ASYMMETRICAL PULSES MODEL FOR TRANSMISSION OF A SIGNAL IN A DYNAMIC SYSTEM

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This study models the behaviour of a dynamic system when a signal is transmitted by it, considering the delay in propagation and the parameters of the system. For this purpose, a delay operator τ *written under the binomial form* $(a + b\tau)^n$ *is used, which acts on the initial state of the system,* n *being the number the system constituents, and* $a + b = 1$, $a \neq b$ *are some parameters of a certain medium. This mathematical expression generates asymmetric pulses, whit a suitable graphical representation, which allows the analysis of the propagation process and the calculation of the involved parameters.*

Keywords: asymmetrical pulses, delay operator, combinatorial model, dynamic system

1. Introduction

Over time, it has become increasingly evident that robust mathematical formalisms constitute a foundational element in the study of diverse systems and phenomena. In the fields of life sciences and socio-economic research, numerous theoretical frameworks have been advanced to improve understanding of the inherent complexity of real-world systems [1, 2]. The integration of computational tools into scientific research has substantially improved these investigations, enabling more advanced modeling and analysis [3, 4]. Dynamic systems are typically characterised by mathematical formulations that describe the time dependence of specific properties as functions of initial conditions, internal connectivity, and various forms of interaction. These systems provide a robust framework for the application of computer-aided analytical methods.

Time-varying properties of materials in physics define them as dynamic media, providing a suitable basis for study in optics, laser physics, and related applications. The coupling of high-intensity laser fields with dynamic media −

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including liquid crystals, ionic solids, and laser-induced plasmas – represents a foundational aspect of ultrafast optics and photonic science [5, 6]. These media respond nonlinearly to electromagnetic excitation, enabling phenomena such as group delay dispersion (GDD), spectral expansion, and temporal pulse shaping through processes involving plasma mirrors and spatio-temporal interactions [7,8]. Plasma mirrors have demonstrated efficacy in enhancing temporal contrast by selectively reflecting high-intensity pulse components while attenuating low-intensity precursors, thereby improving pulse integrity in high-field laser experiments [9, 10, 11]. Liquid crystals and ionic crystals provide enhanced flexibility for spatial and temporal modulation due to to their anisotropic optical characteristics and sensitivity to external electromagnetic fields [5, 12]. When subjected to laser excitation, these materials undergo reorientation processes that enable dynamic beam shaping and adaptive focusing. In contrast, plasmas produced through laser-induced ablation or breakdown evolve on ultrafast timescales and exhibit pronounced nonlinearities, making them highly suitable for spectral control and rapid optical switching applications [6]. Interaction between a delayed laser pulse and a plasma plume generated by laser ablation significantly alters the pulse's propagation dynamics. As the plume evolves, spatial and temporal gradients in electron density and refractive index emerge, leading to temporal delay, scattering, and partial absorption of the pulse. These effects can significantly influence energy deposition and alter the material's response to irradiation. This effect can induce a sudden increase in the ablation rate once a threshold fluence is reached, a behavior that has been related with a transition in the plasma plume’s hydrodynamic expansion from one-dimensional to three-dimensional regime [13].

These effects have contributed significantly to the development of cutting-edge applications, including attosecond pulse generation, coherent waveform control, high-resolution optical imaging, and laser-driven particle acceleration [14, 15, 16, 17]. The capability to manipulate the spatio-temporal characteristics of laser pulses through custom-engineered optical environments enables novel directions in precision photonics, with substantial implications for material processing, biomedical imaging, and quantum optical technologies.

As dynamic media, fatty acids (FA) under controlled conditions offer a flexible platform for investigating certain processes in living matter and for practical applications. Experimental studies have demonstrated that laser irradiation can induce refractive index changes and molecular reorientation in mesomorphic fatty acid films, revealing complex thermo-optical interactions relevant to biological membranes and lipid-based materials [18]. Moreover, pulsed laser techniques have been shown to trigger localized thermal effects and structural modifications in soft matter systems, offering insights into lipid transport, membrane fluidity, and energy dissipation mechanisms [7, 19].

