

RESEARCH ON MATHEMATICAL MODEL AND EXPERIMENT OF INTERNAL GEAR BY POWER SKIVING

Longlong GENG^{1*}, Xingzhao CHEN², Zhengyang HAN³, Chuang JIANG⁴

In order to meet the large demand of internal gears for new energy vehicles and solve the efficient processing problems, a method to machine internal gear by power skiving is put forward. The principle of power skiving to internal gear was analyzed, and mathematical model was established. Then the tooth surface equation of cutter and internal gear was established and derived. An internal gear was simulated to verify the correctness of the model. Finally cut experiment was carried out and the workpiece was measured. The precision of internal gear meets the engineering requirements. The results proved the correctness and effectiveness of model proposed in this paper. It has theoretical and engineering application value for machine of internal gear.

Keywords: Internal gear; Power skiving; Mathematical model; Simulation; Cut experiment

1. Introduction

With the development of low-carbon and environmental protection, new energy vehicles has become an important concern and attracting many experts and scholars to research. New energy vehicles is high-speed, miniaturization, and integration of electric drives, so the requirements of high-speed, high-power, and lightweight transmissions are put forward. Planetary reducer is widely used in new energy vehicles for the advantages of compactness, stable transmission, and large carrying capacity[1]. Internal gears are key components of planetary reducers[2], and their efficient machining is even more important.

Traditionally internal gears are machined by gear shaping. It is single-tooth machining method for the movement of gear shaper cutter during the cutting process[3-5], which resulting in low efficiency and cannot match the demand to

* Corresponding author

¹ Associate Prof., School of Mechatronics Engineering, Henan University of Science and Technology, China, e-mail: gll0379@haust.edu.cn

² Master's student, School of Mechatronics Engineering, Henan University of Science and Technology, China, e-mail: 17550242045@163.com

³ Lecturer, School of Mechatronics Engineering, Henan University of Science and Technology, China, e-mail: hanzhengyang@haust.edu.cn

⁴ Associate Prof., School of Mechatronics Engineering, Henan University of Science and Technology, China, e-mail: jiangchuang0379@163.com

internal gears. Gear skiving is based on the principle of spatial spiral gear meshing, the axes of tool and workpiece are crossed-axis in space. The relative motion speed between the axes is used to achieve cutting during machining. Not only the machining efficiency, but the machining accuracy is very high, so gear skiving is an efficient gear machining method. It is provided a potential solution to efficient and precise machining of internal gears.

Experts and scholars had put forward the method of internal gear by skiving in 1970s in theory, such as Jiang Xuewen [6], Zhai Fenglai [7]. However, due to the limitations of cutting tools and machine tools, it is not used in industry. In recent years, with the development of the design and manufacturing of CNC machine tools and cutting tools, the gear skiving technology has been promoted in the field of gear processing. Yang [8] proposed a method for scraping cylindrical gears; Guo[9,10] introduced the cylindrical gear carving; Cao[11] established a mathematical model of helical cylindrical gear turning and analyzed the tooth surface error. Deng[12,13] studied the machining of face gears and non-orthogonal surface gears. The above research has carried out the research on gear turning for different types of gears, which greatly enriches the gear processing technology. At the same time, domestic scholars have studied the design and technology of skiving tools, and Guo[14,15] designed a skiving cutter to complete the processing of involute cylindrical gears. Li[16] simulated and analyzed the dynamic characteristics of cutting force and temperature during the gear turning process. Guo [17] analyzed the influence of the error of the tooth turning tool on the accuracy of the tooth turning, and Yao [18] studied the simulation of the tooth turning. The above research has carried out in-depth research on the control of process parameters and the forming quality of the tooth surface in the gear turning process. Therefore, domestic experts and scholars have conducted in-depth research on the gear skiving technology, but the gear skiving processing technology of internal gears needs further discussion and research.

On this basis, this paper proposes a method for the processing of internal gear for new energy vehicles, to solve the problem of low efficiency of traditional gear shaping and achieve efficient and high-quality machining of internal gears. In this paper, the principle of internal gear machining will be introduced, then a mathematical model of gear cutting will be established, and the internal gear of the planetary reducer of new energy vehicles will be taken as an example to deduce the tooth surface equation of the internal gear and carry out simulation to verify the established gear cutting model through cutting experiment.

