SIMULATION-BASED SENSITIVITY ANALYSIS OF MANUFACTURING EQUIPMENT AVAILABILITY

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The operation and maintenance is one of the most important territories of the automotive industry. From the mathematical point of view, maintenance is a discrete state space stochastic process without after-effects, so it can be modeled as a Markov-chain. After determination of probability densities of changes of operational states and setting up the transition probability matrix, a matrix-algebraic method can be used for investigating these processes with systems approach analysis. This paper is aimed to discuss the possibilities of the use of simulation-based sensitivity analysis of maintenance systems and processes. The proposed method helps decision making in maintenance management.

Keywords: Maintenance; Availability; Sensitivity analysis; Decision making**.**

1. Introduction

The operation of a technical system is a stochastic process based upon the equipment, its maintenance, its preparation, and also the personnel carrying out repair, and the regulations for the whole process. By Manzini et.al., maintenance is the function that monitors and keeps plant, equipment, and facilities working. It must design, organize, carry out, and check the work to guarantee nominal functioning of the item during working times and to minimize downtimes caused by breakdowns or by the resulting repairs [9].

According to the classical view, the role of maintenance is to fix broken equipment. But nowadays, if the strategic dimension of maintenance is also taken into account, it should cover decisions of maintenance management taken to shape the future maintenance requirements of the organization [7]. A maintenance system and process can be characterized by Mean Time Between Failures (MTBF), Mean Time To Repair (MTTR) and the availability of equipment. Availability may be generically defined as the capability of a system to be ready to perform its functions when required [2].

In recent years, there are several papers and books that discuss new methods from different aspects to help decision making in maintenance management. For example, the purpose of Jardine and Tsang's book is to provide readers with the tools needed for making data-driven decisions [7]. Duer

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presented a modelling method of the operation process of repairable technical objects of various classes [4]. A particular attention was paid to the model of the process which includes a service expert system with an artificial neural network. Duer's paper also included theoretical grounds of the modelling process of the operation of objects in the form of the following models: mathematical (analytical), graphical and descriptive ones. Book of Manzini et.al. identifies the role of maintenance in a production system and the capability of guaranteeing a high level of safety, quality, and productivity in a proper way [9].

The maintenance is often viewed as a necessary short-term investment for the company dealing with equipment failures and enhancing system efficiency while investment in a new technology is considered as a part of long-term competitive strategy. The paper of Nguyen et.al. has highlighted the importance of considering maintenance actions at the tactical level [10].

Shu and Zhao presented a simplified three-step Markov analysis approach that avoids building large Markov models [17]. Their approach simplified the modelling and calculating of Markov-based safety integrity level verification without loss of accuracy. A case study was performed and the results prove that the proposed approach can simplify Markov modelling without loss of accuracy if a proper common cause failure model was adopted. Nguyen et al. modeled the optimization problem as a Markov decision process. First, a model where technological evolution is not considered will be constructed and analyzed.

The maintenance optimization has the potential for substantially reducing the operating costs and for increasing corporate profit by increasing availability and productivity [6]. The approach proposed by Hu and Zhang is based on the analysis of fault coupling features of complex mechanical system, considering both of age and risk factors. This risk based opportunistic maintenance model allows rearranging the optimal individual replacement times to get the minimal cost with acceptable failure risk regarding the whole system, which convert the downtime loss as a negative factor caused by single failure in the system into a favorable opportunity of proactive maintenance for other degraded components. The algorithm used by Hu and Zhang is a numerical procedure, where the life cycles are simulated and the optimal solution is numerically searched. The large number of random simulations by Monte Carlo method ensures the stability of the estimates and guarantees the solution convergence to the optimal one. As all significant combinations of parameters are considered, the optimal solution cannot be missed. The presented method can largely reduce failure loss and maintenance cost, which plays an important role in fault early-warning control [6].

In study of Azadeh et al., an integrated "Health, Safety, and Environment (HSE)" and maintenance systems were presented. Multivariate analysis was used for continuous performance assessment and improvement of these systems. The proposed approach would help policy makers and top managers [2].

