VECTOR CONTROL OF PERMANENT MAGNET SYNCHRONOUS MOTOR BASED ON SLIDING MODE VARIABLE STRUCTURE CONTROL

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To solve the problem existed in the traditional PI controller robustness and chattering problem, this paper proposes a control strategy of synchronous structure. The sliding mode control based on a novel reaching law is investigated in this paper, and this control is verified in permanent magnet synchronous vector control system. Instru the effectiveness and reliability of the method, theoretical analysis, The result of simulations show that the proposed sliding mode can achieve accurate speed control, simulation results show the better robustness and satisfactory dynamic and static performance. Compared with the traditional sliding mode controller, the proposed sliding mode has better tracing performance.

Keywords: Sliding mode variable structure; Variable exponent reaching law; Vector control; Buffeting; Permanent magnet synchronous motor; Mathematical model;

1. Introduction

Permanent magnet synchronous motor (PMSM) has the advantages of simple structure, high power density and high efficiency. It has been widely used in high precision CNC machine tools and robots [3]. The traditional PMSM controller adopts PI regulator, which has simple control algorithm and meet the control requirements in a certain range. However, its design depends on the precise mathematical model, with long response time and weaker robustness [2]. PMSM, however, is a complex variable with strong coupling, nonlinearity and variable parameters. In the practical application, the traditional PI controller is hard to meet the requirements of high performance control due to the influence of external disturbances and internal perturbation. The development of modern control theory provides the possibility for PMSM high performance controller [1].

SMC was an effective nonlinear robust control method proposed by USSR scholars in the 1950s. With complete adaptability and robustness, its biggest advantage is that in sliding mode, the transfer of system state is not affected by the original parameters and external disturbances. Meanwhile, it does not need to

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observe the system accurately, with simple control rate setting method, easy digital realization, fast response and well transient performance [2].

In recent years, with the improvement and development of SMC theory, domestic and foreign researchers have tried to apply SMC to position servo system of various types of motors. Many scholars have explored the application of SMC technology in PMSM speed control system [1]. The SMC is introduced into PMSM position sensorless speed regulation system to improve the speed observation accuracy. Besides, SMC has been directly used for PMSM torque control.

In this work, the traditional PI speed controller and switch table were replaced by a new sliding mode controller with variable exponential reaching law and SVPWM vector control method. Sliding mode controller forced the rotating speed into the set sliding surface, greatly reducing the pulse [6]. The exponential approach law made the state variable of the system move towards the sliding surface at two rates of velocity and exponential when the trajectory of the state variable was far away from the sliding surface, improving the approach speed. When approaching the sliding surface, the exponential approach law velocity was close to zero, which effectively reduced the initial system buffeting of the sliding surface. After the system entered the stable state, it stabilized at the origin and the buffeting phenomenon disappears. This solved the serious buffeting problem inherent in the sliding mode variable structure [7].

2. Variable exponential reaching law principle

The motion of the sliding mode variable structure control consists of two parts:

1) The first is the normal movement phase under continuous control of the system. Its motion trajectories in state space are all located outside the switching surface, or slightly crossover the switching surface.

2) The second is the sliding mode movement phase, when the system moves along the switching surface to the stability point.

Gao Weibin put forward the concept of reaching law in the last century, and designed the exponential reaching law and the state trajectory of system movement phase in the process of sliding mode control. It is widely used at home and abroad. However, the exponential approach law has its own shortcomings: with zonal switching band, the system cannot approach the origin in the switching band, showing a buffeting close to the origin [8].

High-frequency buffeting can stimulate the high-frequency components not considered in system modeling, increasing the burden on the controller. In order to overcome the shortcoming of the exponential reaching law, a new approach law, the variable exponential reaching law, is derived.
\[
\dot{s} = \left[ -\varepsilon |X| \operatorname{sign}(s) - \eta s \right]
\]

\[
\lim_{t \to \infty} |X| = 0, \quad \eta > \varepsilon > 0
\]

The exponential approach law makes the system state variable approach the sliding mode surface at the two kinds of velocity (variable speed and exponential) at the beginning of the system state. When approaching the sliding mode surface, the exponential term approaches zero, and the variable speed \(-\varepsilon |X| \operatorname{sign}(s)\) plays a key role. When the selected state quantity \(|X|\) is infinitely tending toward zero in the process of system stabilization, the sliding mode control law makes the state quantity \(|X|\) enter the sliding surface and move towards the origin. Meanwhile, the control term \(-\varepsilon |X|\) unceasingly reduces in the control law, leading to smaller sliding gain and weaker buffeting. Once the system is stabilized in origin, the coefficient of the sliding-mode buffeting control becomes zero, thus eliminating buffeting. The reaching law can obviously satisfy the generalized sliding mode condition.