2. Principle of internal gear by power skiving

Skiving is based on the principle of staggered shaft meshing. During the machining process, the shaft intersection angle between the cutter and the

workpiece is generated, as shown in Fig.1. Its size is determined by the size of the helix angle between the workpiece and the skiving cutter. a_1 is axis of workpiece, a_c is axis of cutter, ω_1 is the angular velocity of the workpiece, ω_c is the angular velocity of the tool. The tool pitch circle is tangent to the workpiece at the point O , β_1 is the helix angle of the workpiece, β_c the helix angle of cutter, and Σ is the axial intersection angle between cutter and workpiece.

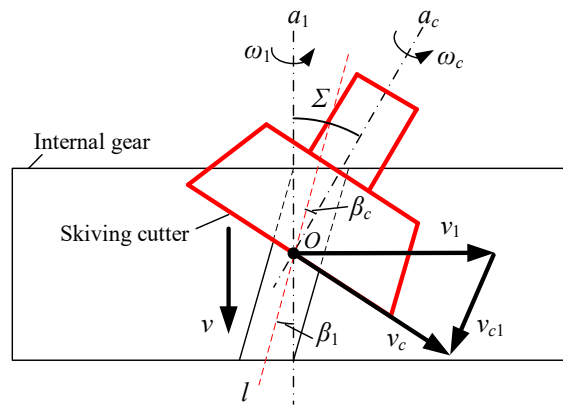


Fig. 1. Principle of internal gear by power skiving

When skiving gear, the tooth direction l of the gear should be in the same direction as the cutter tooth direction, so it should be satisfied

$$\Sigma = \beta_1 - \beta_c \quad (1)$$

Where, the helix angle β_1 is positive, it means left-handed, when the helix angle is negative, it means right-handed.

The skiving motion is composed of two parts, v_{c1} is the cutting motion of the workpiece and skiving cutter at the point O , which is synthesized by the rotation speed of the workpiece v_1 and the rotation speed of the tool v_c ; The other is the feed movement v of the skiving cutter along the axis of the workpiece. The motion between the workpiece and the skiving cutter satisfies the following relationship

$$\omega_1 = \frac{z_1}{z_c} \omega_c \pm 2\pi \frac{v}{L} \quad (2)$$

Where, z_c is the teeth number of skiving cutter, z_1 the teeth number of workpiece, and L the pitch of the workpiece. ‘-’ is applicable to the same helix direction of the cutter and the workpiece, ‘+’ is the opposite.

3. Mathematical model of power skiving

3.1 Mathematical model of skiving cutter

The skiving cutter can be seen as a cylindrical helical gear with a decreasing displacement coefficient from rake face to tool holder, and the cutting edge on rake face can be reduced to an end face involute, as shown in Fig. 2. r_b is the base circle radius of the end face involute, and r_0 is reference circle radius of cutter. The starting point of the involute is the intersection point of base circle with X_1 , and Z_1 the direction coincides with the direction of the cutter axis.

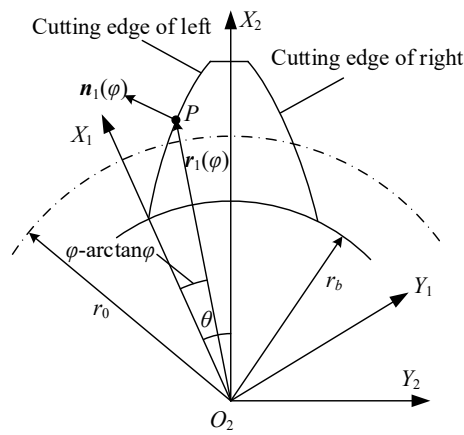


Fig. 2. Mathematical model of skiving cutter

The homogeneous coordinates position vector and normal vector of point P on the cutting edge in S_1 can be expressed as

$$\mathbf{r}_1(\varphi) = \begin{bmatrix} r_b(\cos \varphi + \varphi \sin \varphi) \\ S_a r_b(\sin \varphi - \varphi \cos \varphi) \\ 0 \\ 1 \end{bmatrix} \quad (3)$$

$$\mathbf{n}_1(\varphi) = \begin{bmatrix} S_a^2 \sin \varphi \\ -S_a \cos \varphi \\ 0 \end{bmatrix} \quad (4)$$

