OBSERVER BASED ESTIMATION OF CHAOTIC DYNAMICS WITH EXPONENTIAL NONLINEARITIES

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The observability of a system with respect to its possible outputs is usually evaluated using the rank criterion of the observability matrix or the observability indices. In addition, this paper uses the properties of the differential equations characterizing real-time electrical circuits which base their nonlinearities on the exponential function. Interplay between mathematical tools and practical considerations is done through observability coefficients, observability matrices and high order sliding mode observers for the Colpitts chaotic system, representative for such nonlinearities.

Keywords: observability coefficients, chaotic oscillators, observability matrices, exponential nonlinearities, high order sliding mode observers.

1. Introduction

The aim of the present work is to contribute to the field of synchronization between chaotic systems [1], targeting the chaos-based cryptography [2]. The existent approaches evaluate the observability of a system with respect to its possible outputs using the rank criterion of the observability matrix or the observability indices [3].

In addition to the known approaches, this paper uses the properties of the systems described by differential equations characterizing real-time electrical circuits which base their nonlinearities on the exponential function, mostly consequence of the Ebers-Moll model [4]. Due to its rapid growth or decay to zero, together with the bifurcation parameters [5], which are according to an evolution in bounded space, and time constants, due to the reactive elements in such circuits [6], the exponential function as element of the observability matrix enables or not the practical recovery of the original state space from a single data series. For exemplification we choose the Colpitts oscillator (1), with bifurcation parameters $A = 2g/Q$ and $B = -1/Q,$ due to its popularity in the field.

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$$\begin{cases} \dot{x}_1 = A(-e^{-x_2} + x_3 + 1) \\ \dot{x}_2 = Ax_3 \\ \dot{x}_3 = -\dfrac{1}{2A}(x_1 + x_2) + Bx_3 \end{cases} \tag{1}$$

Since the seminal work of Kennedy [7, 8] the Colpitts system was given much attention in the literature. Its dynamics is studied from the perspective of chaos theory in [9]. The two-way synchronization between chaotic oscillators is analyzed in [10] in terms of the coupling coefficient required to get full synchronization. A secure communication algorithm based on hybrid chaotic dynamics implying the one-way synchronization between two Colpitts chaotic oscillators is proposed in [11].

The remaining of the paper is organized as follows: Section 2 presents some useful concepts and algorithms existent in the literature, Section 3 highlights the main results of the paper, and some conclusions and perspectives are formulated in Section 4. The simulation results are obtained with Matlab-Simulink R2013a and the scripts and the models are available on http://www.elcom.pub.ro/~od/

2. Concepts of observability and observability indices

Some theoretical tools from [3] and used in this work are particularized for the tridimensional Rössler system (2) from [12], with bifurcation parameters (a_1, a_2, a_3) and $x_j = x_j(t); j = \{1,2,3\}$.

$$\begin{cases} \dot{x}_1 = -x_2 - x_3 \\ \dot{x}_2 = -x_1 + a_1 x_2 \\ \dot{x}_3 = a_2 + x_3(x_1 - a_3) \end{cases} \tag{2}$$

The state $y = x_3$ is chosen as output of the system. The output is sometimes called observable in the remaining of the paper. The measurement function is the row vector $Y = [0 \quad 0 \quad 1]$ the unity element indicating the observable. The transformation map between the original state space $R^3(x_1, x_2, x_3)$ and the differential embedding viewed by the variable $y = x_3$, $R_j^3(z_1, z_2, z_3)$, where $(z_1, z_2, z_3) = (y, \dot{y}, \ddot{y})$ is $\Phi_j : R^3(x_1, x_2, x_3) \to R_j^3(z_1, z_2, z_3)$, with $j = 3$, given in (3).

$$\Phi_3 : \begin{cases} z_1 = x_3 \\ z_2 = \dot{x}_3 = a_2 + x_3(x_1 - a_3) \\ z_3 = \ddot{x}_3 = \dot{x}_3(x_1 - a_3) + \dot{x}_1 x_3 \\ \qquad = a_2(x_1 - a_3) + x_3(x_1 - a_3)^2 - x_3(x_2 + x_3) \end{cases} \tag{3}$$

If Φ_j is injective and its Jacobian matrix has a determinant which is non-null on the entire state space the original system is observable at any point in space when the measurement function is given by the variable j, as stated by [3].