A stochastic process which development in the future is influenced by its development in the past only through its development in the present, that is, a stochastic process without aftereffects, is called Markov-process [3]. The fundamental assumption of the Markov model-based simulation is that the probability of a given state transition depends only on the current state of the system and not on any previous ones [1]. The mathematical background of Markov-processes is discussed in books of Bharucha-Reid [3], Karlin and Taylor [8] and Ushakov [19].

Gan and Shi modeled a system and decision process by discrete Markov method, and through a policy-iteration algorithm, the long-term expected cost rate and control policy are optimized [5].

By Semanco and Marton, the simulation of the business and manufacturing processes plays an essential role in modern scientific research and decision-making in management [16].

Pokorádi showed the possibilities of the use of Markov matrix in the case of stationary maintenance processes, and proposed a well-algorithmizable method for mathematical modelling of stationary stochastic industrial process [11]. This modelling method can be used for estimating maintenance cost and the time of availability of equipment.

Sensitivity analysis helps to identify those elements of the system that exercise a high degree of leverage on system behavior [14]. Sokolowski and Banks defined sensitivity analysis as the study of how uncertainty in a model's output can be assigned to the various sources of input uncertainty. The sensitivity analysis is important for several reasons. It can help uncover model errors and identify important bounds on input variables. This analysis can also help identify research priorities and simplify models. Thus, sensitivity analysis plays a significant role as a tool to assess model validity. Thus, sensitivity analysis plays a significant role as a tool to assess model validity [18].

The State Dependent Regression (SDR) is used to identify non-dynamic and non-linear regression models, i.e. in the non-parametric regression context [20]. As such, SDR could well be considered for applications like sensitivity analysis. Applying the SDR approach, Ratto et al. [13] have first developed a nonparametric approach for the efficient estimation of sensitivity indices in the framework Global Sensitivity Analysis (GSA) [15].

Pokorádi showed the adaptation of the linear mathematical diagnostic modeling methodology to set-up the Linear Sensitivity Model of System Reliability (LSMoSR) and Linear Sensitivity Model of System Unreliability (LSMoSU) [12]. These models are analogue modular approach tools that use matrix-algebraic method based upon the mathematical diagnostics methodology of aircraft systems and gas turbine engines. In this paper their possibility of use was demonstrated to investigate the system with complex interconnection sensitivity by a simply example.

The aims of this paper are the followings:

- to apply the method proposed by reference [11] to depict Markovian model of the investigated maintenance process;
- to propose a simulation method to investigate sensitivity of availability of equipment depending on MTBFs and MTTRs;
- to show possibilities of use of proposed method by a case study.

The outline of the paper is as follows: Section 2 presents the stochastic model of investigated maintenance process. Section 3 shows the proposed simulation-based method to determinate sensitivity of availability of operated technical system. Section 4 interprets conclusions about sensitivity of investigated system availability and reliability and the proposed methods. Section 5 summarizes the paper, outlines the prospective scientific work of the Author.

In the introduction to the paper, the author(s) will specify the present stage of the branch researches (by quoting the adequate bibliography) and will specify the purpose of the paper.

2. The model of the investigated process

In this Chapter the stochastic model of the investigated maintenance process will be shown. The general methodology of setting up of this model can be known completely by publication [11].

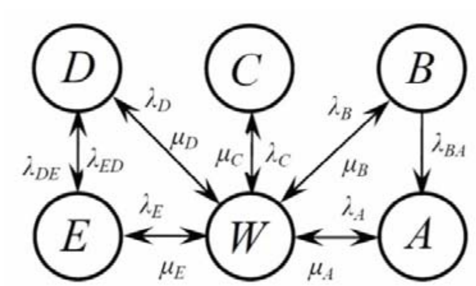

Fig. 1. The Graph Model of Investigated Maintenance Process
W – Equipment is Applicable; ***A*** – ***Type A*** Failure's Repair;
B – ***Type B*** Failure's Repair; ***C*** – ***Type C*** Failures' Repair;
D – ***Type D*** Failure's Repair; ***E*** – ***Type E*** Failure's Repair