Where, $S_a=1$ indicates the left cutting edge, and $S_a=-1$ indicates the right cutting edge. φ is the profile parameter of the involute, and the relationship between the pressure angle α_p at the point of the involute P is as follows: $\varphi = \tan \alpha_p$.

In coordinate system S_2 , X_2 is the symmetry line of the left and right cutting edges, and the coordinates of point P are represented in the S_2 coordinate system

$$\mathbf{r}_2(\varphi) = \mathbf{M}_{21}\mathbf{r}_1(\varphi) \quad (5)$$

Where

$$\mathbf{M}_{21} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.2 Mathematical model of skiving

The machining model of internal gear skiving is established as shown in Fig. 3. The coordinate system S_6 is fixed to the workpiece, and Z_6 the direction coincides with the axis of the workpiece. The coordinate system S_3 , S_4 , S_5 are auxiliary coordinate system, where Z_5 and Z_6 are coincide; the coordinate system S_5 is parallel to S_4 ; Z_3 coincides with Z_2 . φ_c is rotate of the cutter, and φ_1 is rotate of the workpiece. a is the center distance between the cutter and the workpiece, and d_s is the axial position of the tangent contact in the direction Z of S_6 .

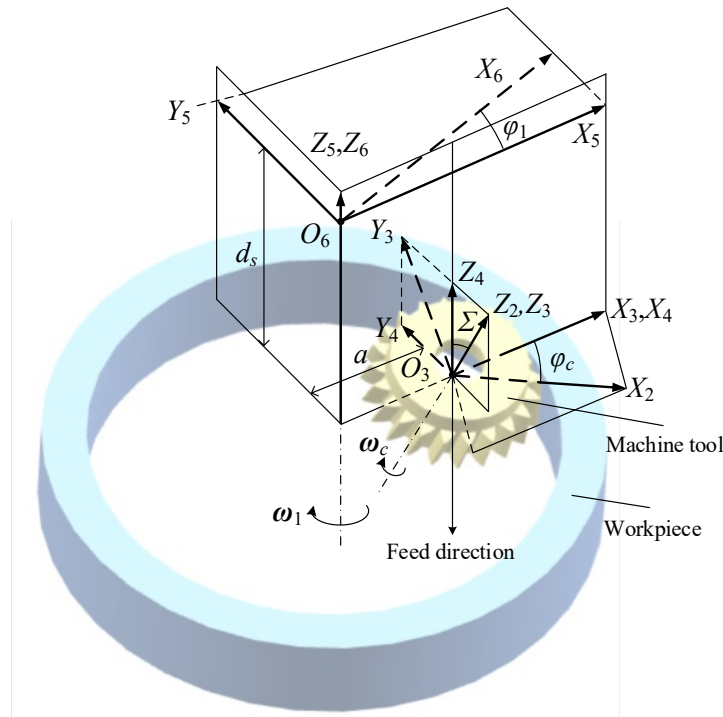


Fig. 3. Mathematical model of internal gear skiving

The tooth surface of internal gear can be obtained by coordinate transformation as

$$\mathbf{r}_6(\varphi, \varphi_c, d_s) = \mathbf{M}_{62}(\varphi_c, d_s) \mathbf{r}_2(\varphi) \quad (6)$$

Where $\mathbf{M}_{62}(\varphi_c, d_s) = \mathbf{M}_{65} \mathbf{M}_{54}(d_s) \mathbf{M}_{43} \mathbf{M}_{32}(\varphi_c)$

$$\mathbf{M}_{32} = \begin{bmatrix} \cos \varphi_c & \sin \varphi_c & 0 & 0 \\ -\sin \varphi_c & \cos \varphi_c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{43} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \Sigma & -\sin \Sigma & 0 \\ 0 & \sin \Sigma & \cos \Sigma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{54} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_s \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{65} = \begin{bmatrix} \cos \varphi_1 & \sin \varphi_1 & 0 & 0 \\ -\sin \varphi_1 & \cos \varphi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

According to eq. (2), the following relationship is satisfied

$$\varphi_1 = \frac{z_1}{z_c} \varphi_c \pm 2\pi \frac{d_s}{L} \quad (7)$$

Where + apply to left-handed internal gear and - apply to the right-handed internal gear.

