ANALYSIS OF THE MODIFICATION PRINCIPLE OF THE PITCH CURVE OF ECCENTRIC GEAR AND IT'S TRANSMISSION CHARACTERISTICS

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In order to obtain different variable-speed transmission characteristics, based on the eccentric gear, this paper divides the eccentric gear into N segments and introduces a modification coefficient m_{1i} *in each segment interval to obtain a multi-segment modified eccentric gear mechanism. The mathematical models of the pitch curve and transmission characteristics of the multi-segment modified eccentric gear are established by using geometric relations and numerical analysis methods, and a visual design software is compiled using MATLAB. By analyzing the factors affecting the transmission characteristics of the multi-segment modified eccentric gear, the following conclusions are obtained: ①The eccentricity affects the variation range of the transmission ratio; ②The modification coefficient* m_{1i} *can change the law of the transmission ratio; ③The more the number of segments, the more modification coefficients* m_{1i} *can be introduced, the more types of transmission ratio characteristics can be obtained, the pitch curve is closer to the free pitch curve, and the adjustability is stronger.*

Keywords: Eccentric gear; Multi - segment modification; Transmission ratio; Variable speed; Matlab

Nomenclature

R	Eccentric circle radius
e	Eccentricity
r_1	Radius of pitch curve of driving gear
φ_1	Rotate angle of driving gear
r_2	Radius of pitch curve of driven gear
φ_2	Rotate angle of driven gear
a	Distance of gear pair
τ	Transmission ratio

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m_{1i} The modification coefficient of the i-th segment, $i \in [1, N]$
N Number of segments of the driving gear

1 Introduction

Gear mechanism[1] is a common transmission mechanism used to transmit large torque and is widely used in agriculture[2], industry[3], aerospace industry[4]. A pair of cylindrical gears can only achieve uniform transmission. To meet certain variable-speed working conditions, the rotation center of the cylindrical gears is usually offset by a certain distance to obtain eccentric gears[5] that can achieve variable-speed transmission[6]. However, the variable-speed transmission characteristics of eccentric gears are relatively single and cannot meet the needs of more variable-speed working conditions.

Non-circular gears[7] are used to achieve a non-linear relationship between the rotation angles of the driving mechanism and the driven mechanism. They integrate the advantages of the link mechanism[8] and the cam mechanism[9] in transmission, and have the kinematic characteristics of a compact structure, smooth transmission, and continuous unidirectional rotation[10-12].

Zhao H.[13] et al used eccentric gear to drive a differential pump, effectively reducing the pulsation rate of the differential pump. Yu G.[14] et al applied eccentric gear to the planting mechanism, the motion trajectory of the obtained planting mechanism can meet the trajectory requirements during seedling planting. Wang Y.[15] et al used eccentric gear to drive winding mechanism, the raw silk can obtain a package shape approximately in the form of a trapezoid through this winding mechanism.

Due to the small number of design parameters of eccentric gears, they cannot well meet the requirements of complex transmission working conditions. Based on the two-segment modification principle[16], this paper proposes to divide the pitch curve into N segments, and introduces different modification coefficients in each segment, multi-segment modified eccentric gears with stronger flexibility in adjusting transmission characteristics are obtained. The numerical analysis method is used to construct the mathematical models of the pitch curves of the gear pair and their transmission characteristics. Since there are many design parameters and the direct analysis and calculation process is complex and cumbersome, a visual design platform is developed using MATLAB. On the basis of analyzing the influence law of the eccentricity on the transmission ratio of eccentric gears， studying the influence of the modification coefficient on the eccentric gear.

This article is organized as follows. Section 2 introduce the evolution method of multi-segment modified eccentric curve and establish the mathematical model of multi-segment modified eccentric curve, Section 3 establish the mathematical model of pitch curve and transmission ratio of multi-segment modified eccentric gear, Section 4 compile a visual design platform, and introduce the functional features of this software. Section 5 analyze the influencing factors affecting the transmission characteristics of multi- segment modified eccentric gear. Section 6 draws the main conclusions.