Fatty acids − cholesterol (FA-Ch) mixtures in thin films and in the liquid crystal state (LC) state may exhibit heterogeneity or display various textures characteristic of the thermotropic LC state of type C. Experimental approaches [18] were concentrated on the biological membrane (BM) and the role of FA while in a mesomorphic LC state on the membrane behaviour in interaction with the environment. In such experimental setups, pulses generated by real sources (for instance a pulsed laser) have to pass through different LC samples that model a simple BM. Experimental measurement showed that pulse shape could support some shrinking or widening during the propagation. Figure 1 presents the evolution of the pulse profile after transmission through a conventional LC cell containing arachidonic acid (AA), a polyunsaturated (20:4, 5, 8, 11, 14 *all cis*) essential fatty acid, which serves together with linoleic and α-linolenic acid as precursors for the synthesis of prostaglandins, thromboxane, leukotrienes, and lipoxins. It is a forerunner of phospholipids, which modulate the hormonal actions, nervous transmission and cellular ions exchanges. Graph show that at low bias voltage U, Ch amount increased the nonlinear contribution of the medium to the pulse width [20, 21]; for $U > 1.6$ V the *low Ch percentage* was more important.

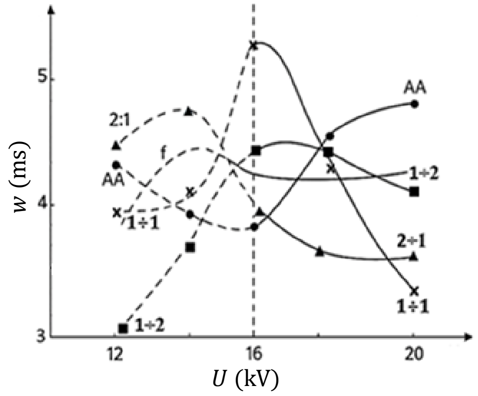

Fig. 1. Pulse width w versus bias voltage U for laser pulse passing through the samples of AA and mixtures with Ch.

By using the experimental data, a graph (Fig.2) and a mathematic expression were generated by Table Curve3D computer program [22]:

$$z = a + b\ln x + c(\ln x)^2 + d(\ln x)^3 + e(\ln x)^4 + fy + gy^2 +$$

$$+hy^3 + iy^4 + jy^5 \qquad (1)$$

with a, b, ..., j some parameters of the medium. This analysis permitted the estimation of Ch percentage without performing the required experiments, but it could not forecast the system behaviour in different other situations. Starting from

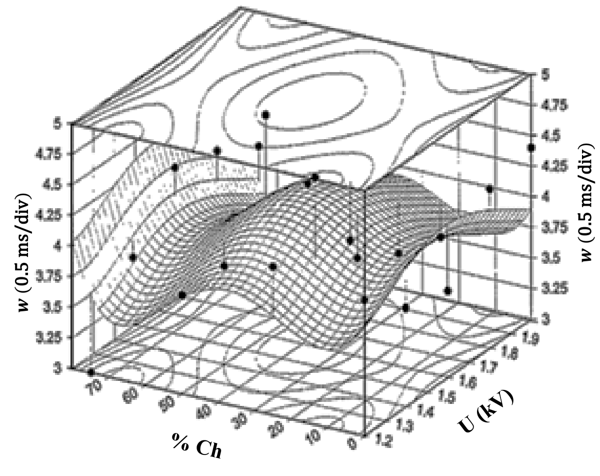

Fig.2. Pulse width w versus bias voltage U and Ch percentage in AA and mixtures with Ch

such practical reasons, we consider realistic a consistent mathematical model based on a general expression. The model of asymmetrical pulses seems to be the required one and do not mitigate the medium influence but gives a most accurate description of the real situations.