Then meshing equation can be expressed according to eq. (6).

$$f(\varphi, \varphi_c, d_s) = \frac{\partial \mathbf{r}_6}{\partial \varphi} \times \frac{\partial \mathbf{r}_6}{\partial \varphi_c} \cdot \frac{\partial \mathbf{r}_6}{\partial d_s} = 0 \quad (8)$$

3.3 Tooth profile points solving

According to tooth surface rotation projection, O_H is the projection point on Z_6 of tooth profile P_i which satisfies eq. (6) as showed in Fig. 4. H_i is the distance from point P_i to the X_6Y_6 plane, and R_i is the radius on the profile where point P_i is located. Then tooth profile P_i can be expressed as

$$\begin{cases} r_{6z} - H_i = 0 \\ r_{6x}^2 + r_{6y}^2 - R_i^2 = 0 \end{cases} \quad (9)$$

A nonlinear equation system with three unknowns $(\varphi, \varphi_c, d_s)$ are formed by eq.(8) and eq.(9). By giving an initial value, a set of exact solutions can be calculated. Substituted it into eq. (6), the tooth profile points can be obtained.

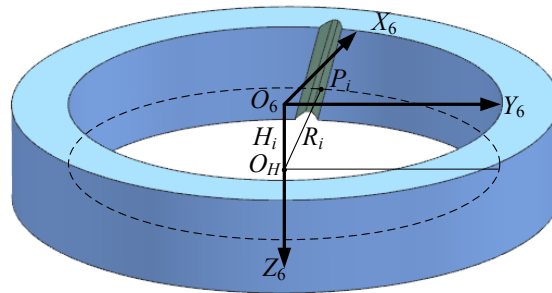


Fig. 4. Position of tooth profile points

4. Numerical examples

According to the established mathematical model of gear skiving, an internal gear of new energy planetary reducer is machined as example; the parameters of internal gear and skiving cutter are shown in Table 1. The tooth profile points of the internal gear are calculated by eq.(6)-eq.(9), and a model is established in UG[19], as shown in Fig. 5.

Table 1

Parameters of Internal gear and skiving cutter	
Item	value
teeth number of workpiece z_1	97
normal module m_n /mm	2.5
pressure angle $a/(\circ)$	17
modification coefficient x /mm	0.2195
tip diameter d_{a1} /mm	251.1
root diameter d_{f1} /mm	262.4
teeth number of skiving cutter z_c	57
helix Angle of cutter $\beta_c/(\circ)$	0
shaft Angle $\Sigma/(\circ)$	18
center distance a /mm	56.24

4.1 Simulated machining

Simulation machining can effectively verify the parameters of tool design, and predict the interference, installation and other problems that may occur in the actual machining process. VERICUT[20] was used to simulate the gear skiving.

The profile of skiving cutter was modeled in UG and imported into VERICUT for simulated machining, as shown in Fig. 6. In VERICUT, an internal gear skiving machine tool is established, the skiving cutter and workpiece are assembled on the tool spindle and the workpiece spindle respectively, and the

machining program is written to carry out the internal gear skiving simulation, as shown in Fig. 7.