2. Multi - segment modified eccentric curve

2.1 eccentric curve

Fig.1 shows the principle of generating an eccentric curve. As can be seen from Fig.1, according to the geometric relationship, the expression of the eccentric curve can be written as

$$r_1(\varphi_1) = \sqrt{R^2 - e^2 \sin^2(\varphi_1)} - e\cos(\varphi_1) \tag{1}$$

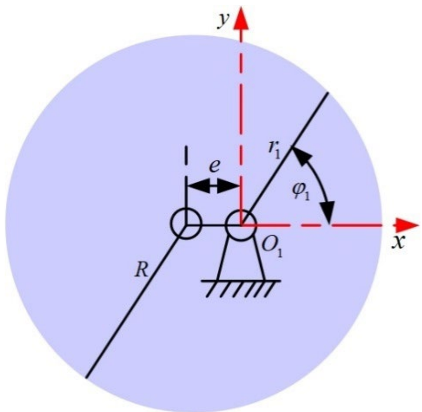

Fig.1 Eccentric curve

2.2 Two - segment Modified Eccentric Curve

Currently, the commonly seen modified non - circular gears are mainly of the two - stage modification type. In Fig.2, black solid line curve is the eccentric curve, the eccentric curve is divided into two segments at the polar angle $\varphi = \pi$. A modification coefficient m_{11} is introduced in the curve segment $\varphi \in (0 \sim \pi)$, so that the original two polar angles $\varphi \in (0 \sim \pi)$ and $\varphi(\pi \sim 2\pi)$ can be adjusted to two new polar angles $\varphi_1(0 \sim \pi/m_{11})$ and $\varphi_2(\pi/m_{11} \sim 2\pi)$, the modification coefficient of the second - stage curve is set as m_{12} . The expression of the two-stage modified eccentric curve can be written as

$$r_1(\varphi_1)=\begin{cases}\sqrt{R^2-e^2\sin^2(m_{11}\varphi_1)}-e\cos(m_{11}\varphi_1) & \left(0\le\varphi_1\le\dfrac{\pi}{m_{11}}\right)\\ \sqrt{R^2-e^2\sin^2\left(m_{12}\left(2\pi-\varphi_1\right)\right)}-e\cos\left(m_{12}\left(2\pi-\varphi_1\right)\right) & \left(\dfrac{\pi}{m_{11}}\le\varphi_1\le2\pi\right)\end{cases} \tag{2}$$

The modified eccentric curve can be obtained by changing the polar angles (Red dotted curve in Fig.2 shows the modified eccentric curve when $m_{11}=2$ is applied, Blue dotted-dashed curve is the modified eccentric curve with $m_{11}=0.6$).

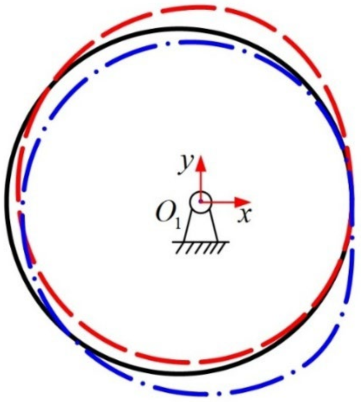

Fig.2 Two-stage modified eccentric curve

To ensure the continuous closure of the pitch curves of two-stage modified eccentric curve, the introduced modification coefficients m_{11} and m_{12} need to satisfy the following expressions:

$$\begin{cases}m_{11}>1/2\\ \dfrac{\pi}{m_{11}}+\dfrac{\pi}{m_{12}}=2\pi\end{cases} \tag{3}$$

2.3 Multi-segment modified eccentric curves

To meet the requirements of more transmission function needs, on the basis of the two-segment modified eccentric section curve, the continuous closed eccentricity is divided into N segments ($N\ge3$)[19], as shown in Fig.3. For each segment of eccentric curve in the figure, a modification coefficient m_{1i} ($i\in[1,N]$) is introduced respectively. By changing the value of m_{1i}, different modified eccentric curves can be obtained.