In order to further weaken the buffeting of the trajectory of the state variables before reaching the origin, the sign function can be smoothed as

\[
\operatorname{sign}(s) = \frac{s}{s + \sigma}, \quad \sigma > 0.
\]

where \(\sigma\) is a small positive constant.

3. Mathematical model of permanent magnet synchronous motor

On condition of ensuring the control performance, the induced potential waveform in the winding is regarded as a sine wave, ignoring the saturation of magnetic core, hysteresis and eddy current loss. The voltage equation of permanent magnet synchronous motor in \(\alpha - \beta\) stationary coordinate system is:

\[
\begin{bmatrix}
    u_\alpha \\
    u_\beta
\end{bmatrix} = \begin{bmatrix}
    R_s + DL_d & \omega_e \left( L_d - L_q \right) \\
    -\omega_e \left( L_d - L_q \right) & R_s + DL_d
\end{bmatrix} \begin{bmatrix}
    i_\alpha \\
    i_\beta
\end{bmatrix} + \begin{bmatrix}
    \left( L_d - L_q \right) \left( \omega_e i_d - i_q \right) + \omega_e \psi_f \\
    -\sin \theta_e \\
    \cos \theta_e
\end{bmatrix}
\]

The voltage equation in \(d - q\) rotating coordinate system is

\[
\begin{bmatrix}
    u_d \\
    u_q
\end{bmatrix} = \begin{bmatrix}
    R_s + DL_d & -\omega_e L_q \\
    \omega_e L_d & R_s + DL_q
\end{bmatrix} \begin{bmatrix}
    i_d \\
    i_q
\end{bmatrix} + \begin{bmatrix}
    0 \\
    \omega_e \psi_f
\end{bmatrix}
\]
The electromagnetic torque equation is:

\[ T_e = \frac{3}{2} p \psi_r i_q \]  

(5)

The equation of motion is

\[ T_e - T_L = J \cdot \frac{d\omega}{dt} \]  

(6)

Where \( u_d \) and \( u_q \) the shaft voltage of \( d \) axis and \( q \) axis respectively, \( i_d \) and \( i_q \) the shaft current of \( d \) axis and \( q \) axis respectively, \( R_s \) is the resistance of stator \( d \); \( \omega_c \) the electrical angular velocity of the rotor; \( D \) the differential operator; \( \psi_a \) the flux linkage of permanent magnet and the stator; \( T_e \) the electromagnetic torque; \( T_L \) the motor load torque; \( P \) the number of motor pole; \( J \) the motor inertia. In this work, \( i_d = 0 \) is set in vector control method for permanent magnet synchronous motor [9].

4. Design of controller

4.1 Controller design

Sliding mode controller is the control based on the phase plane. The basic idea is to design a predetermined sliding surface, and guide the entire state trajectory to sliding surface through controller, ensuring asymptotically stable motion on the sliding surface.

State variable of the control system is set as:

\[
\begin{cases}
  x_1 = \omega_r - \omega \\
  x_2 = \dot{x}_1 = -\dot{\omega}
\end{cases}
\]  

(7)

where \( \omega_r \) is the given speed; \( \omega \) the actual speed of the motor.

After calculating Formulae (4), (5), (6) and (7), we obtain

\[
\begin{cases}
  \dot{x}_1 = -\dot{\omega} = -\frac{p}{J} \left[ \frac{3 p \psi_a i_q}{2} - T_L \right] \\
  \dot{x}_2 = -\ddot{\omega} = -\frac{3 p^2 \psi_a i_q}{2 J}
\end{cases}
\]  

(8)

Set \( b = \frac{3 p^2 \psi_a}{2 J} \) \( M = \dot{i}_q \) we obtain the system state space expression:

\[
\begin{bmatrix}
  x_1 \\
  x_2 
\end{bmatrix} =
\begin{bmatrix}
  0 & 1 \\
  0 & 0 
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  -b 
\end{bmatrix} M
\]  

(9)
In order to achieve non-overshoot stability, the first-order sliding surface is selected, noted as $s$.

$$s = cx_1 + x_2$$  \hspace{1cm} (10)

In order to improve the dynamic quality in normal motion section, a new variable exponential reaching law is used to design the sliding mode variable structure controller of the motor [10].

Set the variable exponential approach law of permanent magnet synchronous motor as

$$\dot{s} = \begin{bmatrix} c\dot{x}_1 + \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\varepsilon |x_1| \text{sign}(s) - \eta s \end{bmatrix}$$  \hspace{1cm} (11)

where $\varepsilon$, $\eta$ are the variable exponential approach law parameters.

