SORET AND HEAT SOURCE EFFECTS ON MHD FLOW OF A VISCOUS FLUID IN A PARALLEL POROUS PLATE CHANNEL IN PRESENCE OF SLIP CONDITION

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The heat and mass transfer characteristics of the unsteady viscous, incom-pressible and electrically conducting fluid flow past a porous parallel plate channel are taken into account in this paper. Based on the pulsatile flow nature, exact solution of the governing equations for the fluid velocity, temperature and concentration are obtained by using two term perturbation technique. The expressions of skin friction, Nusselt number and Sherwood number are also derived. The numerical values of fluid velocity, temperature and concentration are displayed graphically whereas those of shear stress, rate of heat transfer and rate of mass transfer at the plate are presented in tabular form for various values of pertinent flow parameters.

Keywords: MHD fluid, Porous medium, Heat source, Parallel plate channel.

Nomenclature:

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>$h$</td>
<td>distance between two parallel plates</td>
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<tr>
<td>$B_o$</td>
<td>uniform magnetic field</td>
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<tr>
<td>$C$</td>
<td>species concentration</td>
</tr>
<tr>
<td>$C_f$</td>
<td>skin-friction coefficient</td>
</tr>
<tr>
<td>$C_h$</td>
<td>species concentration at the heated wall</td>
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<tr>
<td>$C_c$</td>
<td>species concentration at the cold wall</td>
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<td>$c_p$</td>
<td>specific heat at constant pressure</td>
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<td>$Da$</td>
<td>Darcy parameter</td>
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<tr>
<td>$D_m$</td>
<td>chemical molecular diffusivity</td>
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<td>$Gm$</td>
<td>Solutal Grashof number</td>
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<tr>
<td>$Gr$</td>
<td>thermal Grashof number</td>
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<td>$g$</td>
<td>acceleration due to gravity</td>
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<td>$U$</td>
<td>A scaled velocity</td>
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<td>$Sc$</td>
<td>Schmidt number</td>
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<td>$Sh$</td>
<td>Sherwood number</td>
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<td>$H$</td>
<td>non-dimensional heat source</td>
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<tr>
<td>$j_w$</td>
<td>mass flux</td>
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<td>$K$</td>
<td>permeability of porous medium</td>
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<td>$K^*$</td>
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<td>$Kr$</td>
<td>non-dimensional chemical reaction parameter</td>
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<td>$K_T$</td>
<td>thermal conductivity of the fluid</td>
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<td>$M$</td>
<td>magnetic parameter</td>
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<td>$Nu$</td>
<td>Nusselt number</td>
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<td>$n$</td>
<td>frequency of oscillation</td>
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<td>$Pr$</td>
<td>Prandtl number</td>
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<td>$Q_o$</td>
<td>dimensional heat source</td>
</tr>
<tr>
<td>$q_w$</td>
<td>heat flux</td>
</tr>
<tr>
<td>$\omega$</td>
<td>A scaled frequency</td>
</tr>
<tr>
<td>$\phi$</td>
<td>A scaled concentration</td>
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1. Introduction

The study of MHD flows have stimulated considerable interest due to its important physical applications in solar physics, meteorology, power generating systems, aeronautics and missile aerodynamics, cosmic fluid dynamics and in the motion of Earth’s core, Cramer and Pai [1]. In a broader sense, MHD has applications in three different subject areas, such as astrophysical, geophysical and engineering problems. In light of these applications, free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike, has been studied by Cheng and Minkowycz [2]. The effect of slip condition on MHD steady flow in a channel with permeable boundaries has been discussed by Makinde and Osalusi [3]. Makinde and Mhone [4] investigated on the heat transfer to MHD oscillatory flow in a channel filled with porous medium. Venkateswarlu et al. [5-8] studied the heat and mass transfer effects on unsteady radiative MHD natural convective flow past an infinite vertical porous plate. Recently Turkyimazoglu and Pop [9] investigated analytically Soret and heat source effects on the unsteady radiative MHD free convection flow from an impulsively started infinite vertical plate.