In order to define the Jacobian matrix for Φ_j to obtain observability matrices with accurate interpretation regardless of the linearity or nonlinearity of the transformation map engendered by the considered output, the Lie derivative $L_{f_j}(x) = \sum_{k=1}^{m} f_k \frac{\partial f_j(x)}{\partial x_k}$ of the $j-th$ component of the vector field f and higher-order derivatives $L_{fi}^{n}(x) = L_{fi}(L_{fi}^{n-1}(x))$ are used. The dimension of the map is m. The observability matrix M_3 defined in (4) for system (2) with $y = x_3$ must be full column rank, i.e. $rank(M_3) = 3$, to ensure full observability [3].

$$M_3 = \begin{bmatrix} 0 & 0 & 1 \\ x_3 & 0 & x_1 - a_3 \\ a_2 + 2x_3(x_1 - a_3) & -x_3 & (x_1 - a_3)^2 - x_2 - 2x_3 \end{bmatrix} \tag{4}$$

Matrix M_3 depends on the dynamical variables with the determinant $|M_3| = -x_3^2$, thus being singular at $x_3 = 0$, i.e. it does not allow an inverse. So, the Rössler system is not fully observable when its output is $y = x_3$.

The degree of observability can be locally quantified by the observability indices defined as the absolute value of the rate between the minimum eigenvalue of the matrix $[M_j^T M_j]$ and the maximum eigenvalue of the previously mentioned matrix, computed at the any moment. Nevertheless, in this work, it is not the mean of these local indices that will be computed, but the observability coefficients obtained from the structure of the considered system. The steps of the algorithm particularized in [13] for the Rössler map, are briefly presented below.

(a) Write the fluency matrix. In the Jacobian matrix of the system, replace each constant element by 1, each nonlinear term by $\bar{1}$. Elements which are neither constant, nor variable, will be replaced by 0.

(b) Define by $C_{1,i}$ the column vectors corresponding to each state of the studied system, i corresponding to the measured state variable x_1, x_2 or x_3. A value of 1 indicates the state which was chosen to reconstruct the dynamics of the system. Thus, $C_{1,1}=[1 \quad 0 \quad 0]^T$, $C_{1,2}=[0 \quad 1 \quad 0]^T$, $C_{1,3}=[1 \quad 0 \quad 0]^T$. Matrices $H_{1,i}$ are obtained by replacing the diagonal element of the fluency matrix corresponding to each variable by a dot and multiplying each of its rows by the corresponding element in $C_{1,i}$. Count the number $p_{1,i}$ of linear elements and the number $q_{1,i}$ of nonlinear elements in $H_{1,i}$.

(c) Replace the dot in $H_{1,i}$ by 0, 1 or $\overline{1}$, according to the fluency matrix, and transpose $H_{1,i}$. Count the number of non-null elements of each row, defining the new column vectors $C_{2,i}$.

(d) Obtain the matrices $H_{2,i}$ by replacing each non-null element of $[H_{1,i}]^T$ by a dot and the rest of the elements by their corresponding elements in the fluency matrix multiplied by the corresponding element of the column vector $C_{2,i}$. Count the number $p_{2,i}$ of linear and the number $q_{2,i}$ of nonlinear elements in $H_{2,i}$.

(e) Compute the observability coefficients with formula (5), where $p_{1,i}+q_{1,i}=1$, if $p_{1,i}=0$ and $p_{2,i}+q_{2,i}=1$, if $p_{2,i}=0$.

$$\eta_i=\frac{1}{2}\left[\frac{p_{1,i}}{p_{1,i}+q_{1,i}}+\frac{q_{1,i}}{(p_{1,i}+q_{1,i})^3}+\frac{p_{2,i}}{p_{2,i}+q_{2,i}}+\frac{q_{2,i}}{(p_{2,i}+q_{2,i})^2}\right] \tag{5}$$