The main statistical data of the failures and their repairs are included in During the operation of the investigated manufacturing equipment four main types of failures can be experienced. These (***Type A***; ***B***; ***D*** and ***E***) failures occur more than 94 % of equipment outages. The other failures are modeled by ***Type C*** failure. When ***Type B*** failures are repaired, the servicemen detect frequently that ***Type A*** failure will occur shortly, then ***Type A*** failure repair is carried out as well. During the repair of ***Type D*** and ***Type E*** failures a similar situation can occur. Maybe in these cases the other failure should be repaired too.

Table 1. The Fig. 1 shows weighted directed graph of the operational process. In the graph, the weights of the directed edges show probability densities (failure or repair rates) of changes of operational states.

The system of differential equations that describes the changes in time of the probability of staying in different states can be determined as

$$\begin{aligned}
\frac{dP_w}{d\tau} &= -(\lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E)P_w + \mu_A P_A + \mu_B P_B + \mu_C P_C + \mu_D P_D + \mu_E P_E \\
\frac{dP_A}{d\tau} &= \lambda_A P_W - \mu_A P_A + \lambda_{BA} P_B \\
\frac{dP_B}{d\tau} &= \lambda_B P_W - (\mu_B + \lambda_{BA}) P_B \\
\frac{dP_C}{d\tau} &= \lambda_C P_W - \mu_C P_C \\
\frac{dP_D}{d\tau} &= \lambda_D P_W - (\mu_D + \lambda_{DE}) P_D + \lambda_{ED} p_E \\
\frac{dP_E}{d\tau} &= \lambda_E P_W + \lambda_{DE} P_D - (\mu_E + \lambda_{ED}) P_E
\end{aligned} \tag{1}$$

Because of the investigated process is stationary that is the probabilities do not change in course of the time, the differential coefficients of eq. (1) are:

$$\frac{dP_W}{d\tau} = \frac{dP_A}{d\tau} = \frac{dP_B}{d\tau} = \frac{dP_C}{d\tau} = \frac{dP_D}{d\tau} = \frac{dP_E}{d\tau} = 0 \quad . \tag{2}$$

A further condition of the solution is the

$$\sum_{i=W}^{E} P_i(\tau) = 1 \tag{3}$$

probability of event space ($i \in L$, where L set of Latin letters $W\ A\ B\ C\ D\ E$) This equation expresses that the object of operation has to stay only in one of six states (in the present case, the state space consists of them). Then on the basis of equations (1) – (3) stochastic model of the investigated stationary operation process can be depicted as the following matrix formula:

$$\begin{bmatrix} -(\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E) & \mu_A & \mu_B & \mu_C & \mu_D & \mu_E & 1 \\ \lambda_A & -\mu_A & \lambda_{BA} & 0 & 0 & 0 & 1 \\ \lambda_B & 0 & -(\mu_B+\lambda_{BA}) & 0 & 0 & 0 & 1 \\ \lambda_C & 0 & 0 & -\mu_C & 0 & 0 & 1 \\ \lambda_D & 0 & 0 & 0 & -(\mu_D+\lambda_{DE}) & \lambda_{ED} & 1 \\ \lambda_E & 0 & 0 & 0 & \lambda_{DE} & -(\mu_E+\lambda_{ED}) & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_W \\ P_A \\ P_B \\ P_C \\ P_D \\ P_E \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (4)$$

The methodology of setting up of the model mentioned above can be known profoundly by publication [11]. Table 2 consists of results of equation (5) that is stochastic model of maintenance process using nominal values of Table 1.

Knowing the cost of different repairs, the expected value of the total repairing cost is:

$$RK = T\sum_{i=A}^{E}\frac{k_i P_i}{MTTR_i} \quad , \qquad (5)$$

and the expected value of the work expenditure is:

$$WE = T\sum_{i=A}^{E}\frac{m_i P_i}{MTTR_i} \quad , \qquad (6)$$

where:

T – investigated time-interval;
$MTTR_i$ – Mean Time to the i-th Repair
k_i – mean cost of the i-th repair;
m_i – mean work expenditure of the i-th repair.

