HYDROSTATIC STRESS EFFECT ON YIELD BEHAVIOR OF THE HIGH DENSITY POLYETHYLENE

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The hydrostatic stress is, for polymers, a significant parameter which affects the yield behavior of these materials. In this work, we investigate the influence of this parameter on yield behaviour of the high density polyethylene (HDPE). Some tests on samples with diverse geometries are described in this paper. Uniaxial tests: tensile on notched round bar samples with different curvature radii, compression on cylindrical samples and simple shear on parallelepiped samples were performed. Biaxial tests with various combinations of tensile/compressive and shear loading on butterfly samples were also realized in order to determine the hydrostatic stress for different states of solicitation. The experimental results show that the yield stress is very affected by the hydrostatic stress developed in the material during solicitations.

Keywords: biaxial tests, hydrostatic stress, HDPE, yield behavior

1. Introduction

In contrast with metallic materials, it has been shown that the polymers mechanical answer, in addition to the deviatoric stress tensor, is sensitive to the hydrostatic pressure \([1, 2]\). This sensitivity is more significant on the yield behavior of these materials, which is due to the effect of this pressure on the molecular chains. Similarly, a high positive hydrostatic pressure promotes the development of damage mechanisms, particularly, the nucleating of micro voids in the amorphous phase \([3]\). Therefore, for taking into account the effect of hydrostatic pressure on the multiaxial response of polymers, it must be included in any yield criterion adopted by the behavior laws for this type of material. Expressed as mean stress, this hydrostatic pressure is introduced into the various yield criteria proposed to describe the polymer yield behavior, as the modified von Mises criteria \([4]\), the modified Tresca criteria \([5]\), the Drucker-Prager criteria

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and Raghava criteria [6]. Several studies have investigated the polymers behavior, through mechanical tests by different types of loading: tension-compression tests on polymethylmethacrylate (PMMA) [7], tension-torsion tests on cylinder samples in amorphous PMMA [8] and tensile (or compression) combined with an internal pressure on hollow cylinders in epoxy, polyethylene and polypropylene [9]-[12]. In this work, we are interested in determining the hydrostatic pressure developed during mechanical tests in uniaxial and biaxial loading on semi-crystalline material; the high density polyethylene. The effect of the hydrostatic pressure on the yield stress of this material will be then investigated.

2. Uniaxial tests

2.1. Tensile test

Tensile tests on axisymmetric notched samples with different curvature radii (R = 2, 4, 10 and 80 mm) (fig 1), were performed using an Instron testing machine at constant strain rate ($10^{-3}$ s$^{-1}$). These tests aim to induce different stress levels with low and high triaxialities (stress triaxiality ratio equal to 0.33, 0.44, 0.6 and 0.8) in the material [13].

The yield stress was determined, in each case, from the deviation of the linear elastic response in the true stress-true strain curves (Fig. 2). Their values are equal to 12.7, 10.7, 9 and 8.65 MPa for samples with curvature radii: R = 2, 4, 10 and 80 respectively.

The hydrostatic stress is calculated as follow:

$$\sigma_h = T\sigma_{eq}$$

(1)

Where $T$ is the stress triaxiality ratio and
\[ \sigma_{eq} = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \]  \hspace{1cm} (2)

is the von Mises equivalent stress.

Fig. 3 shows the effect of hydrostatic stress on yield stress in tensile tests. It can clearly see that this yield stress varies linearly with the hydrostatic stress. The highest value is recorded for the largest hydrostatic stress developed in the material, which represents the largest triaxiality rate.
2.1. Compressive and shear tests

Other type of uniaxial tests; axial compression and pure shear are performed respectively on cylindrical and parallelepiped samples (Fig.4). From the HDPE stress–strain response obtained for both tests, given in Fig. (5), the value of the yield stress was determined. Their values are respectively equal to 13 and 7.86 MPa. The hydrostatic stress developed in the material during the shear test is found equal to zero, which represent the lowest value. While for the compression test, the value of this stress is equal to 4.29 in absolute value.

![Fig. 4. a) Cylindrical samples; b) Parallelepiped samples](image)

![Fig. 5 Equivalent stress–strain responses during uniaxial compression and simple shear](image)

3. Biaxial tests

The biaxial tests method used is described in a previous paper [14,15]. Combined tensile/compression and shear loading are applied to butterfly samples using an Arcan device (Fig. 6).
By changing $\alpha$ (angle relative to vertical), the sample central section will be subject to different stress states (table I).