3. Main results

Observability coefficients for the Colpitts oscillator

According to the algorithm in [13] previously described in Section 2, we compute the observability coefficients for the Colpitts oscillator (1). The Jacobian matrix and the fluency matrix for the Colpitts oscillator are expressed in (6).

$$J=\begin{bmatrix} 0 & Ae^{-x_2} & A \\ 0 & 0 & A \\ -\frac{1}{2A} & -\frac{1}{2A} & B \end{bmatrix} \rightarrow F=\begin{bmatrix} 0 & \overline{1} & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{6}$$

Matrices $H_{1,i}$ and the numbers of their linear and nonlinear elements:

$$H_{1,1}=\begin{bmatrix}\bullet & \bar{1} & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix};H_{1,2}=\begin{bmatrix}0 & 0 & 0\\ 0 & \bullet & 1\\ 0 & 0 & 0\end{bmatrix};H_{1,3}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 1 & 1 & \bullet\end{bmatrix}; \quad (7)$$

$$\rightarrow p_{1,1}=1, p_{1,2}=1, p_{1,3}=2; \quad q_{1,1}=1, q_{1,2}=0, q_{1,3}=0;$$

The transposed matrices $[H_{1,i}]^T$ and the row vectors $C_{2,i}$ are expressed in:

$$[H_{1,1}]^T=\begin{bmatrix}0 & 0 & 0\\ \bar{1} & 0 & 0\\ 1 & 0 & 0\end{bmatrix};[H_{1,2}]^T=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 1 & 0\end{bmatrix};[H_{1,3}]^T=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 0 & 0 & 1\end{bmatrix}; \quad (8)$$

$$\rightarrow C_{2,1}=[0 \quad 1 \quad 1]^T; C_{2,2}=[0 \quad 0 \quad 1]^T; C_{2,3}=[1 \quad 1 \quad 1]^T;$$

Consequently the new matrices $H_{2,i}$ and the corresponding numbers of linear and nonlinear elements $p_{2,i}, q_{2,i}$, respectively, are given in (9).

$$H_{2,1}=\begin{bmatrix}0 & \bar{1} & 1\\ \bullet & 0 & 1\\ \bullet & 1 & 1\end{bmatrix};H_{2,2}=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 0 & \bullet & 1\end{bmatrix};H_{2,3}=\begin{bmatrix}0 & \bar{1} & \bullet\\ 0 & 0 & \bullet\\ 1 & 1 & \bullet\end{bmatrix}; \quad (9)$$

$$\rightarrow p_{2,1}=4; p_{2,2}=3; p_{2,3}=2; q_{2,1}=1; q_{2,2}=0; q_{1,3}=1;$$

Eventually, the observability indices are calculated in (10). We recall that an observability index which equals unity indicates that the investigated system is fully observable from the corresponding variable.

$$\eta_1=0.73; \eta_2=1; \eta_3=0.89; \quad (10)$$

Full reconstruction of the Colpitts' dynamics when its output is x_2.

According to the computation of the observability coefficients from (10), choosing the second state of (1) as output, the system is observable at any point in the state space. The corresponding transformation map is given in (11).

$$\Phi_2:\begin{cases} z_1=x_2\\ z_2=\dot{x}_2=Ax_3\\ z_3=\ddot{x}_2=A\dot{x}_3=-\frac{1}{2}(x_1+x_2)+ABx_3\end{cases} \quad (11)$$

The non-null determinant $|M_2|=0.5\cdot A$ of the observability matrix guarantees that it is nonsingular, allowing an inverse. Thus, system (11) can be solved and the estimated states of (1) are in (12). This estimation is valid as a consequence of the linearity of Φ_2.

$$M_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & A \\ -1/2 & -1/2 & AB \end{bmatrix} \rightarrow \hat{X} = \begin{bmatrix} -\hat{z}_1 + 2B\hat{z}_2 - 2\hat{z}_3 \\ \hat{z}_1 \\ \hat{z}_2 / A \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (12)$$