Table 1

Nominal Data of Statistical Analysis

Failure	***A***	***B***	***C***	***D***	***E***
MTBF [hour]	1316.3	892.8	1339.4	1410.1	1396.4
Failure rate λ [hour^{-1}]	7.597 10^{-4}	1.1201 10^{-3}	7.466 10^{-4}	7.0917 10^{-4}	7.1613 10^{-4}
$MTTR_i$ [hour]	7.08	9.63	2.14	8.21	7.62
Repair rate μ [hour^{-1}]	0.14124	0.10384	0.46729	0.1218	0.13123
Mean Repairing Cost [€] k_i	150.2	115.4	98.7	210.8	352.4
Mean Work Expenditure [man-hour] m_i	14.16	14.45	5.35	24.63	17.5
λ_{ij} [hour^{-1}]	–	0.427	–	0.613	0.524

Table 2

Nominal Results of Model

$P_W = 9.7399\ 10^{-1}$	$P_A = 1.1452\ 10^{-2}$	$P_B = 2.0551\ 10^{-3}$
$P_C = 1.5562\ 10^{-3}$	P_D = 5.1048 10-3	P_E = 5.8403 10-3

The expected total repairing cost of the present maintenance system for 10.000 hours is:

51996.0 Euros ,

and expected work expenditure is:

4252.43 man hours .

3. Sensitivity analysis

The stochastic mathematical model set up in the previous Chapter can be used for simulation-based sensitivity investigation of maintenance system. Using the mathematical model relative changing of output parameters can be determined in case of changing of input one (or ones). By literatures [12] and [18], the sensitivity coefficient by input parameter x_i of general function $y = f(x_1, x_2, \ldots, x_n)$ can be determined analytically by

$$K_{x_i} = K_i = \frac{\partial f(x_1; x_2; \ldots x_n)}{\partial x_i} \frac{x_i}{f(x_1; x_2; \ldots x_n)} = \frac{\partial y}{\partial x_i} \frac{x_i}{y} . \qquad (7)$$

This derivative-based approach is very efficient from a computational standpoint; however, it does have one serious flaw. Derivative-based approaches are only valid at the point that they are computed. This is acceptable for linear systems but would be of little value for systems exhibiting nonlinear behavior. There are, however, other methods that can be applied for all systems [18]. Taking into account the specific characteristics of the stochastic model, its analytical sensitivity coefficients cannot be determined using the equation (7). Therefore this differential equation should be transform into difference equation

$$K_{x_i} = K_i = \frac{\Delta y}{\Delta x_i} = \frac{y(x_{i0} + \Delta x_i) - y_0}{y_0} , \qquad (8)$$

where index *0* signs the nominal values.

The values given by Tables 1 and 2 were used as the nominal ones. During the sensitivity analysis, values of independent parameters were modified by 1%, and on the basis of the model results and equation (8), the sensitivity coefficients of the independent parameters were determined.

Firstly, the MTBFs were increased, having simulated improvement of reliability of given structural element of operated system. Comparing the simulation results, we can determine improving the reliability of which component has the maximal effects on availability, repairing cost and work expenditure of the system. The effects of increasing by 1% the MTBFs on the system availability P_w are showed by Fig. 2. Fig. 3 shows the sensitivities of repairing cost and work expenditure of the system.

Secondly, the MTTRs were decreased by 1% to simulate that repairing work are more efficient, for example because of modernized technology or optimal maintenance management.

On the basis of the simulation results, we can determine that changing of repair technology of which failure occurs the maximal increasing of availability or decreasing of repairing cost, and work expenditure of the system. The effects of decreasing by 1% the MTTRs on the system availability are showed by Figure 4. Figure 5 shows the sensitivities of repairing cost and work expenditure of the system.

The diagrams illustrate the measurements and algebraic signs of sensitivities of the above mentioned output characteristics of maintenance system (at the X Axis) depend on simulated changing of investigated independent variables.