The heat transfer enhancement is one of the most important technical aims for engineering systems due to its wide applications in electronics, cooling systems, post accident heat removal, fire and combustion modelling, development of metal waste from spent nuclear fuel, next-generation solar film collectors, heat exchangers technology, applications in the field of nuclear energy and various thermal systems. Sparrow and Cess [10] were one of the initial investigators to consider temperature dependent heat absorption on steady stagnation point flow and heat transfer. Ishak [11] worked mixed convection boundary layer flow over a
horizontal plate with thermal radiation. Analytical solutions for hydromagnetic free convection of a particulate suspension from an inclined plate with heat absorption were presented by Ramadan and Chamkha [12]. Venkateswarlu et al. [13-16] investigated the Soret and chemical reaction effects on the radiative MHD flow from an infinite vertical porous plate.

The following strategy is pursued in the rest of the paper. Section two presents the formation of the problem. The analytical solutions are presented in section three. Results are discussed in section four and finally section five provides a conclusion of the paper.

2. Formation of the problem

We consider the unsteady laminar slip flow of an incompressible, viscous and electrically conducting fluid through a channel with non-uniform wall temperature bounded by two parallel plates separated by a distance \( h \). The channel is assumed to be filled with a saturated porous medium. A uniform magnetic field of strength \( B_0 \) is applied perpendicular to the plates. The above plate is heated at constant temperature. It is assumed that there exist a homogeneous chemical reaction of first order with constant rate \( K \) between the diffusing species and the fluid. Initially i.e. at time \( t \leq 0 \), both the fluid and plate are at rest and at uniform temperature \( T_0 \). Also species concentration within the fluid is maintained at uniform concentration \( C_0 \). Geometry of the problem is presented in Fig. 1. We choose a Cartesian coordinate system \((x, y)\) where \( x \) – lies along the centre of the channel, \( y \) – is the distance measured in the normal section such that \( y = h \) is the channel’s width as shown in the figure below. Under the usual Bousinesq approximation, the equations governing the flow can be written as (see, Adesanya and Makinde [17]):

![Geometry of the problem](image_url)

Continuity equation:
\frac{\partial v}{\partial y} = 0 \quad (1)

Momentum equation:
\frac{\partial u}{\partial t} = \frac{1}{\rho} \frac{dp}{dx} + v \frac{\partial^2 u}{\partial y^2} + g \beta_f (T - T_0) + g \beta_c (C - C_0) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K} u \quad (2)

Energy equation:
\frac{\partial T}{\partial t} = \frac{K_f}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{Q}{\rho c_p} (T - T_0) \quad (3)

Concentration equation:
\frac{\partial C}{\partial t} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m K_f}{T_m} \frac{\partial^2 T}{\partial y^2} - K_s^* (C - C_0) \quad (4)

Assuming that slipping occurs between the plate and fluid, the corresponding initial and boundary conditions of the system of partial differential equations for the fluid flow problem are given below

\begin{align*}
u = \phi_1 \frac{du}{dy}, \quad T = T_0, \quad C = C_0 \quad \text{at} \quad y = 0 \\
u = \phi_2 \frac{du}{dy}, \quad T = T_1 + \varepsilon (T_1 - T_0) \exp (int), C = C_1 + \varepsilon (C_1 - C_0) \exp (int) \quad \text{at} \quad y = a 
\end{align*}

where \( n \) – frequency of oscillation and \( \varepsilon \ll 1 \) is a very small positive constant.

we introduce the following non-dimensional variables
\[ \psi = \frac{x}{h}, \eta = \frac{y}{h}, U = \frac{h}{v} u, P = \frac{h^2}{\rho v^2} p, \gamma = \frac{\phi_1}{h}, \sigma = \frac{\phi_2}{h}, \omega = \frac{h^2}{v} n, \tau = \frac{v}{h^2} t, \theta = \frac{T - T_0}{T_1 - T_0}, \phi = \frac{C - C_o}{C_1 - C_o} \quad (6) \]

