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EXISTENCE AND UNIQUENESS RESULTS FOR A CONFORMABLE BOUNDARY VALUE PROBLEM VIA MEASURE OF NONCOMPACTNESS

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This paper investigates the existence and uniqueness of solutions for boundary value problems involving the conformable derivative. By employing the measure of noncompactness in conjunction with Darbo's fixed point theorem, we establish sufficient conditions that guarantee the existence of continuous solutions. The main contribution of this work lies in extending classical fixed point techniques to the conformable fractional setting, thereby broadening the analytical framework for such problems. An illustrative example is presented to demonstrate the effectiveness of the proposed approach.

Keywords: Conformable fractional derivative, Darbo's theorem, Lipschitz condition
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1. Introduction

In recent years, boundary value problems (BVPs) involving fractional derivatives have attracted significant attention due to their diverse applications in physics, engineering, and control theory. Among the various approaches, the conformable fractional derivative introduced by Khalil et al. [24] stands out as a particularly effective tool, preserving essential properties of the classical derivative such as linearity, the chain rule, and the standard operational rules, while coinciding with the classical derivative in the limit $\alpha = 1$. In contrast to the Riemann–Liouville and Caputo derivatives, which are nonlocal and rely on singular kernels or complicated initial conditions, the conformable derivative offers a simpler, local, and more intuitive approach. Its close connection with the conformable integral facilitates the analysis of fractional differential equations and enables the derivation of existence and uniqueness results through fixed point techniques [5, 7, 9, 10, 14, 16, 23, 27, 28]. Subsequent studies [1, 2, 8] have expanded its theoretical basis and applications, underscoring its utility for both analytical investigations and practical modeling.

Parallel to these developments, the concept of the measure of noncompactness (MNC), originally introduced by Kuratowski and systematically studied in the monograph of Banaś and Goebel [12] has become a fundamental tool in nonlinear analysis. By extending classical fixed point theorems to broader classes of operators, MNC provides a flexible setting for treating problems where compactness cannot be ensured. In particular, Darbo's fixed point theorem [18] and its generalizations to condensing operators in the sense of Sadovskiĭ [26] form the foundation of many recent advances. This approach has been successfully applied to boundary value problems and functional equations (see, e.g., [3, 4, 6, 12, 13, 15, 17, 20, 21, 22, 25]), demonstrating both the strength and adaptability of MNC methods. When

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combined with the conformable fractional perspective, these techniques yield existence and uniqueness results under weaker assumptions than those typically required by compactness-based methods.

In light of these developments and motivations, this paper studies boundary value problems involving the conformable fractional derivative through the lens of the measure of noncompactness. We establish existence and uniqueness results for solutions of the problem

$$\mathbf{T}_{\alpha}^{a}u(t) + f(t, u(t)) = 0, \quad a \leq t \leq b,$$
$$u(a) = u(b) = 0,$$

where f is continuous and $\alpha \in (1, 2]$. The results obtained not only extend the theoretical understanding of conformable fractional BVPs but also demonstrate the effectiveness of MNC techniques in fractional differential equations.

The content of this paper is organized as follows. Section 2 is devoted to some preliminary concepts and auxiliary results, where we recall the basic notions of conformable fractional calculus together with certain properties of the measure of noncompactness that will be used throughout the paper. In Section 3, we establish and prove our main results concerning the existence and uniqueness of solutions to the considered boundary value problem. Section 4 provides an illustrative example to demonstrate the applicability of the obtained results. Finally, Section 5 presents some concluding remarks and possible further investigations for future research.

2. **Preliminaries**

We now recall the definitions and fundamental properties of the conformable fractional derivative and its integral, which will be essential tools for our analysis.

