DECISION SUPPORT PROGRAM FOR SMALL HYDROPOWER PLANTS OPERATION

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There are not so many references regarding the systemic approach of a small hydropower plant behavior between the resource and energy produced and commercial operation; maybe due to the fact that the problem became important just lately, the commercial relevance of the operation being considered only after the restructure of the energetic system, the emergence of energy markets and private investment in the field.

This is the reason why a software which allows to determine the hourly production of energy for the next day is extremely important for the operator of a small hydropower plant. The paper presents the mathematical model of such a program, the necessary data and the results for a study case.

Key words: small hydropower plant, decision program, DAM, CMBC

1. Introduction

Under the conditions of market economy and of public funds severe limitation, the opportunity to promote the construction of hydraulic or hydroelectric works consists in convincing the investors that these are generating profit.

Compliance with the sustainable development requirements at (national, European) macro-system level, through the efficient use of the natural resources, as well as by providing the necessary energy under safety, security and availability conditions, requires meeting a set of objectives that will completely change the society (mentalities, conceptions, relations, technologies), at all levels and in all fields.

Solving the problem from the systemic approach perspective actually involves identifying an optimum implementation technology, in terms of value (financial), for the generation function of the hydrological system, geographically limited by the area of the geographic basin supplying the water stream.

Associated mathematical model enables, during the system initiation phase, the simulation of the system structure parameterization and quantitative dimensioning, constantly correlated with the financial source needed to be allotted and with the business projection in terms of investment return.

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During its design preparing phase, the mathematical model allows reviewing the alternatives in a shorter real time and choosing the most advantageous solutions in terms of the cost-benefit analysis, with regard to:

- the equipping solution of small hydropower plants: type, size and number of hydro-units, depending on the minimum water turbining capacity under optimum efficiency conditions, the installation coefficient etc.
- storing capacity of compensation basin and of loading chambers for the power plants located downstream, in case of cascade development scheme solution (distributed power plants)
- optimum solving of resource restrictions (compensation flow, sanitary flow, downstream uses) but of energy, as well (capacity and level of network voltage, network reliability, users’ behavior in terms of energy)
- efficient solutions regarding data collection, transmission and processing method
- optimized solutions regarding the location of works depending on their influences on the system (availability, ground conditions, distance to the network etc.)
- sensitivity analysis of system performances to changes of certain variables or groups of variables (flow – energy – operational mode; constructive parameters – power parameters – payback period, etc.)
- facilitating the development of the analysis alternatives, providing data necessary in order to forecast the financial resources and the benefits, the performances and consequences of the different alternatives, in short time; thus simplifying the task of decision makers in choosing the final design solution.

During the commercial operation phase, the mathematical model provides data for its efficient operation, correlating the exogenous variables acting on the system (hydrological data), the endogenous variables (power output), the system defining variables (instantaneous unit hydrograph - HUI, consumption curve, turbines efficiency curves, the objective variables (load curve) and the restriction variables defining the system favorable conditions, thus enabling:

- carrying out of the daily offer, on hourly basis, in order to turn to account the power generated on the day ahead market (DAM)
- conducting of forecasts on longer periods for the centralized market of bilateral contracts (CMBC)
- development and management of direct supply contracts to dedicated consumers (retail market)
- efficient dispatching
- invoicing.
2. Mathematical model used in order to determine the generation function of a river basin

Resource (water) processing subsystem is a hydroelectric development of small hydropower plant type (SHPP), of which simplified physical model consists in a compensation basin (small hydraulic reservoir), a pressure headrace providing the conveyance of inflows from the compensation basin towards the small hydropower plant, and the actual hydropower plant, where the hydraulic energy is transformed into electric energy. Simplified scheme of the physical model, completed with the sizes used to define the mathematical model, is shown in fig. 1.

Figure notations are defined as follows:

- $i$ – index of the study period digitization stage, $i = 1, ..., N$;
- $j$ – index corresponding to the digitization of allowable state and decision range, $j = 1, ..., M$;
- $Q_{ui}$ – flow entered into the compensation basin;
- $Q_{ai}$ – active storage existing in the compensation basin upon the stage completion;
- $Q_{a}^{\text{max}}$ – maximum active storage of the compensation basin;
- $Q_{pi}$ – stage flow capability;
- $Q_{Ti}$ – turbine flow;
- $Q_{\text{inst}}$ – installed flow of the small hydropower plant;
- $Q_{\text{min}}$ – turbines minimum operating flow;
- $P_{ij}$ – achieved hourly power.
Main elements of model are:
- **stage variable** – considered to be the time, and as time horizon – the dispatcher day (DD), digitized with time step equal to the basic discounting range for the DD, $\Delta t = 1$ hour; a number $N = 24$ of stages results,
- **state variable** – considered to be the active storage existing in the compensation basin
- **decision variable** – considered to be the energetic flow (used by turbine), $Q_{Ti}$
- state variable range: $0 \leq Q_{ai} \leq Q_{a}^{\text{max}}$
- decision variable range, $Q_{\text{min}} \leq Q_{Ti} \leq Q_{\text{inst}}$
- number of decision variable digitization steps, $M$, resulting the step value, $\Delta Q_T = (Q_{\text{inst}} - Q_{\text{min}}) / M$
- transformation equation: $Q_{iai} = f(Q_{pij}, Q_{Ti})$ or $Q_{ai} = f(Q_{ai-1}, Q_{ui}, Q_{Ti})$
- objective function: $\max \left\{ F = \sum_{i=1}^{N} f_i(Q_{ai}, Q_{Ti}) \right\}$.