In other previous studies upon a dynamical system [23], a certain delay operator denoted as τ has been used for describing the propagation of a phenomenon within a certain medium, under the form of a binomial $(1+\tau)$. A pulsed propagation of the interaction in a system with n elements was generated in MATLAB, by using a combinatorial development of this expression. Considering a laser pulse as a suddenly emerging phenomenon on a limited space-time interval, the dynamics of the phenomena can be described by the action of the delay operator τ and the interaction under the form of pulses with the form $(1+\tau)^n$ acting upon the initial state $S_i = 1$ at $t = 0$. The evolution of the whole system is therefore described by $S = (1+\tau)^n S_i$, expression developed with combinatorial formulas, where n is the number of amplitudes C_n^k reached by the interaction in time t, i.e. the number of the constituents of the dynamic system which receive the initial information. Considering the coefficient C_n^k as an amplitude, it results that the term $C_n^k \tau^k$ can be associated to a certain amplitude C_n^k at the time moment $k\tau$, beginning from the initial moment. Thus, a discrete-time function is generated, which can be extended through interpolation to a continuous function, and the model can be extended to describe a continuous material medium. In [23] the model was used for optimization of the results obtained by a work team involved in any kind of activity and cooperation.

In another study [24], a graphical method for determine some parameters of this model, based on a tangent line in inflection points of the graph has been presented. The value of amplitude versus time in inflection points has been denoted with F, the distance between two inflection points $k_{1,2}$ with D, and the derivative (the slope) of this function in the first inflection point with m, as is depicted in Fig. 3. The tangent line at the graph in the first inflection point k_1 intersects the vertical line passing through the second inflection point k_2 at a height equal to:

$$m = \frac{H-F}{D} = \frac{H-F}{\sqrt{n+2}} \Rightarrow H = F + m\sqrt{n+2} \tag{2}$$

with arbitrarily selected parameters $n = 36$, $a = 0.3$, and $b = 0.7$, satisfying $a + b = 1$.

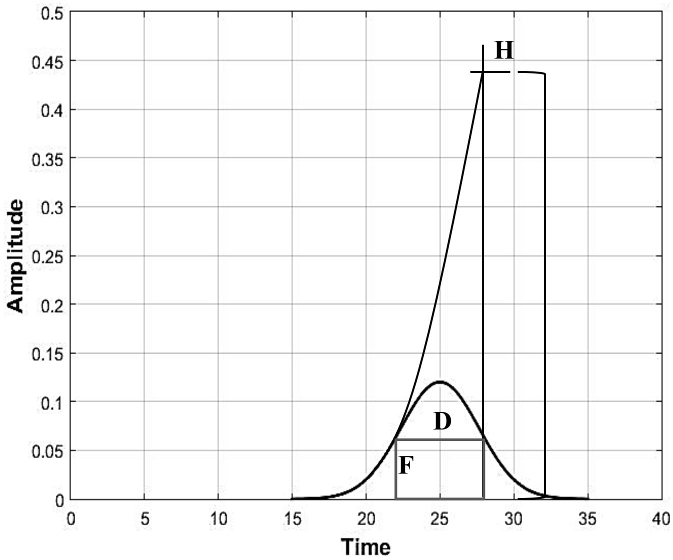

Fig. 3. Pulses amplitude versus time and the tangent in the inflexion point k_1, $n = 36$, $a = 0.3$, $b = 0.7$, $a + b = 1$

These previous studies are continued within this paper by extending the research upon asymmetrical pulses. In this case, the point of maximum amplitude is shifted from the middle of the interval of the symmetrical pulse and can be defined as a function of a difference $a - b$, where the dimensionless parameters a and b refer to amplification without delay τ, and with delay τ, respectively, $a + b = 1$, $a \neq b$. It is shown that the derivative in these points is approximately two times the amplitude of the pulse for high values of n, the difference becoming significant for lower values of n. Our model can be applied in studying complex systems through evolution equations, where discrete pulses generated within short time intervals are described by a specific set of delayed differential equations [25].

The conceptual innovation of employing the notion of "delay time" as a variable within a generalised expansion of a binomial—structured around binomial coefficients and articulated through the principles of combinatorial analysis—constitutes the central and original premise of this model. This foundational idea introduces a mathematically elegant and universally applicable formalism, offering a robust framework for interpreting phenomena across both fundamental and applied scientific disciplines.