4. Discussion of results

On the basis of the modelling presented previously and its results from maintenance management points of view conclusions can be drawn, which can be divided into two main – about the investigated operational system and about the proposed method – groups.

To draw comprehensive conclusions, it is suggested to represent the sensitivities of probabilities of staying in repairing states to MTBF (Fig. 6) and MTTR (Fig. 7) too. Fig. 6 shows by column diagram that the probabilities of staying in repairing states how percentage change if MTBFs increase by 1%. Fig. 7 illustrates essentially the same, but MTTRs decrease by 1%.

Conclusions about the Investigated System

The following conclusions can be deduced about the investigated maintenance system from the results of modelling and analysis. It is important to mention that during determination of sensitivities the absolute values of the parameters should be examined firstly. Thereafter their signum should be determined from "mathematical" and "maintenance" points of view. In the figures, the letter 'δ' sings the relative change of the given parameter by percentages.

1a. System availability is the most sensitive to MTBF of ***Type B*** failure (see Fig. 2).

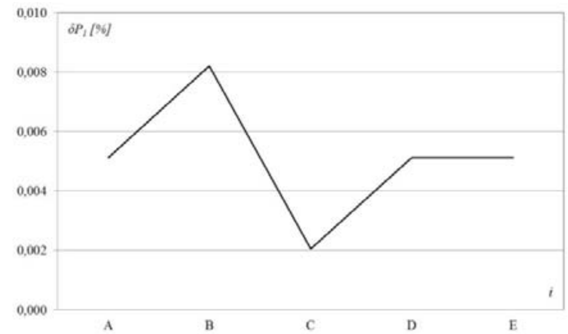

Fig. 2. Sensitivities of Availability depend on the MTBF-s

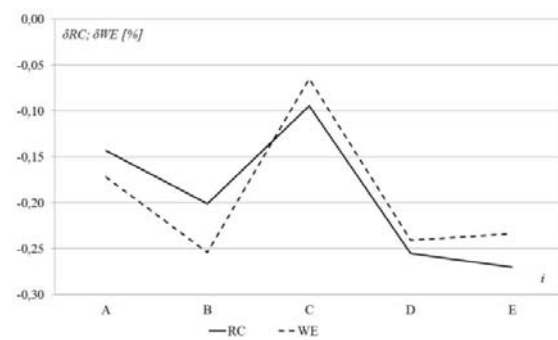

Fig. 3. Sensitivities of Repairing Cost and Work Expenditure depend on the MTBF-s

1b. Repairing cost of the system is the most sensitive to MTBF of ***Type E*** failure (Fig. 3).

1c. Work expenditure of the system is the most sensitive to MTBF of ***Type B*** failure (Fig. 3).

1d. System availability is the most sensitive to MTTR of ***Type A*** failure (Fig. 4).

1e. Repairing cost of the system is the most sensitive to MTTR of ***Type E*** failure (Fig. 5).

1f. Work expenditure of the system is the most sensitive to MTTR of ***Type D*** failure (Fig. 5).

The case of paradoxical conclusions 1e and 1f is that the increment of system availability alters ration of number of different failures. For example: Changing of **Type D** *and* **Type E** *failures have significant reciprocal effects on each other (see Figure 6). Furthermore they have considerable repairing costs* *and work expenditures.* ***Type D*** *failure requires the longest repairing time and* ***Type E*** *failure has the maximum repairing cost (see Table 1.).*