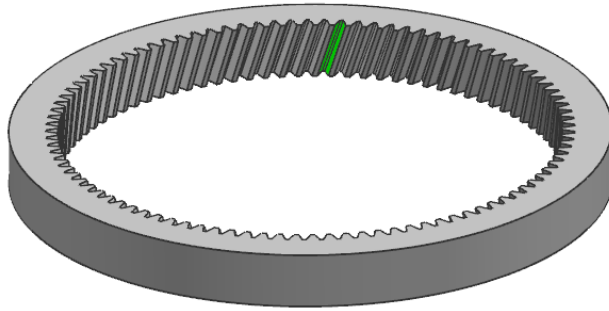


Fig. 5. 3D model of internal gear

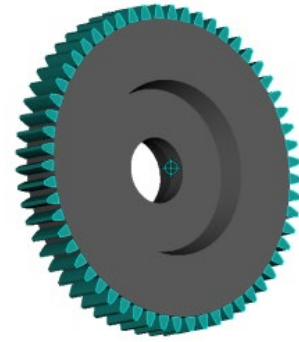


Fig. 6. 3D model of skiving cutter

In the process of simulated machining, there is no interference between skiving cutter, machine tool, and the fixture, etc. The simulated internal gear is shown in Fig. 8.

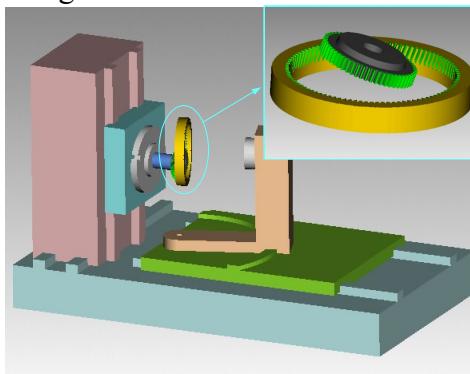


Fig. 7. Simulate of machine

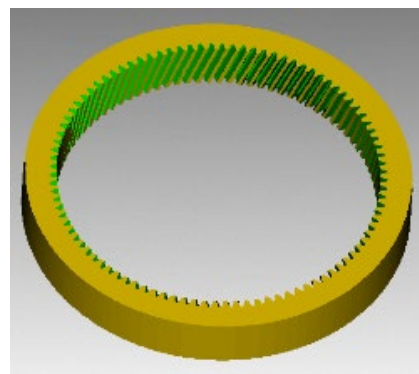


Fig. 8. Simulation result

Comparing Fig. 8 and Fig. 5, the internal gear of simulated and theoretical modeling are consistent in tooth profile and tooth direction, which proves the correctness of the mathematical model of skiving and the tooth CNC program. At the same time, the idle stroke in the machining process is optimized, the processing time is shortened, and the processing efficiency is further improved.

4.2 Cut experiments

The skiving cutter was manufactured according to Table 1, as shown in Fig. 9. The cut experiments were carried out on YK2260NC produced by Luoyang Keda gear Transmission Research Institute Co., Ltd., which is CNC skiving machine for gear. The internal gear was machined according to the optimized skiving program, as shown in Fig.10.