Although models describing symmetrical pulses are well established, both previous research by other authors and our own investigations have encountered specific cases that necessitate further explanation and quantitative analysis [4, 23, 26, 27, 28]. The asymmetric pulse model is employed to reflect the non-ideal conditions commonly encountered in experimental settings. While it shares the same fundamental principles as the symmetric pulse model, it allows for the inclusion of more complex scenarios—such as composite media, clusters, and structural defects—that are often present in real-world systems. Importantly, the fundamental data remains consistent with that of the symmetric model, as demonstrated through the graphical method employed in [24]. Recent studies have shown the relevance of asymmetric pulse modeling, revealing its significant impact on pulse propagation dynamics in nonlinear dispersive media and its influence on spatial and temporal evolution through diffraction effects in broad-band optical systems [4, 26, 27].

2. Basic aspects of asymmetrical pulses. Amplitude asymmetries.

Let us study the case when the generated pulse is not symmetrical, this means when the polynomial is written as

$$S = (a + b\tau)^n S_i = \{C_n^0\, a^n\, \tau^0 + C_n^1\, a^{n-1} b\, \tau^1 + \cdots + C_n^k a^{n-k} b^k\, \tau^k + \\ + \cdots + C_n^n b^n\, \tau^n\} \quad (3)$$

(assuming $S_i = 1$ at $t = 0$). For $a = 0.3$, and $b = 0.7$ the pulse is presented in Fig. 3. The value of H is typically close to $3F$ [24]. However, in the case of asymmetrical pulses generated via the combinatorial model, this relationship no longer holds precisely. A correction term − denoted as $Diff$ (see Fig. 4) − must be introduced to account for the deviation from the value of $3F$.

In order to determine this correction term, we identify the inflection points extending the discrete pulse into a continuous function and starting from the recurrence relation:

$$f(k) = \frac{n-k+1}{k} \cdot \frac{b}{a} \cdot f(k-1) \quad (4)$$

We derive the adjacent terms:

$$f(k-1) = \frac{k}{n-k+1} \cdot \frac{a}{b} \cdot f(k), \quad f(k+1) = \frac{n-k}{k+1} \cdot \frac{b}{a} \cdot f(k) \qquad (5)$$

The first-order differences are:

$$f(k+1) - f(k) = \left(\frac{n-k}{k+1} \cdot \frac{b}{a} - 1\right) f(k) \qquad (6)$$

$$f(k) - f(k-1) = \left(1 - \frac{k}{n-k+1} \cdot \frac{a}{b}\right) f(k) \qquad (7)$$

By using $\alpha = \frac{b}{a}$, the second-order difference at point k becomes:

$$[f(k+1) - f(k)] - [f(k) - f(k-1)]$$
$$= \left\{\left(\frac{n-k}{k+1}\alpha - 1\right) - \left(1 - \frac{k}{n-k+1} \cdot \frac{1}{\alpha}\right)\right\} f(k)$$
$$= \left\{\frac{k^2(\alpha^2+2\alpha+1) + k(-2n\alpha^2 - \alpha^2 - 2n\alpha + 1) + (n^2\alpha^2 + n\alpha^2 - 2n\alpha - 2\alpha)}{(k+1)(n-k+1)\alpha}\right\} f(k) \qquad (8)$$

where $f(k)$,defined as a ratio of factorials, is a nonzero function. The second-order difference vanishes when the numerator of the coefficient within the parentheses becomes zero. This leads to the condition:

$$k^2(\alpha^2 + 2\alpha + 1) + k(-2n\alpha^2 - \alpha^2 - 2n\alpha + 1) +$$
$$+(n^2\alpha^2 + n\alpha^2 - 2n\alpha - 2\alpha) = 0 \qquad (9)$$

The numerator of this expression is composed of nonzero terms for all k in $(0, n)$.