Solution chosen for the study of the SHPP-type hydroelectric development behavior is a simulation - optimization model. During one dispatcher day, the system goes from the initial phase to the final phase through $N = 24$ hourly intervals named stages. Digitization step, $\Delta t = 1$ hour, for the study period (dispatcher day) coincides with the basic discounting interval of the DAM offer, thus accomplishing the proposed objectives accordingly.

**Decision variables** are as follows:

$$ Q_{Tij} = Q_{\text{min}} + \begin{cases} \Delta Q_T, & \text{if } Q_{\text{inst}} > Q_{pij} \geq Q_{\text{min}}, \\ 0, & \text{if } Q_{pij} \leq Q_{\text{min}}, \\ Q_{\text{inst}}, & \text{if } Q_{pij} \geq Q_{\text{inst}}. \end{cases} $$

Transformation equation of the system state is:

$$ Q_{aij} = Q_{aij-1} + Q_{ui} - Q_{Tij}, $$

and the objective function is:

$$ FO_{ij}(Q_{ai-1,j}) = \max_{j=1,\ldots,M} \left\{ Q_{aij} \times C_{ai} + Q_{Tij} \times C_{Ti} \right\} + FO_{i-1,j}(Q_{ai-1,j}), $$

where $C_T$ and $C_a$, in RON/MWh, represent the electricity average hourly selling specific price on the day the offer is prepared, meaning the day before the
dispatcher day. Assumption was made that the energy achieved by using $Q_T$ on every time step, will be sold at the same value as the previous day, respectively $C_T$, and the stock held in the reservoir on the current step, $Q_a$, will be fully used to generate electricity on the next step, therefore it has the value corresponding to this step, respectively $C_a$.

Optimization problem related to system operation can be stated as follows:

Given a system with the evolution described in equation (1), find the decision range: $Q_{Tij}$ that, starting from the initial phase is maximizing the objective function provided by the relation (3), meeting the restrictions for the state variable and decision variable.

3. Steps of computational algorithm

Computational steps for the main alternative, the computational software used in solving the applications was developed for, are shown below.

Step 1. Value $M$ is chosen, representing the number of digitization steps related to the energetic flow variable component $Q_{ij}$ (decision variable). Value of digitization step $\Delta Q_t$ is calculated as follows:

$$\Delta Q_t = (Q_{inst} - Q_{min}) / M, \text{ [m}^3/\text{s]}).$$

(7)

Step 2. Maximum hourly storage capacity $Q_{a}^{max}$ is determined by relation:

$$Q_{a}^{max} = V_{max} / 3600, \text{ [m}^3/\text{s]}).$$

(8)

Step 3. Analysis begins with $i = 1$ and the flow capability of stage $Q_{p1}$ is determined. Provided that the water quantity stored at the beginning and at the end of the analysis period, respectively, is zero (empty basin), the flow capability shall have the following value:

$$Q_{p1} = Q_{a1}.$$ 

(9)

Step 4. Elements defining the system at the end of stage are calculated for every outflow fraction $j\Delta Q_t$, $j = 0,1,...M$:

$$Q_{Tij} = Q_{min} + j\Delta Q_{Tij}$$

$$Q_{a1j} = Q_{p1} - Q_{T1j}$$

$$FO_{1}(Q_{a1j}) = Q_{a1j} \times C_{a1} + Q_{T1j} \times C_{Tij}$$

(10)
Alternatives meeting the restrictions (5): \( Q_{T1j} \geq Q_{p1j}, \ 0 \leq Q_{a1j} \leq Q_{a}^{\text{max}} \) are kept.

Final system states \( Q_{aij}, \ j = 0,1,\ldots, M \), corresponding to the allowed alternatives, along with the values corresponding to the objective function \( FO_1(Q_{aij}) \) are used in order to define the system state during the next stage \( i = 2 \).