Definition 2.1 (Conformable Fractional Derivative [24])**.** *Let $f : [a, b] \to \mathbb{R}$ and $\alpha \in (0, 1]$. The conformable fractional derivative of order α at $t > a$ is defined by*

$$\mathbf{T}_{\alpha}^{a}f(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon(t - a)^{1-\alpha}) - f(t)}{\varepsilon}.$$

If $\mathbf{T}_{\alpha}^{a}f(t)$ exists on (a, b) and $\lim_{t \to a^{+}} \mathbf{T}_{\alpha}^{a}f(t)$ exists, then we define

$$\mathbf{T}_{\alpha}^{a}f(a) = \lim_{t \to a^{+}} \mathbf{T}_{\alpha}^{a}f(t).$$

Remark 2.1. (1) *For $a = 0$, we denote $\mathbf{T}_{\alpha}f(t)$.*
(2) *For $\alpha = 1$, $\mathbf{T}_{1}^{a}f(t) = f'(t)$, reducing to the classical derivative.*

Definition 2.2 (Conformable Fractional Integral [1])**.** *Let $\alpha \in (0, 1]$ and f be integrable on $[a, b]$. The conformable fractional integral is*

$$I_{a}^{\alpha}f(t) = \int_{a}^{t} (x - a)^{\alpha-1} f(x)\, dx.$$

These operators satisfy important properties, including linearity, the power, product, and quotient rules, the inverse relation

$$\mathbf{T}_{\alpha}^{a}(I_{a}^{\alpha}f(t)) = f(t),$$

and the connection to the classical derivative

$$\mathbf{T}_{\alpha}^{a}f(t) = (t - a)^{1-\alpha}f'(t),$$

provided that f is differentiable [1, 8, 24].

Definition 2.3. [1] *For $n < \alpha \leq n+1$, and $\gamma = \alpha - n$, the conformable fractional derivative of a function $f : [a, \infty) \to \mathbb{R}$ of order α is defined by*

$$\mathbf{T}_{\alpha}^{a} f(t) = \mathbf{T}_{\gamma}^{a} f^{(n)}(t),$$

where $f^{(n)}(t)$ exists.

Definition 2.4. [1] *For $n < \alpha \leq n+1$, and $\gamma = \alpha - n$, the conformable fractional integral of a function $f : [a, \infty) \to \mathbb{R}$ of order α is defined by*

$$I_{\alpha}^{a} f(t) = I_{n+1}^{a}((t-a)^{\gamma-1} f)(t) = \frac{1}{n!} \int_{a}^{t} (t-x)^{n} (x-a)^{\gamma-1} f(x)\, dx.$$

Lemma 2.1. [1] *Let $f : [a, \infty) \to \mathbb{R}$ and suppose that $f^{(n)}(t)$ is continuous for $n < \alpha \leq n+1$. Then, for all $t > a$, we have*

$$\mathbf{T}_{a} I_{\alpha}^{a} f(t) = f(t).$$

Theorem 2.1. [1] *Assume that $f : [a, \infty) \to \mathbb{R}$ is $(n+1)$ times differentiable and let $n < \alpha \leq n+1$, with $t > a$. Then, for all $t > a$, we have*

$$I_{\alpha}^{a} \mathbf{T}_{a}^{a} f(t) = f(t) - \sum_{k=0}^{n} \frac{f^{(k)}(a)(t-a)^{k}}{k!}.$$

Having introduced the conformable fractional derivative and integral, we now turn to the concept of measure of noncompactness (MNC), which will be crucial in establishing existence and uniqueness results for boundary value problems involving these operators.

Let (M, ρ) be a complete metric space and denote by $\mathcal{B}(M)$ the family of all bounded subsets of M. A function

$$\mu : \mathcal{B}(M) \to [0, \infty)$$

is called a *measure of noncompactness* if, for all $\Omega, \Omega_1, \Omega_2 \in \mathcal{B}(M)$, it satisfies the following conditions

(i) $\mu(\Omega) = 0$ iff Ω is precompact;
(ii) $\mu(\Omega) = \mu(\overline{\Omega})$; and
(iii) $\mu(\Omega_1 \cup \Omega_2) = \max\{\mu(\Omega_1), \mu(\Omega_2)\}$.

This concept plays a central role in nonlinear analysis, as it provides a quantitative tool to study operator equations in settings where compactness is not available. In particular, measures of noncompactness allow the extension of fixed point theorems to condensing operators, thereby generalizing classical results and enabling the treatment of a wider class of problems. For a detailed account of the theory and further properties of measures of noncompactness, we refer the reader to [6, 12].

The measure of noncompactness in $C[a, b]$ can be formulated as follows [11, 12].

In the space of continuous functions $C[a, b]$ with the norm $\|x\| = \max_{t \in [a,b]} |x(t)|$, measures of noncompactness can be defined using the modulus of continuity.