The equation presented above can be treated as a second-order equation in the variable k, assuming that the parameter n is known. Its solution corresponds to:

$$k_{1,2} = \frac{n\alpha}{\alpha+1} + \frac{1-\alpha}{2(\alpha+1)} \pm \frac{\sqrt{4n\alpha^3 + 8n\alpha^2 + 4n\alpha + (\alpha^2-1)^2 + 8\alpha(\alpha+1)^2}}{2(\alpha+1)^2} \qquad (10)$$

By adding the normalization condition $a + b = 1$, it results

$$k_{1,2} = nb + \frac{a-b}{2} \pm \frac{\sqrt{4nab + (b-a) + 8ab}}{2} \qquad (11)$$

By using a small parameter δ, which affects the transmission process, generating an asymmetry of its shape, so as $a = \frac{1}{2} - \delta$, $b = \frac{1}{2} + \delta$, $a + b = \frac{1}{2} - \delta + \frac{1}{2} + \delta = 1$, it results

$$k_{1,2} = \frac{n}{2} + n\delta - \delta \pm \frac{\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}}{2} \tag{12}$$

and substituting k with $k_{1,2}$

$$f(k+1) - f(k) = \left(\frac{n-k}{k+1}\frac{b}{a} - 1\right) f(k) \tag{13}$$

for the points where the second order difference vanishes, it results

$$f(k_{1,2}+1) - f(k_{1,2}) = \left\{\frac{n-k_{1,2}}{k_{1,2}+1}\left(\frac{\frac{1}{2}+\delta}{\frac{1}{2}-\delta}\right) - 1\right\} f(k_{1,2}) =$$

$$= \frac{\left\{-1+4\delta\mp\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}\right\}/2}{\left\{n(1-4\delta^2)+(2-6\delta+4\delta^2)\pm(1-\delta)\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}\right\}/4} f(k_{1,2}) \tag{14}$$

$$f(k_{1,2}+1) - f(k_{1,2}) = 2\frac{-1+4\delta\mp\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}}{n(1-4\delta^2)+(2-6\delta+4\delta^2)\pm(1-\delta)\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}} \tag{15}$$

The difference D between the inflexion points can be written as

$$D = k_2 - k_1 = \sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)} \tag{16}$$

and the derivative in these inflexion points can be written as

$$m = f(k_{1,2}+1) - f(k_{1,2}) =$$

$$= 2\frac{-1+4\delta\mp\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}}{n(1-4\delta^2)+(2-6\delta+4\delta^2)\pm(1-\delta)\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}} F \tag{17}$$

This means that the tangent line at this graph in the inflection point k_2 “cuts’ the vertical line on time axis in the other inflection point k_1 at a height

$$H = F + mD \tag{18}$$

$$H = F + 2\frac{(-1+4\delta)\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}+\left[n(1-4\delta^2)+(2+2\delta-8\delta^2)\right]}{n(1-4\delta^2)+(2-6\delta+4\delta^2)-(1-\delta)\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}} F \tag{19}$$

$$H = \left\{3 + \frac{\delta\left\{3\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}+8-12\delta\right\}}{n(1-4\delta^2)+(2-6\delta+4\delta^2)-(1-\delta)\sqrt{n(1-4\delta^2)+(2+2\delta-8\delta^2)}}\right\} F \tag{20}$$

which can be approximated to 3 (F being normalized to unity), for large values of n and small values of δ. When $\delta = 0$, the pulse is perfectly symmetrical; when

$\delta \neq 0$, the pulse becomes asymmetrical, with the degree of asymmetry increasing as δ moves away from zero. It acts as a *supplementary parameter* that enables the study of how small deviations from symmetry affect the behaviour of the pulse. The correction term, denoted as $Diff = H - 3$, represents the difference in relation to the expected amplitude concentration for symmetrical pulses. Its variation as a function of n is plotted in Fig. 4 for small δ=0.05, and in Fig. 5 for high δ=0.30, respectively, for slightly asymmetrical pulses and the previously selected working parameters. This difference becomes significant at small values of n, and negligible for high values of n.

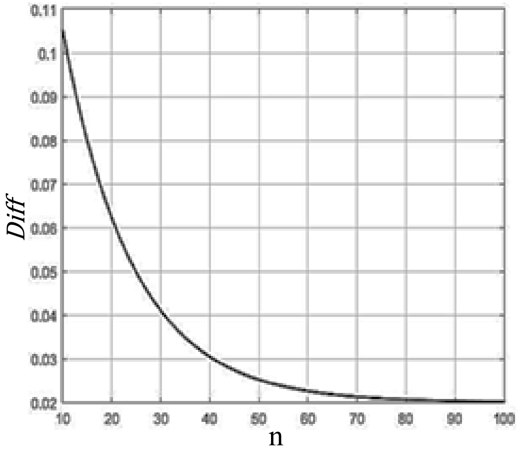

Fig. 4. Correction term $Diff = H - 3$ versus number of system elements n, δ=0.05, F normalized to unity