A four order sliding mode observer [14] is adapted in (13) to estimate $\hat{Z} = [\hat{z}_1 \quad \hat{z}_2 \quad \hat{z}_3]^T$, with $M = 10^5$ and $E_2 = \dot{z}_4 = -0.5Az_2e^{-z_1} - z_3 + Bz_4$. The additional state $\hat{z}_4 = \dot{\hat{z}}_3$ was added in order to avoid chattering in the $\hat{Z}$ estimates.

$$\begin{cases} \dot{\hat{z}}_1 = v_1 = \hat{z}_2 - 5M^{1/4}\left|\hat{z}_1 - y\right|^{3/4} sign(\hat{z}_1 - y) \\ \dot{\hat{z}}_2 = v_2 = \hat{z}_3 - 1.5M^{1/3}\left|\hat{z}_2 - v_1\right|^{2/3} sign(\hat{z}_2 - v_1) \\ \dot{\hat{z}}_3 = v_3 = \hat{z}_4 - 3M^{1/2}\left|\hat{z}_3 - v_2\right|^{1/2} sign(\hat{z}_3 - v_2) \\ \dot{\hat{z}}_4 = E_2 - 1.1M sign(\hat{z}_4 - v_3) \end{cases} \qquad (13)$$

For parameters $(g,Q) = (4.46,1.38)$ and initial conditions $(x_1,x_2,x_3)|_{t=0} = (0.1,0.8,0.9)$ the recovery of the original states x_1 and x_3 of system (1) is shown in Fig.1, for observer (13) initialized at $(z_1,z_2,z_3)|_{t=0} = (0.3,0.4,\ 0.8,0.2)$.

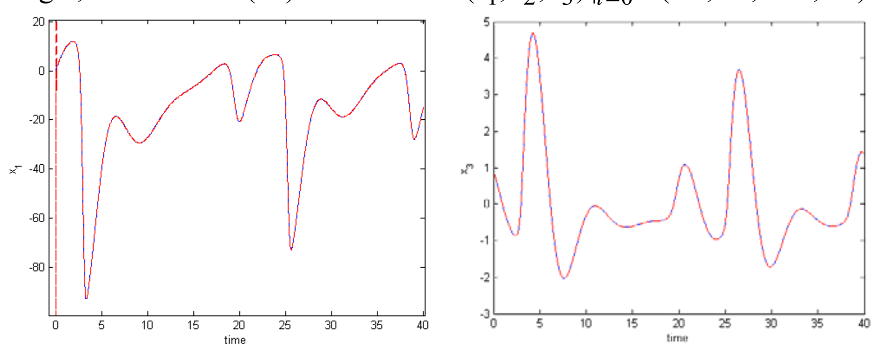

Fig. 1. The first (left) and the third (right) state of the Colpitts oscillator when the output is its second state. Original signals in solid line, estimated signals in dashed line.

The Colpitts dynamics observed from the perspective of its third state

When the output of system (1) is x_3, the available information are in (14). The observability matrix for system (14), given in (15), has the determinant $|M_3| = 0.25A^{-1}e^{-x_2}$, thus being singular for $e^{-x_2} = 0$. So, system (14) has multiple solutions or none at all whenever the signal x_2 satisfies $e^{-x_2} = 0$.

$$\Phi_3 : \begin{cases} z_1 = x_3 \\ z_2 = \dot{x}_3 = -\dfrac{1}{2A}(x_1 + x_2) + Bx_3 \\ z_3 = \ddot{x}_3 = -0.5B(x_1 + x_2)A^{-1} + (B^2 - 1)x_3 + 0.5e^{-x_2} - 0.5 \end{cases} \tag{14}$$

$$M_3 = \begin{bmatrix} 0 & 0 & 1 \\ -0.5A^{-1} & -0.5A^{-1} & B \\ -0.5BA^{-1} & -0.5BA^{-1} - 0.5e^{-x_2} & B^2 - 1 \end{bmatrix} \tag{15}$$

The equation $e^{-x_2} = 0$ implies $x_2 = -\infty$, which cannot be true, neither in theory, nor in simulation, much less in analog implementation. Chaotic systems have an evolution which is bounded in space. See [15] for an application where the boundedness of chaotic evolutions is exploited to simplify an adaptive controller structure, removing explicit knowledge of the nonlinearities of the systems involved from the controller. Second, in simulation, the maximum value reached by the variable x_2 is much smaller, so that $e^{-x_2} \neq 0$, at any point in the state space. In analog circuitry x_2 is the voltage drop on a capacitor, which cannot reach infinity. Consequently, system (14) can be solved, M_3 being nonsingular, and solutions are given in (16).