Equations (2), (3) and (4) reduce to the following non-dimensional form
\[ \frac{\partial U}{\partial \tau} = -\frac{dP}{d\psi} + \frac{\partial^2 U}{\partial \eta^2} + Gr \theta + Gm \phi - \left[ M + \frac{1}{Da} \right] U \quad (7) \]
\[ \frac{\partial \theta}{\partial \tau} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial \eta^2} - H \theta \quad (8) \]
\[ \frac{\partial \phi}{\partial \tau} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial \eta^2} + Sr \frac{\partial^2 \theta}{\partial \eta^2} - K_r \phi \quad (9) \]

Here \( Gr = \frac{g \beta_f (T_1 - T_0) h^4}{v^3} \) is the thermal buoyancy force, \( Gm = \frac{g \beta_c (C_1 - C_0) h^3}{v^3} \) is the concentration buoyancy force, \( M = \frac{\sigma B_0^2 h^2}{\rho v} \) is the magnetic parameter, \( Da = \frac{K}{h^2} \) is the Darcy parameter, \( Pr = \frac{\rho c_p \nu}{K_r} \) is the Prandtl number, \( H = \frac{Q h^2}{\rho c_p v} \) is the heat source parameter, \( Sr = \frac{D_m K_f (T_1 - T_0)}{T_m \nu (C_1 - C_0)} \) is the Soret number, \( Sc = \frac{\nu}{D_m} \) is the Schmidt number and...
\( Kr = \frac{K_r}{\nu} \) is the chemical reaction parameter respectively.

Initial and boundary conditions, in non-dimensional form, are given by

\[
\begin{align*}
U &= \gamma \frac{dU}{d\eta}, \quad \theta = 0, \quad \phi = 0 \text{ at } \eta = 0 \\
U &= \sigma \frac{dU}{d\eta}, \quad \theta = 1 + \varepsilon \exp(i\omega \tau), \quad \phi = 1 + \varepsilon \exp(i\omega \tau) \text{ at } \eta = 1
\end{align*}
\]

(10)

Following Adesanya and Makinde[17], for purely an oscillatory flow we take the pressure gradient of the form

\[ \lambda = -\frac{dP}{d\eta} = \lambda_0 + \varepsilon \exp(i\omega \tau) \lambda_1 \]

(11)

where \( \lambda_0 \) and \( \lambda_1 \) are constants and \( \omega \) is the frequency of oscillation.

Given the velocity, temperature and concentration fields in the boundary layer, the shear stress \( \tau \), the heat flux \( q \), and mass flux \( j \) are obtained by

\[
\begin{align*}
\tau_{in} &= \mu \left[ \frac{\partial U}{\partial \eta} \right], \\
q &= -K_r \left[ \frac{\partial T}{\partial \eta} \right], \\
j &= -D_n \left[ \frac{\partial C}{\partial \eta} \right]
\end{align*}
\]

(12)

In non-dimensional form the skin-friction coefficient \( Cf \), heat transfer coefficient \( Nu \) and mass transfer coefficient \( Sh \) are defined as

\[
\begin{align*}
Cf &= \frac{\tau_{in}}{\rho(v/h)^2}, \\
Nu &= \frac{hq_{in}}{K_r(Tc-To)}, \\
Sh &= \frac{hj_{in}}{D_n(C_1-C_0)}
\end{align*}
\]

(13)

Using non-dimensional variables in equation (6) and equation (12), in equation (13), we obtain the physical parameters

\[
\begin{align*}
Cf &= \left[ \frac{\partial U}{\partial \eta} \right], \\
Nu &= -\left[ \frac{\partial \theta}{\partial \eta} \right], \\
Sh &= -\left[ \frac{\partial \phi}{\partial \eta} \right]
\end{align*}
\]