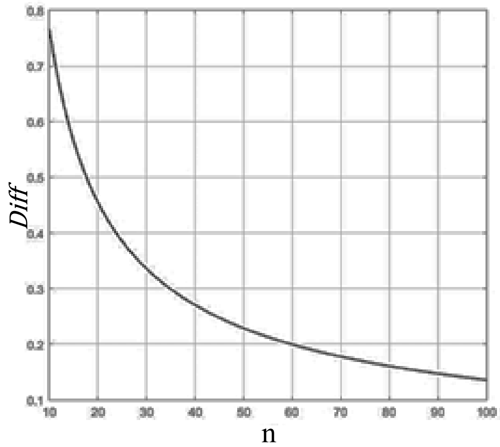

Fig. 5. Correction term $Diff = H - 3$ versus number of system elements n, δ=0.30, F normalized to unity

3. Asymmetrical discrete pulses for variable time interval

In a symmetrical pulse as related to n, by considering an asymmetry in passing from lower power of τ to higher power of τ, the difference in time when k is incremented can be written as

$$D = 1 + \Delta(k) \tag{21}$$

which means that variation of time interval Δ is a function of k.

This corresponds with a distinct set of amplitudes, generated by instant amplitudes C_n^k at time moments kT, with a certain *supplementary* delay time Δ when passing from an instant amplitude to the subsequent one, step by step, with an elementary time d.

If $\Delta = kd$ (*linear* dependence, d being an elementary time), it can be written

$$D = 1 + kd \tag{22}$$

and it results at the left inflection point:

$$D_{\text{left}} = 1 + d\left(\frac{n}{2} - \frac{\sqrt{n}}{2}\right) \tag{23}$$

and at the right inflection point:

$$D_{\text{right}} = 1 + d\left(\frac{n}{2} + \frac{\sqrt{n}}{2}\right) \tag{24}$$

The ratio of time derivatives in the left and right inflection points is inversely proportional to the ratio of the corresponding elementary time intervals:

$$R = \frac{1+\frac{d}{2}(n+\sqrt{n})}{1+\frac{d}{2}(n-\sqrt{n})} \tag{25}$$

By analysing the time intervals and key temporal markers, such as the pulse maximum and inflection points, the following expressions that allow for the determination of n using the appropriate ratios can be established:

(i) The midpoint of the significant time interval corresponding to the pulse maximum is $T \cdot \frac{n}{8}$

(ii) The time difference between the total duration and the pulse maximum is $T \cdot \frac{n^2}{8}$

(iii) The total time can be written $n + d \cdot \frac{n^2}{2} + d \cdot \frac{n}{2}$. With the second and third terms estimated graphically and n already computed, the time constant T can be determined.

(iv) With $d \cdot \frac{n}{8}$ previously evaluated and n known, the time increment d of the elementary time interval between adjacent steps, can likewise be determined.

A key validation criterion for this form of the combinatorial model − built upon the linear contribution of the elementary time interval between successive steps − is the verification of the time derivative ratio at the left and right inflection points. The graph of amplitude versus time has typically the shape presented in figure 6 for increment of elementary time $d = 1$, and $n = 36$; the ratio R of derivatives Left/Right inflexions points versus the increment of elementary time d is represented in figure 7.

It should be noticed that, in general, the maximum of ratio R as a function of d varies from approximately 2 ($n = 9$) to 1.33 ($n = 49$). In case of extremely *low values of* n, a maximum ratio R close to 3 is achieved. Left and right angles of derivatives in left and right inflexion points of $60°/30°$ for extremely low values of n, typical to $50°/40°$ for n close to 50 can be adjusted.