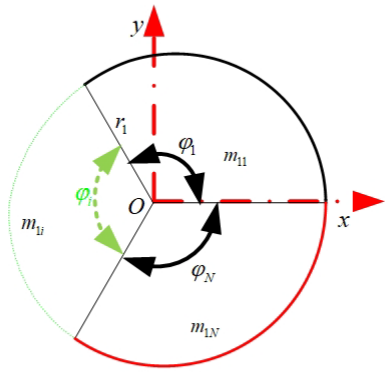

Fig.3 Muti-stage modified eccentric curve

According to geometric relationship and numerical analysis, a mathematical equation of multi-segment modified eccentric curve can be established:

$$r_1(\varphi_1)=\begin{cases}\sqrt{R^2-e^2\sin^2(m_{11}\varphi_1)}-e\cos(m_{11}\varphi_1) & \left(0\le\varphi_1\le\dfrac{2\pi}{Nm_{11}}\right)\\ \sqrt{R^2-e^2\sin^2(m_{12}\left(\varphi_1-\dfrac{2\pi}{Nm_{11}}\right)+\dfrac{2\pi}{N})}-e\cos(m_{12}\left(\varphi_1\dfrac{2\pi}{Nm_{11}}\right)+\dfrac{2\pi}{N}) & \\ \left(\dfrac{2\pi}{Nm_{11}}\le\varphi_1\le\dfrac{2\pi}{Nm_{11}}+\dfrac{2\pi}{Nm_{12}}\right) & \\ \ldots\ldots & \\ \sqrt{R^2-e^2\sin^2(m_{1i}\left(\varphi_1-\sum_{i=1}^{j}\dfrac{2\pi}{Nm_{1i}}\right)+\dfrac{2\pi(j-1)}{N})}-e\cos(m_{1i}\left(\varphi_1-\sum_{i=1}^{j}\dfrac{2\pi}{Nm_{1i}}\right)+\dfrac{2\pi(j-1)}{N}) & \\ \left(\sum_{i=1}^{j-1}\dfrac{2\pi}{Nm_{1i}}\le\varphi_1\le\sum_{i=1}^{j}\dfrac{2\pi}{Nm_{1i}}\right) & \\ \ldots\ldots & \\ \sqrt{R^2-e^2\sin^2\left(m_{1N}\left(2\pi-\varphi_1\right)\right)}-e\cos\left(m_{1N}\left(2\pi-\varphi_1\right)\right) & \left(\sum_{i=1}^{N-1}\dfrac{2\pi}{Nm_{1i}}\le\varphi_1\le 2\pi\right)\end{cases} \tag{4}$$

To ensure the closure of the multi-segment modified eccentric curve, the modified coefficients m_{1i} of each segment need to satisfy the following equation:

$$\begin{cases}m_{1i}>1/N\\ \sum_{i=1}^{N}\dfrac{2\pi}{m_{1i}}=2\pi N\end{cases} \tag{5}$$

Verification of the continuity of the pitch curves of multi-segment modified eccentric curve at the segmentation points:

When $\varphi_1=0$ and $\varphi_1=2\pi$, according to Eq.4, the pitch curve radius vectors $r_1(0)=r_1(2\pi)=R-e$ at the two positions can be obtained.

When $\varphi_1 = \sum_{i=1}^{j} \frac{2\pi}{Nm_{1i}}$, $r_1(j+1) = r_1(j) = \sqrt{R^2 - e^2 \sin^2(\frac{2\pi j}{N})} - e\cos(\frac{2\pi j}{N})$.