(14)

### 3. Solution of the problem

We assume the trial solutions for velocity, temperature and concentration of the fluid as (see, Venkateswarlu et al. [18-19] and Siva Kumar et al. [20]):

\[
\begin{align*}
U(\eta, \tau) &= U_0(\eta) + \varepsilon \exp(i\omega \tau) U_1(\eta) + o(\varepsilon^2) \\
\theta(\eta, \tau) &= \theta_0(\eta) + \varepsilon \exp(i\omega \tau) \theta_1(\eta) + o(\varepsilon^2) \\
\phi(\eta, \tau) &= \phi_0(\eta) + \varepsilon \exp(i\omega \tau) \phi_1(\eta) + o(\varepsilon^2)
\end{align*}
\]

(15) (16) (17)

Substituting equations (15) – (17) into equations (7) – (9), then equating the harmonic and non–harmonic terms and neglecting the higher order terms of \( o(\varepsilon^2) \), we obtain

\[
U_0^* \left[ M + \frac{1}{Da} \right] U_0 = -[Gr\theta_0 + Gm\phi_0 + \lambda_0]
\]

(18)
\[ U_1^* = \left[ M + \frac{1}{Da} + i \omega \right] U_1 = -\left[ Gr \theta_l + Gm \phi_l + A_1 \right] \]  

(19)

\[ \theta_0^* - Pr H \theta_0 = 0 \]  

(20)

\[ \theta_l^* - Pr (H + i \omega) \theta_l = 0 \]  

(21)

\[ \phi_0^* - Sc Kr \phi_0 = -Sc Sr \theta_0^* \]  

(22)

\[ \phi_l^* - Sc (Kr + i \omega) \phi_l = -Sc Sr \theta_l^* \]  

(23)

where the prime denotes the ordinary differentiation with respect to \( \eta \).

Initial and boundary conditions, presented by equation (10), can be written as

\[
\begin{align*}
U_0 &= \gamma \frac{dU_0}{d\eta}, \quad U_1 = \gamma \frac{dU_1}{d\eta}, \quad \theta_0 = 0, \theta_1 = 0, \quad \phi_0 = 0, \quad \phi_1 = 0 \quad \text{at} \quad \eta = 0 \\
U_0 &= \sigma \frac{dU_0}{d\eta}, \quad U_1 = \sigma \frac{dU_1}{d\eta}, \quad \theta_0 = 1, \theta_1 = 1, \quad \phi_0 = 1, \quad \phi_1 = 1 \quad \text{at} \quad \eta = 1
\end{align*}
\]

(24)

The analytical solutions of equations (18) – (23) with the boundary conditions in equation (24), are given by

\[
U_0 = A_{22} \exp(-m_5 \eta) + A_{21} \exp(m_5 \eta) + A_b + \frac{A_b \sinh(m_5 \eta)}{\sinh(m_1)} - \frac{A_b \sinh(m_3 \eta)}{\sinh(m_1)} 
\]

(25)

\[
U_1 = A_{39} \exp(-m_6 \eta) + A_{38} \exp(m_6 \eta) + A_{25} + \frac{A_{23} \sinh(m_2 \eta)}{\sinh(m_4)} - \frac{A_{24} \sinh(m_2 \eta)}{\sinh(m_4)} 
\]

(26)

\[
\theta_0 = \frac{\sinh(m_5 \eta)}{\sinh(m_1)} 
\]

(27)

\[
\theta_1 = \frac{\sinh(m_2 \eta)}{\sinh(m_1)} 
\]

(28)

\[
\phi_0 = \frac{A_b \sinh(m_5 \eta)}{\sinh(m_1)} - \frac{A_b \sinh(m_3 \eta)}{\sinh(m_1)} 
\]

