

EXPERIMENTAL STUDY OF THE BOGIE VERTICAL VIBRATION - CORRELATION BETWEEN BOGIE FRAME ACCELERATIONS AND WHEELSETS ACCELERATIONS

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The paper presents the correlation between the vertical accelerations in wheelsets and the vertical accelerations in the bogie frame, and the identification of the type of dependence between them. The analysis is performed for two pairs of experimental data series, which include the standard deviation of accelerations measured in the axles boxes and on the bogie frame against the two wheelsets, during circulation at a constant velocity. The correlation characteristics are determined through the scatter plots and based on Spearman correlation coefficient. In order to identify the type of dependency between the standard deviation (STD) of acceleration in wheelsets and the standard deviation of accelerations in the bogie frame, three regression functions were analyzed. The practical conclusion of this research is that the relationship between the STD wheelset accelerations and STD bogie frame accelerations can be described through a second degree polynomial regression curve.

Keywords: bogie, vibration, correlation coefficient, regression analysis

1. Introduction

The dynamic performances in the rail vehicle - ride quality, ride comfort and safety, depends on the vibration behaviour to which it is subjected during running. The vibrations in the railway vehicle are mainly brought about by the track irregularities - track geometric irregularities, irregularities in the rolling surfaces and discontinuities of the rails, which mostly come from the construction imperfections, track exploitation, change in the infrastructure due to the action of the environment factors or soil movements [1]. Running on a track with irregularities will generate vibrations of the wheelsets [2 - 5], which are transmitted to the bogies and the car body, so that the dynamic response of the entire vehicle is affected by the track irregularities [6 - 9].

Recent studies, based on numerical simulations or measured data, have shown that the dynamic response in the vehicle is correlated with the track irregularities [10 - 14], which creates the premises of developing certain

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monitoring methods of the track quality or the condition in the vehicle suspension. For instance, the condition monitoring of the suspension gives a series of benefits to the rail operators. When defects in the operation of the suspension components are detected at an early stage, the further deterioration in vehicle performance is prevented and safety increased. Repairs or timely replacement of the faulty components increases the reliability and availability of the railway vehicle. Last but not least, the costs associated to the vehicle maintenance can be greatly reduced [15 - 19].

Correlation method has been used to point out the connection between the lateral and vertical axle box acceleration and differently processed track geometry parameters based on a real measurement run on straight track [13]. Also, the correlation between the high-frequency vibration of the axle box acceleration and the geometry of the rail welds has been applied to develop an approach for real-time health detection of rail welds [14].

Developing a monitoring method for the condition of the vehicle primary suspension can be based on the correlation between the wheelsets vibrations and the vibrations in the bogie frame. To this purpose, it is initially necessary to know the characteristics of the correlation and the dependence type in the parameters being monitored. Starting from here, the paper is looking into the correlation between two pairs of experimental data series, which include the standard deviation of accelerations (STD accelerations) measured in the axles boxes and on the bogie frame against the two wheelsets, during circulation at a constant velocity. The analysis consists in establishing the characteristics of the correlation – direction and shape, and the association degree and the dependence type between the STD bogie frame acceleration and STD wheelsets acceleration.

2. Measurement bogie frame and wheelsets vibrations

The experimental data processed in this paper have been derived from a Minden-Deutz bogie fitted passenger train, for long and middle traffic. The maximum speed of the vehicle is 140 km/h. The vertical accelerations of the wheelsets and the bogie frame during running on a current track have been measured, on a section in alignment and vertical alignment. Recordings of the accelerations at constant speed 117 km/h for sequences of 20 seconds at the sampling frequency of 2048 Hz were made. The accelerometers were installed on a side of the bogie, with one accelerometer on each axle box and one accelerometer on the bogie frame against each wheelset. The position of the four accelerometers is shown in figure 1.

To perform the experimental determinations, a measurement chain was formed in which they were integrated the components of the measurement, acquisition and processing system for the vertical accelerations – namely four

4514 Brüel & Kjær piezoelectric accelerometers and the set of the NI cDAQ-9174 chassis for data acquisition and the NI 9234 module for acquisition and synthesizing the data from accelerometers, and the NL-602U type GPS receiver for monitoring and recording the vehicle velocity (figure 2).

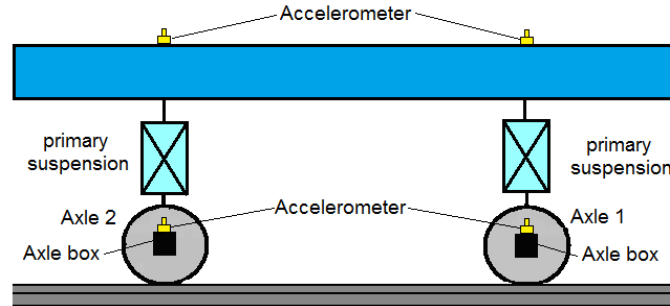


Fig. 1. Mounting the accelerometers on the bogie frame and the axle boxes.