3 Mathematical model of multi-segment eccentric gear and its transmission characteristics

3.1 Establishment of the Curve Equation of Multi-segment modified Eccentric gear

To facilitate the establishment of the multi-segment modified eccentric curve equation, a coordinate system xO_1y is established with the rotation center O_1 of the driving gear as the origin. The pitch curve equation of the driving gear is the modified eccentric curve Eq.4. According to the gear meshing principle and transmission characteristics[18], the equation of the pitch curve of the driven non-circular gear can be expressed as

$$\begin{cases} r_2(\varphi_2) = a - r_1(\varphi_1) \\ \varphi_2(\varphi_1) = \int_0^{\varphi_1} \frac{1}{\tau} d\varphi_1 = \int_0^{\varphi_1} \frac{r_1(\varphi_1)}{a - r_1(\varphi_1)} d\varphi_1 \end{cases} \tag{6}$$

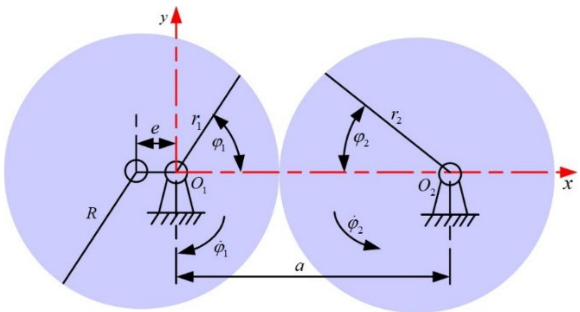

Fig.4 Non-circular gear mechanism

On the premise that the modified coefficients m_{1i} of each section satisfy Eq.5, according to the closed-loop condition[19] of the pitch curve of non-circular gears, it can be known that the multi-segment modified eccentric gear pair can achieve accurate transmission and needs to satisfy the following formula:

$$2\pi = \int_0^{2\pi} \frac{r_1(\varphi_1)}{a - r_1(\varphi_1)} \mathrm{d}\varphi_1 = \int_0^{\frac{2\pi}{Nm_{11}}} \frac{\left(\sqrt{R^2 - e^2\sin^2(m_{11}\varphi_1)} - e\cos(m_{11}\varphi_1)\right)}{a - \left(\sqrt{R^2 - e^2\sin^2(m_{11}\varphi_1)} - e\cos(m_{11}\varphi_1)\right)} d\varphi_1$$

$$+ \sum_{i=2}^{N} \int_{\sum_{j=1}^{i-1}\frac{2\pi}{m_{1j}}}^{\sum_{j=1}^{i}\frac{2\pi}{m_{1j}}} \frac{\sqrt{R^2 - e^2\sin^2(m_{1i}\left(\varphi_1 - \sum_{k=1}^{i}\frac{2\pi}{Nm_{1k}}\right) + \frac{2\pi(i-1)}{N})} - e\cos(m_{1i}\left(\varphi_1 - \sum_{k=1}^{i}\frac{2\pi}{Nm_{1k}}\right) + \frac{2\pi(i-1)}{N})}{a - \sqrt{R^2 - e^2\sin^2(m_{1i}\left(\varphi_1 - \sum_{k=1}^{i}\frac{2\pi}{Nm_{1k}}\right) + \frac{2\pi(i-1)}{N})} - e\cos(m_{1i}\left(\varphi_1 - \sum_{k=1}^{i}\frac{2\pi}{Nm_{1k}}\right) + \frac{2\pi(i-1)}{N})} d\varphi_1 \tag{7}$$

Firstly, use the optimization methods of numerical analysis[20] and one-dimensional search to determine the interval of the center distance a. Then, use the golden section method[21] to obtain the exact value of the center distance a in the determined interval. After obtaining the center distance from the above formula and substituting it into Eq.6, the pitch curve equation of the driven gear can be obtained, where the driven gear rotation angle φ_2 can be solved by the trapezoidal method.