For a bounded set $X \subset C[a,b]$ and $x \in X$, given $\varepsilon > 0$ and $M > 0$, define

$$\begin{aligned}
\omega^M(x,\varepsilon) &= \sup\{|x(t)-x(s)| : t,s \in [0,M], |t-s| < \varepsilon\},\\
\omega^M(X,\varepsilon) &= \sup\{\omega^M(x,\varepsilon) : x \in X\},\\
\omega_0^M(X) &= \lim_{\varepsilon\to 0} \omega^M(X,\varepsilon),\\
\omega_0(X) &= \lim_{M\to\infty} \omega_0^M(X),\\
X(t) &= \{x(t) : x \in X, t \in \mathbb{R}\}.
\end{aligned}$$

Then the measure of noncompactness of X is

$$\mu(X) = \omega_0(X) + \limsup_{t\to\infty} \operatorname{diam} X(t), \quad \text{where } \operatorname{diam} X(t) = \sup\{|x(t)-y(t)| : x,y \in X\}.$$

Theorem 2.2 (Darbo Fixed Point Theorem [18])**.** *Let X be a Banach space, and let D be a nonempty, bounded, closed, and convex subset of X. If $T : D \to D$ is a continuous operator that is* condensing*, i.e., for any bounded subset A of D,*

$$\alpha(T(A)) \leq k\,\alpha(A)$$

for some constant $k \in [0,1)$, where α is a measure of noncompactness, then T has at least one fixed point in D.

Theorem 2.3 (Uniqueness via Banach's Contraction Principle [19])**.** *Assume, in addition to the hypotheses of Theorem 2.2, that there exists $q \in [0,1)$ such that*

$$\|T(u)-T(v)\| \leq q\,\|u-v\| \qquad \text{for all } u,v \in D.$$

Then the fixed point of T in D is unique.

Our primary focus in this work is the following conformable fractional BVP

$$\begin{cases} \mathbf{T}_\alpha^a u(t) + f(t,u(t)) = 0, & a \leq t \leq b,\\ u(a) = u(b) = 0, \end{cases} \tag{1}$$

where f is continuous and $\alpha \in (1,2]$. Using the measure of noncompactness and Darbo's fixed point theorem (Theorem 2.2), we aim to establish existence and uniqueness of solutions to (1), with uniqueness guaranteed under the contraction condition (Theorem 2.3).

2.1. Integral Representation and Green's Function

In this section, we develop the integral representation of solutions via Green's function associated with the conformable derivative, followed by main results on existence and uniqueness.

Consider the boundary value problem

$$\begin{cases} \mathbf{T}_\alpha^a u(t) + y(t) = 0, & a \leq t \leq b,\\ u(a) = u(b) = 0, \end{cases} \tag{2}$$

where $\mathbf{T}_\alpha^a$ denotes the conformable derivative of order α, and y is given.

Lemmas 2.2 and 2.3 are derived from the theoretical results established in [2], adapted specifically for the compatible boundary value problem in this paper. We omit the proofs here for brevity, as they employ analogous arguments to those in [2].

Lemma 2.2. *A function $u(t)$ is a solution of the BVP (2) if and only if it satisfies the integral equation*

$$u(t) = \int_a^b G(t,s)y(s)ds,$$

where the Green function G is given by

$$G(t,s) = \frac{(s-a)^{\alpha-2}}{b-a} \times \begin{cases} -(b-a)(t-s) + (b-s)(t-a), & a \le s \le t \le b, \\ (b-s)(t-a), & a \le t \le s \le b. \end{cases}$$

Lemma 2.3. *The Green function G satisfies:*

(i) $G(t,s) \ge 0$ *for all* $a \le t, s \le b$,

(ii) *For each fixed* s, $\max_{t\in[a,b]} G(t,s) = G(s,s)$,

(iii) $G(s,s)$ *has a unique maximum at*

$$s = \frac{a + (\alpha-1)b}{\alpha},$$

with

$$\max_{s\in[a,b]} G(s,s) = \frac{(b-a)^{\alpha-1}(\alpha-1)^{\alpha-1}}{\alpha^\alpha}.$$

3. **Main Results**

We now present the main results concerning the existence and uniqueness of solutions for the boundary value problem (1). We begin by stating the following conditions

(H1): There exists a nonnegative constant L such that

$$|f(t,x) - f(t,y)| \le L|x-y|, \quad x, y \in \mathbb{R}.$$

(H2): Define

$$\zeta := \frac{(b-a)^\alpha(\alpha-1)^{\alpha-1}}{\alpha^\alpha} L < 1.$$

(H3): There exists a positive number r_0 such that

$$\frac{\zeta}{L}\big(Lr_0 + \gamma\big) \le r_0,$$

where $\gamma = \max_{a\le t\le b} |f(t,0)|$.