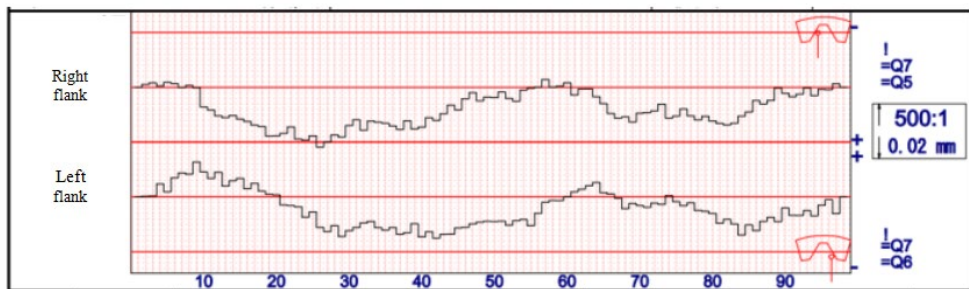
Fig. 9. Skiving cutter

Fig. 10. Machine of internal gear

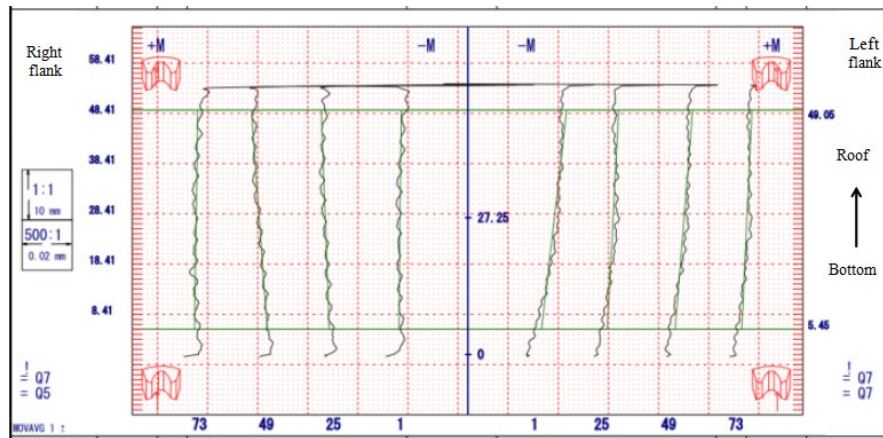
After the processed workpiece is chamfered. The processing accuracy is tested by the Gleason 650GMS, as shown in Fig.11. The measurement results are shown in Fig.12.



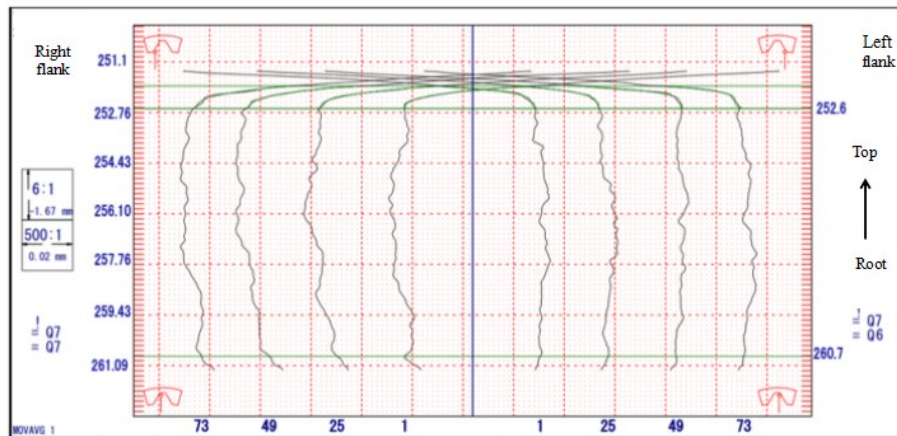
Fig. 11. Measurement of internal gear



(a) pitch deviation



(b) helix deviation



(c) profile deviation

Fig. 12. Measurement results

The pitch deviation, helix deviation and profile deviation of internal gear are showed in Fig.12. According to the calculation of CHINA GB-2001, the pitch deviation accuracy of the processed workpiece is 5 and 6 grades, the helix deviation accuracy is 5 and 7 grades, and the profile deviation accuracy is 6 and 7 grades respectively. The reasons for the above errors include the motion error of the machine tool, the installation error of the workpiece, the profile error of the tool, etc., but all within the range of error requirements and meet the requirements of pre-machining accuracy. The measurement results verify the correctness of the established model.

5. Conclusions

In this paper, a method for internal gears by skiving was proposed to meet major engineering requirements. The mathematical model by skiving was

established. Theoretical analysis and experimental verification were carried out, and the results demonstrated the feasibility of this method. This paper provides theoretical and technical support for the efficient gear skiving of internal gears.

At the same time, the parameters optimization of gear skiving will be carried out in the follow-up to further improve the service life and processing efficiency of skiving cutter.

Acknowledgement

The research is funded by Henan Provincial Science and Technology Research Project (No.232102220057, No.242102220084).