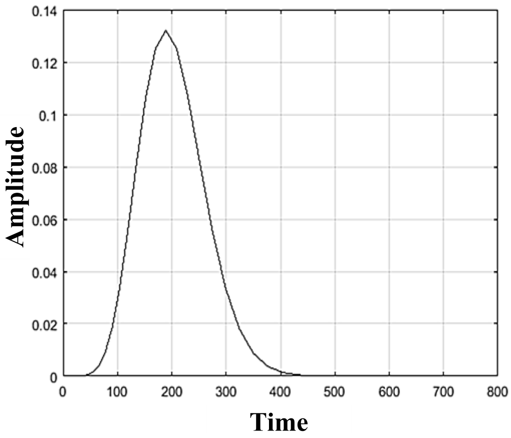

Fig. 6. Amplitude versus time for increment of elementary time interval $d = 1, n = 36$.

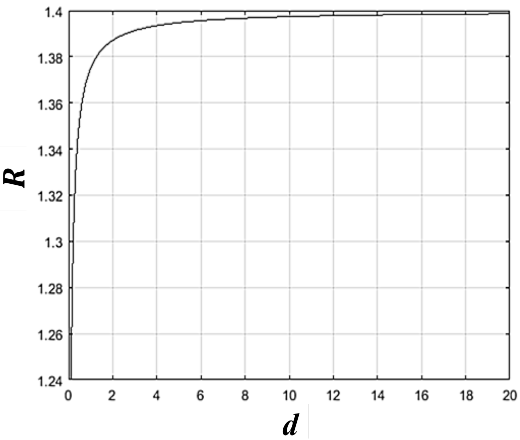

Fig. 7. Ratio R of derivatives in Left/Right inflexion points versus the increment d of elementary time interval.

Using an asymmetrical binomial pulse *with a constant time difference* between two adjacent amplitudes cannot produce significant asymmetry in the slopes at the left and right inflection points.

For a parabolic variation of the elementary time interval

$$D = 1 + kd + k^2 e \tag{26}$$

no significant variations for the ratio R can be found.

The derivatives at inflection points reveals a deviation from symmetric behaviour, particularly for *lower values of* n,by incorporating additional parameters. The ratio between slopes at left and right inflection points is a reliable criterion for validating the combinatorial model, applied to pulses of variable duration [27, 28].

4. Conclusion

In practice, neither coming pulses, nor the pierced media are ideal. Therefore, asymmetrical pulses, variable time intervals, nonlinear or inhomogeneous media have to be studied. In laser physics and applications, the signal can be experimentally sampled for giving useful information regarding the medium of propagation. Our model achieves a theoretical sampling for obtaining predictable information. In the economic activity, to find optimal solutions leads to a better management and needs modern scientific methods. Such methods must control every moment the obtained work results. Asymmetrical pulses could model and optimize the activity of an inhomogeneous work team, on a constant imposed time, or on a variable time interval. Thus, the values of parameters a, b, n, etc. can be adjusted from the very beginning, or along the way.

The delay in propagation of the signal under the form $(a + b\tau)^n$, developed with combinatorial analysis, gives information on the status of the dynamic processes, for every time. The mathematical framework developed for modelling asymmetrical pulses in interaction with a dynamic system presents a robust combinatorial approach grounded in delay operators and binomial expansions. By introducing expressions of the form $(a + bt)^n$ we can capture both the temporal distribution of amplitudes and the displacement of the peak relative to the centre of the time interval. This asymmetry is critical for accurately representing processes where the medium influences signal propagation, such as in biological systems, optoelectronic devices, or many body systems from nature and society. Through interpolation, discrete functions are extended to continuous forms, enabling graphical representation and geometric analysis.

In engineering, biology, medicine, and the emerging social sciences of the future, physics and mathematics is expected to play a major role [19, 20, 29, 30]. As a direction for future research, we have in view to use the proposed model in an integrated, modern, computer-controlled experimental setup to investigate phenomena occurring at the cell surface. Since real cell membranes are heterogeneous, the formation of distinct domains within the membrane significantly influences the transmission of signals inside and outside the cell.

The study consolidates flexible theoretical frameworks capable of integrating both symmetric and asymmetric features in the dynamic evolution of a phenomenon, and it opens paths for applying the model in fields such as signal processing in different media and the simulation of phenomena in complex socio-economic systems.

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