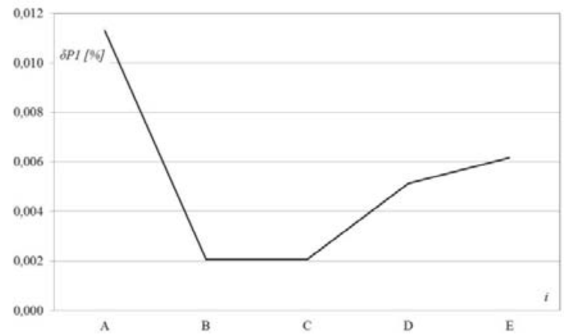

Fig. 4. Sensitivities of Availability depend on the MTTR-s

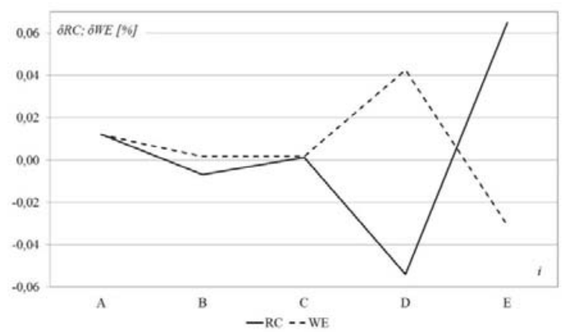

Fig. 5. Sensitivities of Repairing Cost and Work Expenditure depend on the MTTR-s

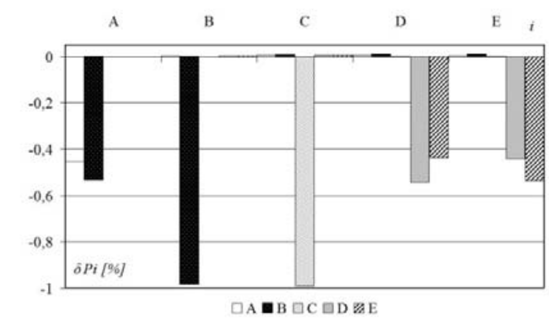

Fig. 6. Sensitivities of Probabilities of Staying in Repairing States depend on the MTBF-s

1g. A significant reduction of repairing cost can only be achieved by reducing of MTTR of ***type D*** failure (Fig. 5).

1h. A significant reduction of work expenditure can only be achieved by reducing of MTTR of failure type E (Fig. 5).

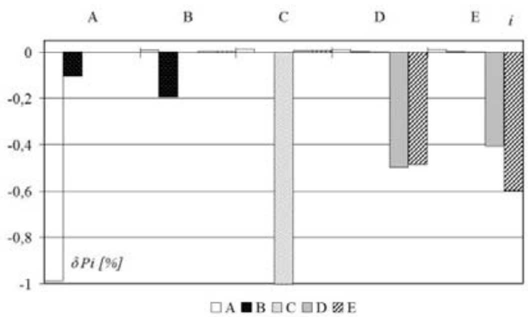

Fig. 7. Sensitivities of Probabilities of Staying in Repairing States depend on the MTTR-s

4.2. Generalized Conclusions about the Proposed Method

2a. The decision proved to be correct that not any significant failures were modelled by only one, ***Type C*** failure.

The Figures 2 - 5 show that output parameters of the system have the minimal sensitivities to MTBF and MTBF of **Type C** *failure. At the same time, Figures 6 and 7 illustrate that the changing of MTBF and MTTR of* **Type C** *failure practically do not have effects on probabilities of staying in other repairing states.*

2b. The proposed simulation method can be used to analyze sensitivity of maintenance processes.

The results of the simulation show that the investigated maintenance system contains complex interconnections and effects. It is possible that using "linear approach", these relationships cannot be depicted and decision makers draw erroneous conclusions and make inaccurate decisions.

2c. The proposed simulation method can be developed.

For example, knowing production proceeds, the sensitivity of revenue can be determined. (Due to the lack of data we have waived in this study.)

5. Closing Remarks

This study applied a stationery Markovian model for simulation-based sensitivity analysis of the investigated maintenance process. The discussed methodology is able to increase the efficiency of maintenance system. The results of analysis can be used for determining the optimal repairing and developing tasks of the operating system from point of view of maintenance management. The proposed method is capable of supporting the decision making in maintenance management. In this paper possibility of use of simulation-based sensitivity analysis was demonstrated by a case study.

The Author's planed prospective scientific research related to this field of applied mathematics and decision making in maintenance management includes the study of methodologies of mathematical tools for example Monte-Carlo Simulation-based analysis of maintenance systems and processes.

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