Theorem 3.1. *Under the conditions (H1)-(H3), the boundary value problem (1) admits at least one solution.*

Proof. Define the operator T on $C[a,b]$ by

$$(Tu)(t) = \int_a^b G(t,s) f(s,u(s))\, ds, \quad t \in [a,b].$$

Then, $u(t)$ is a solution of the BVP (1) if and only if it is a fixed point of T. The proof is divided into three steps.

Let

$$B_{r_0} = \{u \in C[a,b] : \|u\| \le r_0\}$$

denote the closed ball in $C[a,b]$ of radius r_0 centered at 0, where $\|u\| = \sup_{t\in[a,b]} |u(t)|$.

Step 1: Continuity of T.

Using Lemma 2.3 and condition (H1), for any $\varepsilon > 0$ there exists $\delta = \varepsilon/\zeta$ such that for all $u, v \in B_{r_0}$ with $\|u - v\| < \delta$, we have

$$\|Tu - Tv\| < \varepsilon.$$

In detail,

$$|Tu(t) - Tv(t)| = \left| \int_a^b G(t,s)f(s,u(s))ds - \int_a^b G(t,s)f(s,v(s))ds \right|$$
$$\leq \frac{(b-a)^{\alpha-1}(\alpha-1)^{\alpha-1}}{\alpha^\alpha} \int_a^b |f(s,u(s)) - f(s,v(s))|ds$$
$$\leq \frac{(b-a)^{\alpha-1}(\alpha-1)^{\alpha-1}}{\alpha^\alpha} L(b-a)\|u-v\| = \zeta\|u-v\|.$$

Hence,

$$\|Tu - Tv\| = \sup_{t\in[a,b]} |Tu(t) - Tv(t)| \leq \zeta\|u-v\| < \varepsilon.$$

Step 2: T maps B_{r_0} into itself.
For any $u \in C[a,b]$ and $t \in [a,b]$, using (H1), (H2) and Lemma 2.3 for G, we have

$$|Tu(t)| = \left| \int_a^b G(t,s)f(s,u(s))ds \right|$$
$$\leq \int_a^b G(t,s)\big(|f(s,u(s)) - f(s,0)| + |f(s,0)|\big)ds$$
$$\leq \int_a^b G(t,s)(L\|u\| + \gamma)ds$$
$$\leq \frac{(b-a)^{\alpha-1}(\alpha-1)^{\alpha-1}}{\alpha^\alpha}(b-a)(L\|u\| + \gamma)$$
$$= \frac{\zeta}{L}(L\|u\| + \gamma).$$

By condition (H3), T maps B_{r_0} into itself.
Step 3: Measure of noncompactness estimate.
From Step 1, for all $u, v \in B_{r_0}$,

$$|Tu(t) - Tv(t)| \leq \zeta\|u-v\| = \zeta \sup_{s\in[a,b]} |u(s) - v(s)|.$$

This gives

$$\limsup_{t\to\infty} \operatorname{diam}(TU)(t) \leq \zeta \limsup_{t\to\infty} \operatorname{diam}(U)(t).$$

Furthermore, for all $|t_1 - t_2| < \varepsilon, t_1, t_2 \in [a,b]$, continuity of G implies

$$|G(t_1,s) - G(t_2,s)| < \varepsilon, \quad \forall s \in [a,b].$$

Combining this with (H1) and (H2), for any $u, v \in B_{r_0}$,

$$|Tu(t_1) - Tv(t_2)| \leq \left| \int_a^b G(t_1,s)f(s,u(s))ds - \int_a^b G(t_2,s)f(s,v(s))ds \right|$$
$$\leq \int_a^b |G(t_1,s) - G(t_2,s)||f(s,u(s))|ds$$
$$+ \int_a^b G(t_2,s)|f(s,u(s)) - f(s,v(s))|ds$$
$$\leq \varepsilon(b-a)(L\|u\| + \gamma) + \zeta\|u-v\|.$$