(29)

\[
\phi_1 = \frac{A_b \sinh(m_5 \eta)}{\sinh(m_1)} - \frac{A_b \sinh(m_3 \eta)}{\sinh(m_1)} 
\]

(30)

By substituting equations (25) – (30) into equations (15) – (17), we obtained solutions for the fluid velocity, temperature and concentration and are presented in the following form

\[
U(\eta, \tau) = \left[ U_{22} \exp(-m_5 \eta) + U_{21} \exp(m_5 \eta) + U_b + \frac{U_b \sinh(m_5 \eta)}{\sinh(m_1)} - \frac{U_b \sinh(m_3 \eta)}{\sinh(m_1)} \right] + \\
\varepsilon \exp(i\omega \tau) \left[ U_{39} \exp(-m_6 \eta) + U_{38} \exp(m_6 \eta) + U_{25} + \frac{U_{23} \sinh(m_2 \eta)}{\sinh(m_4)} - \frac{U_{24} \sinh(m_2 \eta)}{\sinh(m_4)} \right] 
\]

(31)

\[
\theta(\eta, \tau) = \left[ \frac{\sinh(m_5 \eta)}{\sinh(m_1)} \right] + \varepsilon \exp(i\omega \tau) \left[ \frac{\sinh(m_2 \eta)}{\sinh(m_2)} \right] 
\]

(32)

\[
\phi(\eta, \tau) = \left[ \frac{A_b \sinh(m_5 \eta)}{\sinh(m_1)} - \frac{A_b \sinh(m_3 \eta)}{\sinh(m_1)} \right] + \varepsilon \exp(i\omega \tau) \left[ \frac{A_b \sinh(m_5 \eta)}{\sinh(m_1)} - \frac{A_b \sinh(m_3 \eta)}{\sinh(m_1)} \right] 
\]

(33)
3.1 Skin friction: From the velocity field, the skin friction coefficient at the plate can be obtained, which in non-dimensional form is given by

\[
C_f = \left[ A_2 m_5 \exp(m_5 \eta) - m_5 A_2 \exp(-m_5 \eta) + \frac{A_6 m_5 \cosh(m_5 \eta)}{\sinh(m_5)} - \frac{A_5 m_1 \cosh(m_1 \eta)}{\sinh(m_1)} \right] + e \exp(i \omega) \left[ A_3 \exp(m_6 \eta) - A_3 \exp(-m_6 \eta) + \frac{A_2 m_2 \cosh(m_2 \eta)}{\sinh(m_2)} - \frac{A_2 m_4 \cosh(m_2 \eta)}{\sinh(m_4)} \right]
\]

(34)

3.2 Nusselt number: From the temperature field, we obtained the rate of heat transfer coefficient which is given in non-dimensional form as

\[
Nu = - \left[ \frac{m_1 \cosh(m_1 \eta)}{\sinh(m_1)} \right] - e \exp(i \omega) \left[ \frac{m_2 \cosh(m_2 \eta)}{\sinh(m_2)} \right]
\]

(35)

3.3 Sherwood number: From the concentration field, we obtained the rate of mass transfer coefficient which is given in non-dimensional form as

\[
Sh = \left[ \frac{A_2 m_3 \cosh(m_3 \eta)}{\sinh(m_3)} - A_3 m_3 \cosh(m_3 \eta) \right] + e \exp(i \omega) \left[ \frac{A_2 m_2 \cosh(m_2 \eta)}{\sinh(m_2)} - A_4 m_4 \cosh(m_4 \eta) \right]
\]

(36)