3.2 Transmission ratio

Since the radial ratios of each position of the multi-segment modified eccentric gear pair are time-varying, the transmission ratio of its gear pair is also time-varying. According to the meshing transmission theory of gear pairs, a mathematical model of the transmission ratio of the multi-segment modified eccentric gear pair is established as

$$\tau = \frac{a-r_1}{r_1} = \begin{cases} \dfrac{a-\left(\sqrt{R^2-e^2\sin^2(m_{11}\varphi_1)}-e\cos(m_{11}\varphi_1)\right)}{\sqrt{R^2-e^2\sin^2(m_{11}\varphi_1)}-e\cos(m_{11}\varphi_1)} & \left(0\le\varphi_1\le\dfrac{2\pi}{Nm_{11}}\right) \\ \dfrac{a-\left(\sqrt{R^2-e^2\sin^2(m_{12}\left(\varphi_1-\dfrac{2\pi}{Nm_{11}}\right)+\dfrac{2\pi}{N})}-e\cos(m_{12}\left(\varphi_1\dfrac{2\pi}{Nm_{11}}\right)+\dfrac{2\pi}{N})\right)}{\sqrt{R^2-e^2\sin^2(m_{12}\left(\varphi_1-\dfrac{2\pi}{Nm_{11}}\right)+\dfrac{2\pi}{N})}-e\cos(m_{12}\left(\varphi_1\dfrac{2\pi}{Nm_{11}}\right)+\dfrac{2\pi}{N})} & \left(\dfrac{2\pi}{Nm_{11}}\le\varphi_1\le\dfrac{2\pi}{Nm_{11}}+\dfrac{2\pi}{Nm_{12}}\right) \\ \cdots\cdots & \\ \dfrac{a-\left(\sqrt{R^2-e^2\sin^2(m_{1i}\left(\varphi_1-\sum\limits_{i=1}^{j}\dfrac{2\pi}{Nm_{1i}}\right)+\dfrac{2\pi(j-1)}{N})}-e\cos(m_{1i}\left(\varphi_1-\sum\limits_{i=1}^{j}\dfrac{2\pi}{Nm_{1i}}\right)+\dfrac{2\pi(j-1)}{N})\right)}{\sqrt{R^2-e^2\sin^2(m_{1i}\left(\varphi_1-\sum\limits_{i=1}^{j}\dfrac{2\pi}{Nm_{1i}}\right)+\dfrac{2\pi(j-1)}{N})}-e\cos(m_{1i}\left(\varphi_1-\sum\limits_{i=1}^{j}\dfrac{2\pi}{Nm_{1i}}\right)+\dfrac{2\pi(j-1)}{N})} & \left(\sum\limits_{i=1}^{j-1}\dfrac{2\pi}{Nm_{1i}}\le\varphi_1\le\sum\limits_{i=1}^{j}\dfrac{2\pi}{Nm_{1i}}\right) \\ \cdots\cdots & \\ \dfrac{a-\left(\sqrt{R^2-e^2\sin^2\left(m_{1N}\left(2\pi-\varphi_1\right)\right)}-e\cos\left(m_{1N}\left(2\pi-\varphi_1\right)\right)\right)}{\sqrt{R^2-e^2\sin^2\left(m_{1N}\left(2\pi-\varphi_1\right)\right)}-e\cos\left(m_{1N}\left(2\pi-\varphi_1\right)\right)} & \left(\sum\limits_{i=1}^{N-1}\dfrac{2\pi}{Nm_{1i}}\le\varphi_1\le 2\pi\right) \end{cases} \quad (8)$$

4. Design software

To analyze the shapes and transmission characteristics of multi-segment modified eccentric gear, based on the established mathematical model, a visual analysis and design software is established using Matlab[22], as shown in Fig.5.