REFERENCES

- [1] *W. Cai, X.G. Wu, M. H. Zhou, et al*, Review and development of electric motor systems and electric power trains for new energy vehicles, *Automotive Innovation*, Vol. **4**, Iss. 1, 2021.
- [2] *X.Y. Chen*, Video illustration of new energy vehicle structure and principle, Beijing: China Machine Press, 2020.
- [3] *C.H. CUI, H. ZHAO*, Development of simulation system for internal gear shaping, *Tool Application Technology*, Iss. 7:32-36, 2020.
- [4] *B. Li, J. He, F.Y. Zheng*, Research on three-axis linkage shaping technology for non-circular internal gears, *China Mechanical Engineering*, Vol. **27**, Iss. 11: 1455-1460, 2016.
- [5] *J.Z. Mao, H.M. Li, Z.M. Wu*, Research on the machining method of oblique slotting variable tooth thickness internal gears, *Journal of Mechanical Engineering*, Vol. **32**, Iss. 02: 82-89, 1996.
- [6] *X.W. Jiang*, Skiving of inner gear teeth, *Machine*, Iss.04: 46-49, 1974.
- [7] *F.L. Zhai*, New processing technology of internal helical gear-skiving, *Machine worker*, Iss. 03:6-7, 1982,
- [8] *J. Yang, H.J. Song*, Research practice of a new type of gear machining technology, *Journal of Mechanical Transmission*, Iss. 12:163-167, 2014.
- [9] *X.D. Huang, H. Yang, E.K. Guo, et.al*, Advanced gear cutting theory and method, *Machine Design and Manufacturing Engineering*, Iss. 12: 163-167, 2014.
- [10] *E.K. Guo, R.J. Hong, X.D. Liu, et.al*, Research on the cutting mechanism of cylindrical gear power skiving, *International Journal of Advanced Manufacturing Technology*, Vol. **79**, Iss. (1-4): 541-550, 2015.
- [11] *C.J. Wang, X.M. Cao, H. Xu, et.al*, Processing mathematical model and tooth surface error analysis of helical cylindrical gears skiving, *Journal of henan university of science and technology*, Vol. 43, Iss. 4:15-20, 2022.
- [12] *Z.Y. Han, C. Jiang, X.Z. Deng*, Research on machining nonorthogonal face gears by power skiving with tooth flank modification based on a six-axis machine tool, *International Journal of Advanced Manufacturing Technology*, Vol. **121**, Iss. (3-4): 2735-2746, 2022.
- [13] *Z.Y. Han, C. Jiang, X.Z. Deng*, Machining and meshing analysis of face gears by power skiving, *Journal of Advanced Mechanical Design Systems and Manufacturing*, Vol. **16**, Iss. 1: 1-16, 2022.
- [14] *E.K. Guo, R. J. Hong, X.D. Huang, et.al*, Research on the design of skiving tool for machining involute gears, *Journal of Mechanical Science and Technology*, Vol. **28**, Iss. 2: 5107-5115, 2014.

- [15] *E.K. Guo, N.F. Ren, X.D. Liu, et.al*, An efficient tapered tool having multiple blades for manufacturing cylindrical gears with power skiving, *International Journal of advanced manufacturing technology*, Vol. **102**, Iss. (9-12): 2823-2832, 2019.
- [16] *G. Li, Y.P. Chen, Y. Yang*, Simulation Analysis of Cutting Force and Temperature Dynamic Characteristics During Gear Skiving Process, *Journal of Mechanical Transmission*, Vol. **46**, Iss. 7: 121-125, 2022.
- [17] *W.C. Guo, Y. Yang, S.M. Mao, et.al*, Analysis of the Influence of Eccentricity Error of Gear Skiving Cutting Tool Movement on Gear Skiving Accuracy, *Journal of Mechanical Transmission*, Vol. **44**, Iss. 7: 89-94, 2020.
- [18] *J.C. Yao, Y.X. Yang, Y. Jiang, et.al*, Research and Application of Skiving Virtual Cutting Simulation Technology, *Tool Engineering*, Iss. 04: 75-80, 2022.
- [19] *Z.Z Li, R.X. Hu*, *UG NX 12Version Practical Tutorial in Chinese*, Beijing: Posts & Telecom Press, 2023.
- [20] *X.M. Liao, Y.M. Gu*, Construction and Simulation of Five-Axis Machine Tool Based on VERICUT Software, *Mould manufacturing technology*, Vol.25, Iss. 9: 33-35, 2025.