$$\begin{cases} \hat{x}_1 = -2B\hat{z}_2 + \ln(2) + \ln(\hat{z}_1 - B\hat{z}_2 + \hat{z}_3 + 0.5) + 2B\hat{z}_1 \\ x_2 = -\ln(2) - \ln(\hat{z}_1 - B\hat{z}_2 + \hat{z}_3 + 0.5) \\ \hat{x}_3 = \hat{z}_1 \end{cases} \tag{16}$$

The argument of the natural logarithm has to be greater than zero in order to obtain only real values, in (16). So,

$$2(\hat{z}_1 - B\hat{z}_2 + \hat{z}_3 + 0.5) > 0 \Leftrightarrow e^{-x_2} > 0 \tag{17}$$

The estimates from (16) are obtained in simulation in Fig. 2. The bifurcation parameters are $(g, Q) = (4.46, 1.38)$, initial conditions are $(x_1, x_2, x_3)|_{t=0} = (0.1, 0.8, 0.9)$ and $(z_1, z_2, z_3)|_{t=0} = (0.3, 0.4, \ 0.8, 0.2)$. Although condition (17) is, from a theoretical point of view, always fulfilled, in simulation, due to truncations and rounding specific to computation, e^{-x_2} is not positive over the entire domain. So, although the estimation of e^{-x_2} and $\hat{x}_1 + \hat{x}_2$ is pretty accurate as it can be observed on Fig. 2 (right), when e^{-x_2} is asymptotically close to zero, the signals $\hat{x}_1$ and x_2 cannot be correctly estimated as seen in Fig. 2 (left). Due to the dependence that exists between the measured variable x_3 and the unknown state x_2, i.e. $\dot{x}_2 = Ax_3$, another possible approach in order to estimate the dynamics of the transmitter from its output would be to integrate the measured variable x_3 and use the settings for the measured state x_2 described above. The integration constant is very difficult to obtain in practice. Therefore, this is not a reliable solution.

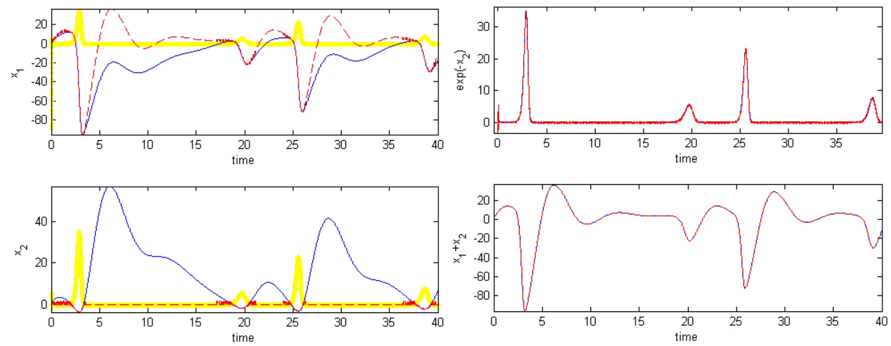

Fig. 2. The third state as output. Original signals in solid line, estimated signals in dashed line. In bold $\exp(-x_2)$. Left: x_1 and $\hat{x}_1$ (top), x_2 and $\hat{x}_2$ (bottom). Right: $\exp(-x_2)$ and $\exp(-\hat{x}_2)$ (top), x_1+x_2 and $\hat{x}_1+\hat{x}_2$ (bottom).