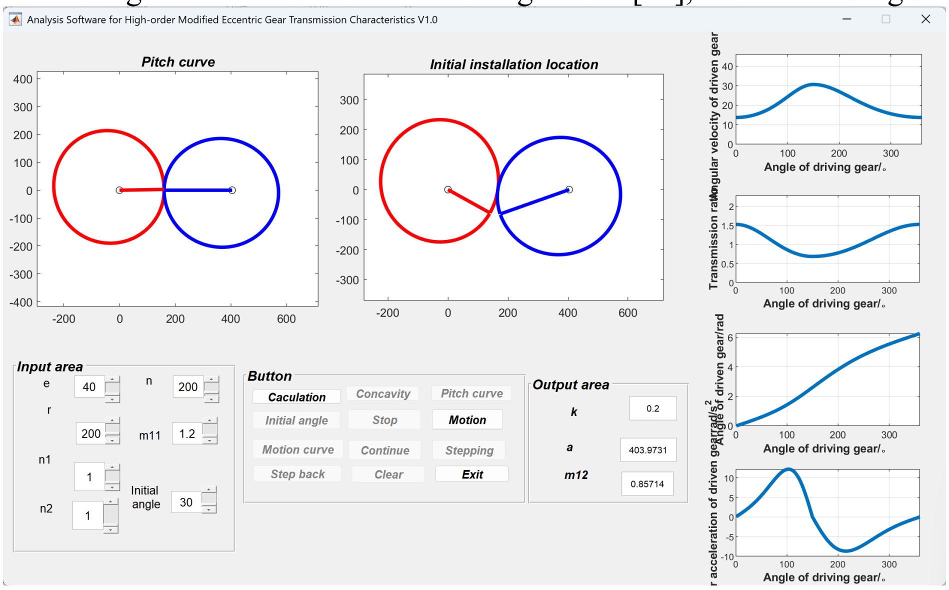

Fig.5 Design software

Fig.6 shows the functional flow chart of the visualization design platform. Designers can input the parameters of eccentric gears and the parameters of the modification principle of each section on the interface of the design platform, and output relevant parameters in real time through the button control method, including the pressure angle curve, contact ratio curve, transmission ratio curve of multi-section modified eccentric gears and the motion characteristics of the driven gear, and conduct the motion simulation of the pitch curve of multi-section modified eccentric gears to preliminarily verify the rationality and accuracy of the design, and save the design calculation results. If the output results of the input design parameters do not meet the design requirements, relevant design parameters can be adjusted in real time for optimal design.

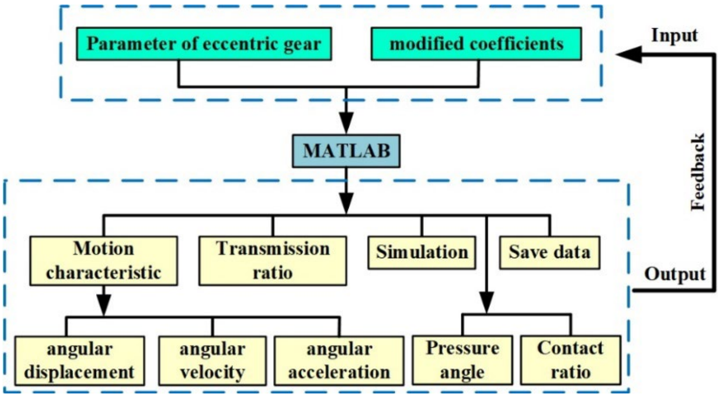

Fig.6 Functional flowchart of visual design platform

5. Analysis of Influencing Factors of Transmission Ratio

5.1 Eccentricity

Fig.7 shows the transmission ratios corresponding to different eccentricities. For the eccentric gear mechanism, the adjustable design parameter is only the eccentricity e. By changing the eccentricity, different transmission characteristics can be obtained. When e = 0 mm, it is a common cylindrical gear with a constant transmission ratio of 1. As the eccentricity increases, the variation range of the transmission ratio expands, but the transmission ratio curve is always symmetric about polar angle $\varphi_1 = 180^o$, However, within one transmission cycle, the average value of its transmission ratio is always 1.