**Step 5.** Stage \( i = 2 \) is started and the flow capability \( Q_{p2j} \) is calculated for every value \( Q_{a1j} \) characterizing the final system state upon the previous stage \( i = 1 \):

\[
Q_{p2j}(Q_{a1j}) = Q_{a2} + Q_{a1j}.
\]

(11)

**Step 6.** All the alternatives of getting through stage \( Q_{a2j} \) corresponding to the values, the decision variable \( Q_{Tij} \) may take, are calculated for every alternative \( Q_{p2j}(Q_{aij}) \) defining the initial state of the system in stage \( i = 2 \). There are kept the alternatives meeting the set of restrictions:

\[
Q_{Tij} = Q_{\text{min}} + j \Delta Q, \ Q_{a2j} = Q_{p2j}(Q_{aij}) - Q_{T1j}, \ j = 0,1,\ldots, M.
\]

(12)

Alternative maximizing the objective function is kept for every initial state \( Q_{p2j}(Q_{aij}) \):

\[
FO_2(Q_{aij}) = \max \{ (Q_{a2,j} \times C_{a2} + Q_{T2,j} \times C_{T2}) + FO_1(Q_{ai,j}) \}.
\]

(13)

At the end of stage \( i = 2 \), the optimum alternative of the system state, set by the maximum value of the objective function, corresponding to the allowed turbined flow alternatives, is given by the values of the flows stored in the compensation basin.

**Step 7.** Same steps of stage \( i = 2 \) are taken for all stages \( i = 3,\ldots, 23 \) going through all the sequences described for stage \( i = 2 \).

- Flow capability corresponding to every alternative of the final state \( Q_{ak-1,j} \) is determined as follows:

\[
Q_{pk}(Q_{ak-1,j}) = Q_{ak} + Q_{ak-1,j}, \ k = 3,\ldots, 23
\]

(14)

- All the potential passes through the stage corresponding to decision variable \( Q_{Tkj} \), where \( j = 0,1,\ldots, M \), are determined for every alternative \( Q_{pk}(Q_{ak-1,j}) \), being calculated the values of \( Q_{akj} \):

\[
Q_{ak,j} = Q_{pk}(Q_{ak-1,j}) - Q_{Ti,j}
\]

(15)
For every initial state of stage \( i = k \), it is chosen the alternative \( Q_{akj} \) that maximizes the objective function:

\[
FO_k(Q_{ak-1,j}) = \max \left\{ Q_{ak,j} \times C_{ak} + Q_{Tk,j} \times C_{Tk} \right\} + FO_{k-1}(Q_{ak-1,j})
\]  

(16)

**Step 8.** For stage \( i = 24 \), considering the condition regarding the processing of the whole quantity of water entered into the system during the study period \( (Q_{a24} = 0) \), the computational elements defining the system at the end of stage shall be:

\[
\begin{align*}
Q_{T24,j} &= Q_{p24,j}(Q_{p23,j}) \\
Q_{a24,j} &= 0 \\
FO_{24,j}(Q_{a24,j}) &= \max \left\{ Q_{T24,j} \times C_{T24} \right\} + FO_{23}(Q_{a23,j})
\end{align*}
\]  

(17)

**Step 9.** It is determined the optimum alternative of getting the system from the initial state \( Q_{a0} = 0 \) to the final state \( Q_{a24} = 0 \) during one dispatcher day \( (i = 1, \ldots, 24) \), under the influence of the inflow \( Q_{ui} \) reaching the compensation basin:

\[
Q_{uzd} = \sum_{i=1}^{N} Q_{ui}.
\]  

(18)

System optimal operating value is achieved by identifying, for each stage, the energetic flow alternative \( Q_{Tij} \) corresponding to the final state \( Q_{Tij} \) that ultimately led to the obtaining of the objective function maximum value:

\[
Q_{Tij}^{opt} = \sum_{i=1}^{N} Q_{Tij}.
\]  

(19)

Generated power output on a dispatcher day is determined:

\[
E_{ZD} = \sum_{i=1}^{N} P_i = K \times Q_{TZD}^{opt} = \sum_{i=1}^{N} (K \times Q_{Ti}^{opt}).
\]  

(20)

Income achieved by programming the optimized operation of the small hydropower plant is given by relation:

\[
V_{ZD} = \sum_{i=1}^{N} P_i \times C_{Hi}
\]  

(21)

**Step 10.** Analysis of results.
4. Case study

Consider a system consisting in a hydroelectric development of small hydropower plant type. Producer has entered into a sale-purchase agreement of negotiated price.

The goal is to determine the optimum operating conditions for the dispatch day (DD), to prepare the DAM offer and the CMBC offer necessary in order to turn to account the electricity estimated to be produced, under efficiency and accepted risk conditions.