**Table I**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Direction of the crosshead movement of the tensile machine</th>
<th>Stress states</th>
</tr>
</thead>
<tbody>
<tr>
<td>$90^\circ$</td>
<td>Positive</td>
<td>Tensile</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>Negative</td>
<td>Compression</td>
</tr>
<tr>
<td>$00^\circ$</td>
<td>Positive (or negative)</td>
<td>Shear</td>
</tr>
<tr>
<td>$0^\circ&lt;\alpha&lt;90^\circ$</td>
<td>Positive</td>
<td>Combined tensile / shear</td>
</tr>
<tr>
<td>$0^\circ&lt;\alpha&lt;90^\circ$</td>
<td>Negative</td>
<td>Combined compression / shear</td>
</tr>
</tbody>
</table>

From the load-displacement curves obtained in biaxial tests presented in Fig.7, the strength value $F$ corresponding to the yield stress was determined at the end of the elastic response of the curves. The normal and the shear yield stresses $\sigma_{yy}$, $\sigma_{xx}$ and $\sigma_{xy}$ respectively, developed in the sample central section, were evaluated using the analytical expressions derived by Doyoyo and Wierzbicki [16]:

$$\sigma_{yy} = \nu \sigma_{xx}$$  \hspace{1cm} (3)

$$\sigma_{xx} = \frac{F}{S} \left[ \frac{\beta \sin \alpha}{\left( 1 - \beta \cos^2 \alpha + \beta \right)} \right]$$  \hspace{1cm} (4)

$$\sigma_{xy} = \frac{F}{S} \left[ \frac{\beta \cos \alpha}{\left( 1 - (1 - \beta) \sin^2 \alpha + \beta \right)} \right]$$  \hspace{1cm} (5)
Where $x$ and $y$ being the coordinates perpendicular and parallel to the central section, respectively, $\alpha$ the loading angle and $S$ is the section of the central section. The constant $\beta$ is defined by the formula: $\beta = 2/(1-\nu)$ in which $\nu$ is the elastic Poisson’s ratio.

![Graph a)](image1.png)

![Graph b)](image2.png)

**Fig. 7** Load-displacement responses of the butterfly-shaped sample for different loading angles: (a) positive vertical displacement (combined shear-tension), (b) negative vertical displacement (combined shear-compression)

The equivalent yield stress is calculated in the sense of von Mises as function of $\sigma_{xy}$, $\sigma_{xx}$, and $\sigma_{yy}$. The evolution of this yield stress vs hydraulic stress is shown in **Fig. 8** and **9** for combined tensile-shear tests and combined
compressive-shear tests respectively. In Fig. 9 the yield and hydrostatic stresses are represented by negative values in order to specify the compression test (negative displacement of the crosshead of the tensile machine according to y).

It can be observed a linear evolution in both cases, except that in the second test case the slope of the line is higher than in the first case. Also, the hydrostatic pressure values and those corresponding to yield stress are significant.

![Fig.8 Yield stress evolution as a function of hydrostatic stress for combined tensile and shear tests](image)

![Fig.9 Yield stress evolution as a function of hydrostatic stress for combined compressive and shear tests](image)

**4. Discussion**

In this work, the investigation is focused on hydrostatic stress determination, and its influence on HDPE yield stress has been explored by an experimental procedure. The results have shown that the yield stress increase linearly with hydrostatic stress. In fact, it was recorded for all tests that yield stress-hydrostatic stress data are well described by straight lines.
However, by comparing the slope of these straight lines, it was find that the triaxial stress state developed during tensile tests on notched samples, the hydrostatic stress effect on the yield stress is more sensitive compared to the biaxial stresses state developed during the combined tension / shear loading.

Indeed the value of this slope in the first case is equal to 0.56 while in the second it is 0.4. This result shows that the hydrostatic stress induced by the stress triaxiality is an important parameter that affects the polymers behavior and mainly on the yield behavior.

Similarly, the higher yield stress value is recorded in the triaxial stress case and it's equal to 12.7 MPa. This value is obtained in the tensile test on axisymmetric notched sample with 2mm radius. In the central section of this sample, the hydrostatic stress developed (10.39Mpa) is mainly due to the high stress triaxiality rate which is equal to 0.8 [13]. It is also noted that the lower value of the yield stress (4.29MPa) is that recorded in the simple shear test, because hydrostatic pressure is equal to zero.

By analyzing the biaxial tests results in combined compression/shear loading, it can seen that the sensitivity of pehd yield behavior, is much more sensitive to hydrostatic stress than in combined tension / shear loading. Effectively, the slope value of the line describing the variation of the yield stress as a function of the hydrostatic stress is higher and is equal to 1.003. This shows that a small increase in the hydrostatic stress in the material induces a sudden change in the yield stress. Also, the maximum value of the yield stress (17,55MPa) is obtained for this type of test with a loading angle $\alpha = 70$.

We would like to specify another interesting result, which is the difference of pehd yield behavior in compression in relation to tensile. Indeed, the higher yield stress values are obtained in the simple compression test and combined compression- shear respectively. This is due to the action of the hydrostatic pressure developed for each stress state on van der Waals secondary connections in the three directions of space [7, 17]. Thus, an increase in hydrostatic stress prevents the molecular mobility and thus increases the yield stress values.

4. Conclusion

This paper comes in the context of exploring the hydrostatic stress sensitivity of polymers and mainly semi-crystalline polymers; since the material of study is HDPE which is considered as a model of this class of polymers. Our contribution is to demonstrate this sensitivity by a rich experimental procedure using different forms of samples with uniaxial and biaxial testing. The results obtained show that the hydrostatic stress varies with solicitation mode and its influence on the pehd yield behavior is more noticeable in compression than in
tensile. Indeed, the yield stress changes linearly with the hydrostatic stress and the maximum values are recorded during combined compression - shear tests.

REFERENCES