Here \( m_1 = \sqrt{Pr_H}, m_2 = \sqrt{Pr(H + \omega)}, m_3 = \sqrt{ScKr}, m_4 = \sqrt{Sc(Kr + \omega)}, m_5 = \sqrt{M + \frac{1}{Da}}, \)

\[
m_6 = \sqrt{M + \frac{1}{Da}} + i \omega A_1 = ScSr, A_2 = \frac{m_1 A_1}{m_1^2 - m_2^2}, A_4 = 1 + A_2, A_4 = \frac{m_2 A_1}{m_2^2 - m_4^2}, A_6 = 1 + A_4,
\]

\[
A_6 = \frac{Gm A_2 - Gr}{m_2^2 - m_5^2}, A_7 = \frac{Gm A_3}{m_2^2 - m_5^2}, A_8 = \frac{m_2 A_1}{m_2^2 - m_5^2}, A_9 = \frac{Gm A_3}{m_2^2 - m_5^2}, A_{10} = \frac{Gm A_1}{m_2^2 - m_5^2}, A_{11} = A_3 = (A_3 + A_{10}).
\]

\[
A_{12} = \frac{\sigma m_1 \coth(m_1) - 1}{1}, A_{13} = A_3 \left[ \frac{\sigma m_3 \coth(m_3) - 1}{1} \right], A_{14} = A_2 - (A_2 + A_4), A_{15} = 1 + A_4,
\]

\[
A_{16} = \frac{\gamma m_5}{1}, A_{17} = 1 + \gamma m_5, A_{18} = 1 - \sigma m_5, A_{19} = A_4 A_7 \exp(-m_3) - A_5 A_8 \exp(m_3), A_{20} = A_1 A_7 \exp(-m_5) - A_4 A_9 A_{21} = \frac{A_2 A_7}{A_9} - A_{22} = A_{22} - A_4 A_6 A_{23} = \frac{Gm A_4 - Gr}{m_2^2 - m_5^2}, A_{24} = A_3 \left[ \frac{Gm A_5}{m_2^2 - m_5^2} \right],
\]

\[
A_{25} = \frac{\sigma m_2 \coth(m_2) - 1}{1}, A_{26} = \frac{\gamma m_2 A_{23}}{\sinh(m_2)}, A_{27} = \frac{\gamma m_2 A_{24}}{\sinh(m_2)}, A_{28} = A_{26} - (A_{25} + A_{27}), A_{29} = A_{23} \left[ \frac{\sigma m_2 \coth(m_2) - 1}{1} \right],
\]

\[
A_{30} = A_3 \left[ \frac{\sigma m_4 \coth(m_4) - 1}{1} \right], A_{31} = A_2 - (A_{25} + A_{30}), A_{32} = 1 + A_4 A_6 A_{33} = A_3 - A_{34} = 1 + \gamma m_6, A_{35} = 1 + \gamma m_6,
\]

\[
A_{36} = A_{36} A_{34} \exp(-m_6) - A_{36} A_{35} \exp(m_6), A_{37} = A_{28} A_{34} \exp(-m_6) - A_{31} A_{32}, A_{38} = A_{37}, A_{39} = A_{38} - A_{33} A_{38} - A_{32}/A_{32}
\]

4. Results and discussion

In order to obtain the physical significance of the problem, we have plotted the fluid velocity, temperature and concentration for different values of physical parameters. The numerical values of the skin friction coefficient, the rate of heat transfer coefficient and the rate of mass transfer coefficient are presented in tabular...
form for varies values of physical parameters. In the present study following default parameter values are adopted for computations: $\tau = \pi / 2, \lambda = 0.5, Gr = 5, Gm = 5, M = 2, Da = 1, \gamma = 0.1, \sigma = 0.1, Pr = 0.71, H = 5, Sc = 0.78, Sr = 5, Kr = 0.5, \omega = 1, \varepsilon = 0.001$. Hence all the graphs and tables are corresponding to these values unless specifically indicated on the appropriate graph or table.