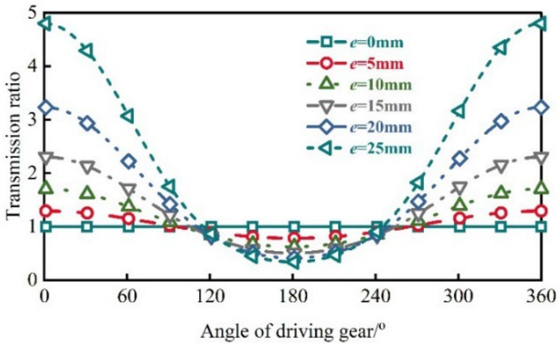

Fig.7 Transmission ratios corresponding to different eccentricities

5.2 Modification coefficient

Fig.8 shows the transmission ratios of two-segments modified eccentric gear corresponding to different modification coefficient m_{11}. It can be seen from the transmission ratio curves in the figure that the variation range of the transmission ratio is not affected by the modification coefficient, and the modification coefficient only affects the polar angle at which the amplitude of the transmission ratio is located. When introducing a modification coefficient m_{11} greater than 1 to the first section curve, the polar angle corresponding to the first section curve increases from 180 deg to $180/m_{11}$ deg, and the extreme value of the corresponding transmission ratio curve appears at $180/m_{11}$ degrees. When introducing a modification coefficient m_{11} less than 1 to the first pitch curve, the polar angle corresponding to the first pitch curve is reduced from 180 deg to $180/m_{11}$ deg, and the extreme value of the corresponding transmission ratio curve appears at $180/m_{11}$ deg.

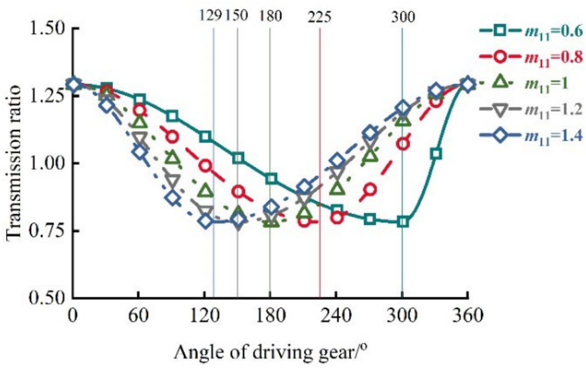

Fig.8 Transmission ratios corresponding to different modification coefficient

5.3 comparison of the transmission ratio

Fig.9 shows the comparison of the transmission ratios of eccentric gear and 4-segment modified eccentric gear. Compared with the eccentric gear which can only change the variation range, by introducing different modification coefficients m_{1i} in each segment of the multi-segment modified eccentric gear to obtain different transmission characteristics, it can adapt to more different variable speed working conditions. This is because after introducing different modification coefficients m_{1i}, the shape of the pitch curve changes. The more segments there are, the stronger the adjustability is, and it is closer to a free curve.

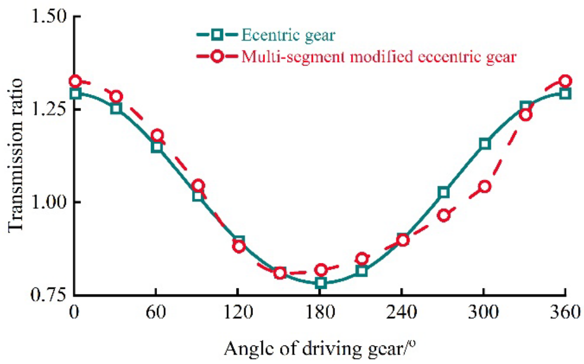

Fig.9 Comparison of the transmission ratios

6. Conclusion

The generation mechanism of multi-segment modified eccentric gears is deduced from the evolution principle of two-segment modified eccentric gears. By introducing corresponding modification coefficients in each segment, eccentric gear mechanisms with countless different transmission characteristics can be theoretically obtained. This generation mechanism can provide reference for the extended design of other types of non-circular gears.

The transmission characteristics of multi-segment modified eccentric gears are analyzed, indicating that different transmission characteristics can be obtained by introducing different modification coefficients in different intervals; the more the number of segments, the more modification coefficients can be introduced, the closer the shape of the pitch curve is to the free pitch curve, and the stronger the adjustability of the gear transmission characteristics.

A visualization design platform for multi-segment modified eccentric gear mechanisms is established, providing a convenient design method for the optimal design of this type of non-circular gear pair.

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