Approximate recovery of Colpitts' dynamics observed by its state x_1

The investigator does not know the other two states, either directly, or through the derivatives of the output $y = x_1$. The embedding of the original dynamics seen from the perspective of this output is expressed in (18). The corresponding observability matrix is given in (19).

$$\Phi_1 : \begin{cases} z_1 = x_1 \\ z_2 = \dot{x}_1 = A(-e^{-x_2} + x_3 + 1) \\ z_3 = \ddot{x}_1 = A\dot{x}_2 e^{-x_2} + A\dot{x}_3 = -\dfrac{1}{2}(x_1 + x_2) + A^2 x_3 e^{-x_2} + ABx_3 \end{cases} \tag{18}$$

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & Ae^{-x_2} & A \\ -1/2 & -1/2 - A^2 x_3 e^{-x_2} & A(Ae^{-x_2} + B) \end{bmatrix} \tag{19}$$

The elements of the matrix M_1 depend on the unknown states x_2 and x_3, which cannot be recovered from the knowledge of the output x_1 and its derivatives. As Ae^{-x_2} rapidly decays to zero, with increasing x_2, the matrix (19) can be rewritten as in (20), which is nonsingular with the determinant $|M_1| = A/2 \neq 0$. Nevertheless, the recovery of the information is not complete, due to the regions of the state space where the approximation $Ae^{-x_2} = 0$ does not hold. The solution $\hat{X} = (M_1^{aux})^{-1}\hat{Z}$ is valid due to the linearity of the new map.

$$M_1^{aux} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & A \\ -1/2 & -1/2 & AB \end{bmatrix} \rightarrow \hat{X} = \begin{bmatrix} \hat{z}_1 \\ -\hat{z}_1 + 2B\hat{z}_2 - 2\hat{z}_3 \\ \hat{z}_2 / A \end{bmatrix} \tag{20}$$

Results of the estimation of the states of system (1) with $\hat{X} = (M_1^{aux})^{-1}\hat{Z}$, are presented in Fig. 3. The initial conditions are the same as for previous results. The shifts of $2AB$, respectively 1, were removed, given that they are well known by the legal receiver and the channel is noise free.

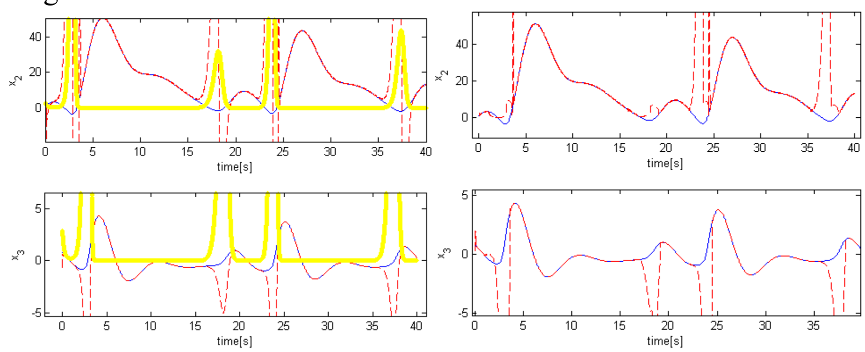

Fig. 3. The states x_2 and x_3 of the Colpitts oscillator when $y = x_1$ for $(g,Q) = (4.46, 1.38)$. Approximation of $A\exp(-x_2)$ to zero (left) and estimation with the gradient algorithm (right). Originals in solid line, estimated in dashed line, Ae^{-x_2} in bold.

Comparing the estimations obtained by approximating $A\exp(-x_2)$ to zero (left) and the estimations using the gradient algorithm (right), one can observe that supplementary difficulties appear when exponential nonlinearities are neglected.

4. Conclusions

One of the applications for chaotic systems is chaos-based encryption. The structure of the transmitter is well known, and the key is generally constituted by the bifurcation parameters and the initial conditions of the system. The synchronization between the transmitter and the receptor is essential in (secret) communications. The output of the transmitter must be chosen so that it ensures, at the receiving end, the most accurate estimation of its dynamics. The Colpitts oscillator was considered, being representative for the chaotic circuits which base their nonlinearity on the exponential function. When the scalar data series corresponding to its second state, the output of the transmitter, is the only available to the receiver, he can accurately estimate the original dynamics. This was proven by computation of the observability coefficients, also by sliding mode observers. The exponential function enables or not the practical recovery of the original state space from a single data series, represented by the first or the third state of the Colpitts transmitter.

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