The effects of thermal Grashof number $Gr$ and solutal Grashof number $Gm$ on the velocity $U$ of the flow field are presented in Figs. 2 and 3. Physically, thermal Grashof number $Gr$ signifies the relative strength of thermal buoyancy force to viscous hydrodynamic force in the boundary layer. Solutal Grashof number $Gm$ signifies the relative strength of species buoyancy force to viscous hydrodynamic force in the boundary layer. A study of the curves shows that thermal Grashof number $Gr$ and solutal Grashof number $Gm$ accelerates the velocity of the flow field at all points. This is due to the reason that there is an enhancement in thermal buoyancy force and concentration buoyancy force.

Fig. 4 shows the variation of fluid velocity $U$ with the Darcy parameter $Da$. The graph shows that an increase in the Darcy parameter increases the fluid flow except at the flow reversal points at the heated wall. Fig. 5 depicts the influence of pressure gradient $\lambda$ on the fluid velocity $U$. It is observed that, the fluid velocity $U$ increases on increasing the pressure gradient $\lambda$. Fig. 6 depicts the influence of magnetic field intensity on the variation of fluid velocity. It is noticed that, an increase in the magnetic parameter $M$ decreases the fluid velocity $U$ due to the resistive action of the Lorenz forces except at the heated wall where the reversed flow induced by wall slip caused an increase in the fluid velocity. This implies that magnetic field tends to decelerate fluid flow.

Figs. 7 and 8 shows the fluid velocity profile variations with the cold wall slip parameter $\gamma$ and the heated wall slip parameter $\sigma$. It is observed that, the fluid velocity $U$ increases on increasing the cold wall slip parameter $\gamma$ thus enhancing the fluid flow. The cold wall slip parameter did not cause any appreciable effect on the heated wall. An increase in the heated wall slip parameter $\sigma$ decreases the fluid velocity minimally at the cold wall and increasing the heated wall slip parameter causes a flow reversal towards the heated wall. It is observed that $\sigma = 0$ corresponds to the pulsatile case with no slip condition at the heated wall in Fig 8.

Figs. 9 to 11, demonstrate the plot of fluid velocity $U$, temperature $\theta$ and concentration $\phi$ for a variety of heat source parameter $H$. It is seen that, the fluid velocity and concentration increases on increasing the heat source parameter whereas temperature decreases on increasing the heat source parameter. This implies that heat source tend to accelerate the fluid velocity and concentration whereas it has a reverse effect on temperature.
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Fig. 2: Influence of Grashof number on velocity profiles.

Fig. 3: Influence of solutal Grashof number on velocity profiles.

Fig. 4: Influence of Darcy parameter on velocity profiles.

Fig. 5: Influence of pressure gradient on velocity profiles.

Fig. 6: Influence of magnetic parameter on velocity profiles.

Fig. 7: Influence of cold wall slip parameter on velocity profiles.
Figs. 12 to 14, shows the plot of fluid velocity $U$, temperature $\theta$ and concentration $\phi$ of the flow field against different values of Prandtl number $Pr$. Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. It is evident, that the velocity $U$ and concentration $\phi$ increases on increasing Prandtl number $Pr$ whereas temperature $\theta$ decreases on increasing $Pr$. Thus higher Prandtl number leads to faster cooling of the plate. This is because radiation and heat source have tendency to reduce fluid temperature.

Fig. 8: Influence of heated wall slip parameter on velocity profiles.

Fig. 9: Influence of heat source parameter on velocity profiles.

Fig. 10: Influence of heat source parameter on temperature profiles.

Fig. 11: Influence of heat source parameter on concentration profiles.
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Fig. 12: Influence of Prandtl number on velocity profiles.  

Fig. 13: Influence of Prandtl number on temperature profiles.

Fig. 14: Influence of Prandtl number on concentration profiles.

Fig. 15: Influence of chemical reaction parameter on velocity profiles.

Fig. 16: Influence of chemical reaction parameter on concentration profiles.