The control law of the sliding mode controller with variable exponential reaching law can be obtained by combining the above equations.

$$U = \frac{1}{b} \begin{bmatrix} cx_2 + \varepsilon |x_1| \text{sign}(s) + \eta s \end{bmatrix}$$  \hspace{1cm} (12)

From the approach law and the control rate of the controller, the moving point of the system moves exponentially towards the sliding surface in the motion outside the sliding surface. When approaching the sliding surface, the system enters the switching belt. The motion of the sliding surface is proportional to the absolute value of the error $|x| = |e| = |\omega_r - \omega|$, so the amplitude will be smaller and the ideal situation will eventually stabilize to the origin. The error is zero, and the buffeting mode will be eliminated. Generally, $\varepsilon$ can be set relatively small, while $\eta$ can be set with a larger value, so that the system will enter the sliding surface at a faster speed. Meanwhile, when entering the sliding surface, the sliding surface switch belt has smaller speed. Without large buffeting, it will approach to the origin faster.

### 4.2 Stability analysis

As the dynamic process of the system is composed of sliding mode, the system stability can be ensured when the sliding mode is approached. Based on Lyapunov stability theory, Lyapunov function can be set as [10]

$$V(x) = \frac{1}{2} s^2 \quad s \neq 0$$  \hspace{1cm} (13)

Then

$$\dot{V}(x) = s \dot{s} \quad s \neq 0$$  \hspace{1cm} (14)

Because the reachability condition of SMC is $s\dot{s} < 0$, it can guarantee $\dot{V}(x) < 0$. That is, the system can enter the sliding mode. It can be obtained by Formulae (10) and (11):
\[
s\dot{s} = s \left[ -\varepsilon \left| x_1 \right| \text{sign}(s) - \eta s \right], \tag{15}
\]
where \( \varepsilon, \eta > 0 \), and \( s \cdot \text{sign}(s) > 0 \), so it can ensure \( s\dot{s} < 0 \).

Once entering the sliding mode \( (s = 0, \dot{s} = 0) \), the system enters the sliding mode movement stage. After being combined with the system state equation, the differential equation of motion of SMDC system is obtained:

\[
C x_1 + \dot{x}_1 = 0 \tag{16}
\]

By solving this equation, we can obtain

\[
x_1 = \omega_t - \omega = C_0 e^{-ct}, \tag{17}
\]

Where \( C_0 \) is a constant. If \( t \to \infty \), \( x_1 \) tends to zero along the index, namely achieving no overshoot speed tracking and system stability. The quality of the system is entirely decided by the parameters in switch belt, irrelevant to system parameters and disturbances. Stabilization time is only relevant with \( c \), with good robustness and fastness. In summary, SMC control system can achieve global stability [11].

5. Simulation test

Using Matlab/Simulink simulation, motor parameters are set as \( L_d = L_q = 8.5mH, R_s = 2.875\Omega, \psi_f = 0.175Wb \) and \( P = 4, J = 0.008kg \cdot m^2 \). The system adopts the control scheme in Fig. 1; speed regulator adopts sliding mode control strategy or PI control; current loop adopts PI vector control.

![Fig. 1 Structure of control system](image-url)
The motor speed waveform is shown in Fig. 2. In this figure, the speed of the permanent magnet synchronous motor is increasingly fast, with the variable exponential approaching sliding mode control. When the stable value is reached, there is no overshoot. With the speed loop of PI control, the motor speed has long response time, leading to overshoot. Fig. 2 (c) shows the motor speed waveform under the control of traditional sliding mode, with a longer response time compared with Fig. 2 (a).

(a) Motor speed under the sliding mode control of exponential approach law

(b) Motor speed waveform under PI control
Figs. 3(a) and (b) show Phase-trajectory diagram of variable-index sliding mode and PI respectively. Fig.3(c) shows phase-trajectory diagram of traditional sliding mode controller; the buffeting of the variable-index sliding mode controller has been obviously weakened, compared with the PI controller and traditional sliding mode controller. The buffeting near the origin is significantly reduced after reaching the sliding mode surface.
Fig. 3 shows the phase-trajectory diagram of the exponential sliding mode

6. Conclusions

In this work, a novel variable-index sliding-mode variable structure controller was designed and applied to the vector control system of permanent magnet synchronous motor (PMSM). The feasibility and effectiveness of this control strategy were proved by simulations. The control method overcomes the shortcomings of traditional PI control, such as long response time, large overshoot and strong robustness. It also has strong inhibition on external interference and noise. The robustness of the system is greatly enhanced, solving the problem of severe buffeting in traditional sliding mode control. The switching frequency of
controller is reduced to easing the burden on the controller, thus enhancing the stability of the system.

REFERENCES