The following are considered known parameters:

- Parameters of the hydroelectric development are:
  \( Q_{inst} = 1.5 \text{ m}^3/\text{s} \), \( Q_{min} = 0.5 \text{ m}^3/\text{s} \), \( P_{inst} = 1.8 \text{ MW} \), \( V_{BC} = 5400 \text{ m}^3 \), namely: installed flow, minimum energetic flow, installed power, active storage of the compensation basin.

- Daily average inflows transmitted by INHGA (National Institute of Hydrology and Water Management) within the daily forecast for the intake section and the compensation flow are:
  \( Q_{af} = 1.25 \text{ m}^3/\text{s} \) and \( Q_s = 0.35 \text{ m}^3/\text{s} \);
  wherefrom a daily average flow likely to be caught into the compensation basin results \( Q_u = 0.9 \text{ m}^3/\text{s} \).

- Value coefficients \( C_f \) and \( C_a \) are set based on the hourly average prices achieved the day before the dispatched day and are filled in table 1.

- Consumption curve, including the hourly average powers required by the consumer for the dispatcher day. Hourly power \( P_c \) and hourly flow \( Q_c \) necessary in order to generate this energy are shown in table 1.

In order to solve the problem, system parameters are determined and its evolution throughout one dispatcher day is computed hourly, the step of the IBD - the base settlement interval. It is to be mentioned that the minimum flow of every stage is given within the computational software by the flow \( Q_c \) necessary to cover the consumption curve. Results achieved are shown in table 2, and the power curves are shown in figure 2, respectively: the total hourly production of electricity (a), from which for DAM offer (b) and for CMBC offer (c).
### Table 1

Value coefficients, hourly power and flow as per the energy required by consumer

<table>
<thead>
<tr>
<th>Hours</th>
<th>$C_T$ [RON/MWh]</th>
<th>$C_o$ [RON/MWh]</th>
<th>$P_t$ [MW]</th>
<th>$Q_o$ [m³/s]</th>
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<tr>
<td>00-01</td>
<td>180</td>
<td>159</td>
<td>0.768</td>
<td>0.640</td>
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<td>01-02</td>
<td>159</td>
<td>140</td>
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<td>0.625</td>
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<tr>
<td>02-03</td>
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<td>0.75</td>
<td>0.625</td>
</tr>
<tr>
<td>03-04</td>
<td>119</td>
<td>119</td>
<td>0.75</td>
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</tr>
<tr>
<td>04-05</td>
<td>119</td>
<td>159</td>
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<td>0.580</td>
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<tr>
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<td>169.5</td>
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</tr>
<tr>
<td>06-07</td>
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<td>0.750</td>
</tr>
<tr>
<td>07-08</td>
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<td>204</td>
<td>0.93</td>
<td>0.775</td>
</tr>
<tr>
<td>08-09</td>
<td>204</td>
<td>206</td>
<td>0.996</td>
<td>0.830</td>
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<tr>
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<td>204.5</td>
<td>1.05</td>
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</tr>
<tr>
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<tr>
<td>16-17</td>
<td>194</td>
<td>239</td>
<td>0.84</td>
<td>0.700</td>
</tr>
<tr>
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<td>0.84</td>
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<td>179</td>
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### Table 2

Offer for the energy markets

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<th>IBD</th>
<th>DAM</th>
<th>CMBC</th>
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</tr>
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<td>2</td>
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</tr>
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<tr>
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<tr>
<td>IBD</td>
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<td>CMBC</td>
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</tr>
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<tr>
<td>23</td>
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<td>48.96</td>
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</tr>
<tr>
<td>Total</td>
<td>4.68</td>
<td>984.6</td>
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</table>

**a)** total hourly production of electricity

**b)** hourly production of electricity for DAM offer
b) hourly production of electricity for CMBC offer

Fig. 2. Graphic representation of the energy market offer: total hourly production of electricity (a), from which on DAM (b) and on CMBC (c)

5. Results and conclusions

Following mentions can be made in terms of the achieved results:

DAM offer consists in quantity-price pairs for each hourly interval. Risk analysis is closely related to the way the offer price for each IBD (hour) is set, this influencing the acceptance in the order of merit; a high "appetite" for risk allows turning to account the energy at the marginal price the market sets under the trading process. Contract energy traded through CMBC is turned to account at the contract price, therefore there is no need for it to be mentioned in the offer.

The case study has demonstrated the application of the mathematical model about the preparation of the offers for the two electricity markets: DAM and CMBC.

Opportunities to apply the presented methodology address the development of existing small hydropower plants trading on DAM the energy estimated to be produced, but also the small hydropower plants under design stage. Therefore it can be considered to represent a decision support program both for the operator and for the designer of a small hydropower plant, as well.

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