Fig. 17: Influence of Soret number on velocity profiles.
Figs. 15 and 16 demonstrate the effects of chemical reaction parameter $Kr$ on the velocity and species concentration. It is observed that, both velocity $U$ and species concentration $\phi$ decreases on increasing the chemical reaction parameter $Kr$. Figs. 17 and 18 depict the influence of Soret number $Sr$ on the fluid velocity $U$ and species concentration $\phi$ of the flow field. It is noticed that, velocity $U$ and species concentration $\phi$ is found to increases on increasing the Soret number $Sr$.

The nature of velocity $U$ in presence of foreign species such as Hydrogen ($Sc = 0.22$), Helium ($Sc = 0.30$), Water vapour ($Sc = 0.60$), Ammonia ($Sc = 0.78$) is shown in Fig. 19. Physically, Schmidt number $Sc$ signifies the relative strength of viscosity to chemical molecular diffusivity. It is observed that $U$ increases on increasing $Sc$. The flow field enhances the fluid velocity $U$ in presence of heavier diffusing species.

The numerical values of skin friction $Cf$ are presented in tabular form in tables 1 to 3. It is clear that, the skin friction $Cf$ increases on increasing the thermal Grashof number $Gr$, solutal Grashof number $Gm$, Darcy parameter $Da$, pressure gradient $\lambda$, Prandtl number $Pr$, heat source parameter $H$, Schmidt number $Sc$ and Soret number $Sr$ whereas it decreases on increasing the magnetic parameter $M$ and chemical reaction parameter $Kr$ at both cold and heated plates. Skin friction coefficient decreases at the cold plate but increases at the heated plate with an increase in the cold wall slip parameter $\gamma$ and heated wall slip parameter $\sigma$.

The numerical values of the heat transfer coefficient $Nu$ are presented in tabular form in table 4. It is clear that, the Nusselt number $Nu$ increases at the cold plate but decreases at the heated plate on increasing the Prandtl number $Pr$ and heat source parameter $H$. 
Soret and heat source effects on MHD [...] porous plate channel in presence of slip condition

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The numerical values of the mass transfer coefficient $Sh$ are presented in tabular form in table 5. It is clear that, Sherwood number $Sh$ increases at the cold plate but decreases at the heated plate on increasing the Soret number $Sr$. Sherwood number $Sh$ decreases at both cold and heated plates with an increase in the chemical reaction parameter $Kr$. Mass transfer coefficient $Sh$ increases at both cold and heated plates on increasing the Prandtl number $Pr$, heat source parameter $H$ and Schmidt number $Sc$.

**Table 4**

**Effect of Pr and H on heat transfer coefficient**

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**Table 5**

**Effect of Pr, H, Sc, Sr, and Kr on mass transfer coefficient**
Soret and heat source effects on MHD [...] porous plate channel in presence of slip condition 185

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5. Conclusions

From the present investigation the following conclusions can be drawn:

- Cold wall slip parameter, heat source parameter and Soret number are tend to accelerate the fluid velocity whereas magnetic parameter and chemical reaction parameter have a reverse effect on it. Increasing heated wall slip parameter causes a flow reversal towards the heated plate.
- Heat source parameter and Prandtl number are tending to retard the fluid temperature.
- Heat source parameter and Soret number and are tend to accelerate the species concentration whereas chemical reaction parameter has a reverse effect on it.
- Heat source parameter and Soret number are tends to accelerate the skin friction whereas magnetic parameter and chemical reaction parameter have a reverse effect on it at both cold and heated plates. Skin friction decreases at the cold plate but increases at the heated the plate on increasing the cold wall slip parameter and heated wall slip parameter.
- Heat transfer coefficient increases at the cold plate but decreases at the heated plate on increasing the heat source parameter and Prandtl number.
- Mass transfer coefficient increases at both cold and heated plates on increasing the heat source parameter and Schmidt number where as chemical reaction parameter has a reverse effect on it. Soret number accelerates the mass transfer coefficient at the cold plate whereas it has a reverse effect at the heated plate.

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REFERENCES


