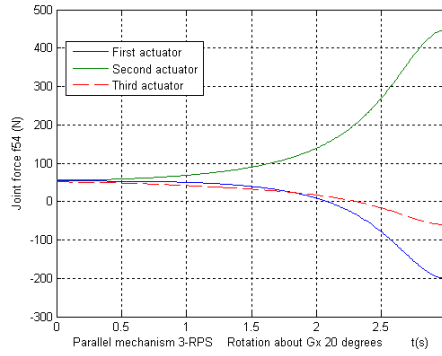


Fig. 7 Internal joint forces  $f_{54}^i$  from three legs

Considering that there are no external forces on the moving platform, a dynamic simulation is based on the computation of the orthogonal internal joint forces  $f_{32}^i, f_{43}^i, f_{54}^i$  ( $i = A, B, C$ ). For the first example, the moving platform moves along the vertical  $z_0$  direction with variable acceleration while all the other positional parameters are held equal to zero (Fig. 3), (Fig. 4). We remarks that the second joint force vanish during this symmetrical evolution, so  $f_{43}^i = 0$ . If the platform rotates about  $x_G$  axis, the internal joint forces are also calculated by the program and plotted versus time as follows:  $f_{32}^i$  (Fig. 5),  $f_{43}^i$  (Fig. 6) and  $f_{54}^i$  (Fig. 7).



#### 4. Conclusions

Based on the principle of virtual work, this approach can partially eliminate the forces of internal joints and establishes a direct determination of the time-history evolution for some internal forces or torques in joints.

Choosing appropriate serial kinematical circuits connecting many moving platforms, the present approach can be applied in forward and inverse mechanics of various types of parallel mechanisms, complex manipulators of higher degrees of freedom and particularly *hybrid structures* with increased number of components of the mechanisms.

#### REFERENCES

- [1] *D.Stewart*, A Platform with Six Degrees of Freedom, Proc. Inst. Mech. Eng., 1, 15, 180, pp. 371-378, 1965
- [2] *J-P.Merlet*, Parallel robots, Kluwer Academic, 2000
- [3] *R.Clavel*, Delta: a fast robot with parallel geometry, Proceedings of 18<sup>th</sup> International Symposium on Industrial Robots, Lausanne, pp. 91-100, 1988
- [4] *L-W.Tsai*, Robot analysis: the mechanics of serial and parallel manipulator, Wiley, 1999
- [5] *J-M.Hervé, F. Sparacino*, Star. A New Concept in Robotics, Proceedings of the Third International Workshop on Advances in Robot Kinematics, Ferrara, pp.176-183, 1992
- [6] *J.Angeles*, Fundamentals of Robotic Mechanical Systems: Theory, Methods and Algorithms, Springer, 2002
- [7] *J.Wang, C.Gosselin*, A new approach for the dynamic analysis of parallel manipulators, Multibody System Dynamics, Springer, 2, 3, pp. 317-334, 1998
- [8] *K-M.Shah, D-K,Lee*, Kinematic analysis of a three-degrees-of-freedom in parallel actuated manipulator, IEEE Journal of Robotics and Automation, 4, 3, pp. 354-360, 1988
- [9] *S-M.Song, M-D.Zhang*, A Study of Reaction Force Compensation Based on Three-Degrees-of-Freedom Parallel Platforms, Journal of Robotic Systems, 12, 2, pp. 783-794, 1995
- [10] *Y-F Fang, Z. Huang*, Kinematics of a three-degrees-of-freedom in parallel actuated manipulator mechanism, Mechanism and Machine Theory, Elsevier, 32, 7, pp. 1273-1287, 1997
- [11] *A.Sokolov, P. Xirouchakis*, Kinematics of a 3-DOF parallel manipulator with an R-P-S joint structure, Robotica, Cambridge University Press, 23, pp. 207-217, 2005
- [12] *S.Staicu*, Modèle dynamique en robotique, UPB Scientific Bulletin, Series D: Mechanical Engineering, 61, 3-4, pp. 5-19, 1999
- [13] *S.Staicu*, Recursive modelling in dynamics of Delta parallel robot, Robotica, Cambridge University Press, 27, 2, pp. 199-207, 2009
- [14] *S.Staicu*, Power requirement comparison in the 3-RPR planar parallel robot dynamics, Mechanism and Machine Theory, Elsevier, 44, 5, pp. 1045-1057, 2009
- [15] *B.Dasgupta, T.S.Mruthyunjaya*, A Newton-Euler formulation for the inverse dynamics of the Stewart platform manipulator, Mechanism and Machine Theory, Elsevier, 34, pp. 711-725, 1998
- [16] *Z.Geng, L.S.Haynes, J.D.Lee, R.L. Carroll*, On the dynamic model and kinematic analysis of a class of Stewart platforms, Robotics and Autonomous Systems, Elsevier, 9, pp. 237-254, 1992
- [17] *C-D Zhang, S-M. Song*, An Efficient Method for Inverse Dynamics of Manipulators Based on Virtual Work Principle, Journal of Robotic Systems, 10, 5, pp. 605-627, 1993

- [18] *A. Sokolov, P. Xirouchakis*, Dynamics of a 3-DOF parallel manipulator with R-P-S joint structure, *Mechanism and Machine Theory*, Elsevier, 42, pp. 541-557, 2007
- [19] *S.Staicu, X-J. Liu, J.Li*, Explicit dynamics equations of the constrained robotic systems, *Nonlinear Dynamics*, Springer, 58, 1-2, pp. 217-235, 2009
- [20] *S. Staicu*, Dynamics analysis of the Star parallel manipulator, *Robotics and Autonomous Systems*, Elsevier, 57, 11, pp. 1057-1064, 2009