Hence,

$$w^M(Tu,\varepsilon) \leq \zeta w^M(u,\varepsilon), \quad \text{and thus} \quad w_0^M(Tu) \leq \zeta w_0^M(u).$$

Taking $M \to \infty$, we get

$$w_0(Tu) \leq \zeta w_0(u),$$

and together with the earlier supremum estimate,

$$\alpha(TB_{r_0}) \leq \zeta \alpha(B_{r_0}).$$

By Theorem 2.2 on condensing operators and measure of noncompactness, T has at least one fixed point in $B_{r_0} \subset C[a,b]$, which corresponds to a solution of the BVP (1).

This completes the proof. □

Corollary 3.1. *Theorem 3.1 guarantees not only the existence but also the uniqueness of solutions to the BVP* (1).

Proof. From the inequality established in Step 1 and under condition (H2), the operator $T : C[a,b] \to C[a,b]$ is a contraction with contraction constant $\zeta < 1$. Therefore, by the Banach fixed point theorem (Theorem 2.3), T possesses a unique fixed point in $C[a,b]$. Consequently, the BVP (1) admits a unique solution in $C[a,b]$.

This completes the proof. □

4. **Example**

This section provides a detailed example to illustrate the applicability of the obtained results.

The following Example 4.1 demonstrates the applicability and effectiveness of our previously obtained theoretical results (e.g., Theorem 3.1). This example represents a situation where the considered conformable boundary value problem concretely satisfies the existence and uniqueness conditions determined by measure of noncompactness techniques.

Example 4.1. *Consider the following boundary value problem:*

$$\mathbf{T}_{3/2}u(t) + \big(t^2 + \cos t + \arctan u(t)\big) = 0, \quad t \in [0,1], \tag{1}$$

$$u(0) = u(1) = 0. \tag{2}$$

Here, the nonlinear function and the parameters are identified as

$$f(t,x) = t^2 + \cos t + \arctan x, \quad a = 0, \quad b = 1, \quad \alpha = \frac{3}{2}\,.$$

Observe that

$$|f(t,x) - f(t,y)| \leq |\arctan x - \arctan y| \leq |x - y|,$$

which implies that the Lipschitz constant is $L = 1$.

We calculate the constants ζ and γ as follows:

$$\zeta = \frac{(1-0)^{3/2}(\frac{3}{2}-1)^{\frac{3}{2}-1}}{(\frac{3}{2})^{3/2}} \times 1 \approx 0.3849 < 1,$$

and

$$\gamma = \max_{t\in[0,1]} |f(t,0)| = \max_{t\in[0,1]} |t^2 + \cos t| = 1 + \cos 1 \approx 1.5403.$$

To ensure that the closed ball B_{r_0} is invariant under the associated operator, we require

$$\frac{\zeta}{L}(Lr_0 + \gamma) \leq r_0.$$

Since $L = 1$*, this gives*

$$r_0 \geq \frac{\zeta\gamma}{1-\zeta} \approx 0.963.$$

Hence, we set $r_0 = 0.963$.

Since conditions (H1)-(H3) are satisfied, Theorem 3.1 guarantees the existence of at least one solution $u(t) \in B_{r_0}$ *for the given problem. Furthermore, as the contraction constant* $\zeta \approx 0.3849$ *is strictly less than* 1*, Corollary 3.1 ensures the uniqueness of this solution. This demonstrates the applicability and effectiveness of our proposed MNC method for this specific conformable BVP.*

5. **Conclusions**

In this study, we established existence and uniqueness results for boundary value problems involving the conformable derivative under Lipschitz-type assumptions on the nonlinear operator. The approach is based on the measure of noncompactness, which allows handling operators that are not compact. By applying Darbo's fixed point theorem, we obtained existence results for condensing operators, while uniqueness is ensured by the Lipschitz condition with constant $L < 1$. This combined approach extends classical fixed point results to fractional-order settings. Future research may consider more general boundary conditions, systems of conformable differential equations, higher-order operators, and applications to physical and engineering models where fractional processes are relevant.

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