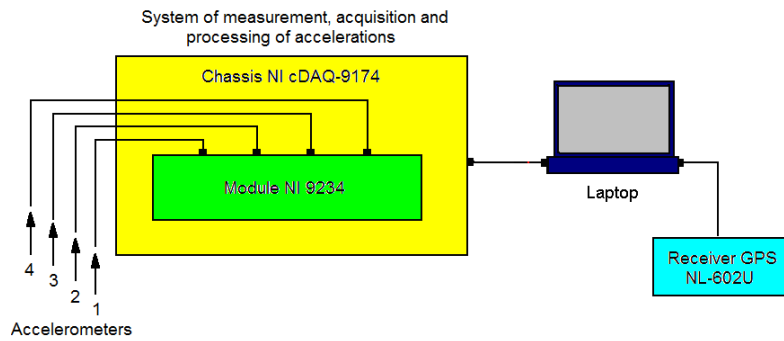


Fig. 2. Scheme of the experimental measurement chain.

3. Analysis of the correlation between bogie frame accelerations and wheelsets accelerations

This section deals with the correlation between the vibrations in the bogie frame and the wheelsets vibrations, by using the data series of the following variables - STD accelerations measured in the axles boxes ($STDa_{w1}$ and $STDa_{w2}$) and STD accelerations measured on the bogie frame against the two wheelsets ($STDa_{b1}$ and $STDa_{b2}$) at velocity of 117 km/h for 30 measuring sequences. We have four data series forming two pairs of data series such as $STDa_{w1}(STDa_{w1,1}, STDa_{w1,2}, \dots, STDa_{w1,n})$ and $STDa_{b1}(STDa_{b1,1}, STDa_{b1,2}, \dots, STDa_{b1,n})$, respectively $STDa_{w2}(STDa_{w2,1}, STDa_{w2,2}, \dots, STDa_{w2,n})$ and $STDa_{b2}(STDa_{b2,1}, STDa_{b2,2}, \dots, STDa_{b2,n})$, for $n = 30$.

Figure 3 features the values of the STD accelerations corresponding to each couple of data series. For the first wheelset, the STD accelerations are scattered between 1.12g and 2.15g, while for the second wheelset, between 1.1g and 2.33g. For the bogie frame, the STD acceleration ranges between 0.65g and

1.05g – above the wheelset 1, and between 0.67g and 1.08g – above the wheelset 2. The dispersion of the STD accelerations is due to the amplitude variability of the track defects alongside. The STD wheelset accelerations is noticed to be circa 2 times higher than the STD bogie frame accelerations.

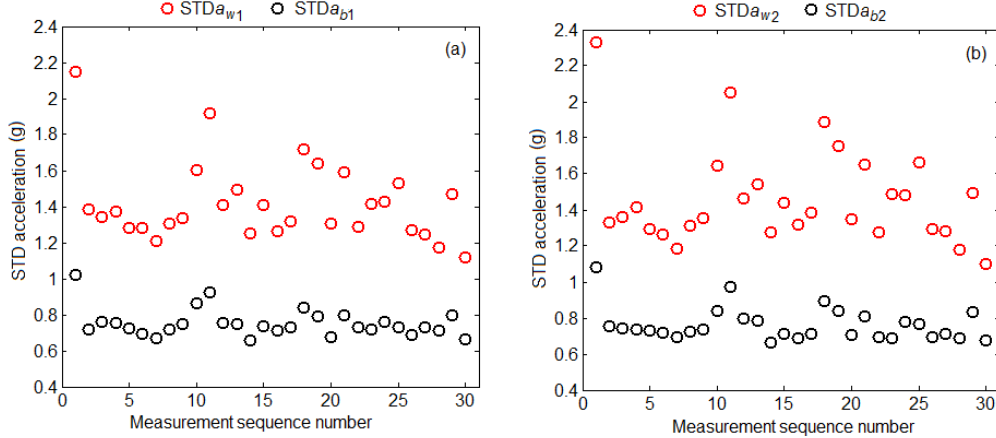


Fig. 3. STD accelerations at the velocity of 117 km/h for 30 measurement sequences: (a) pair of data series $STDa_{w1}$ - $STDa_{b1}$; (b) pair of data series $STDa_{w2}$ - $STDa_{b2}$.

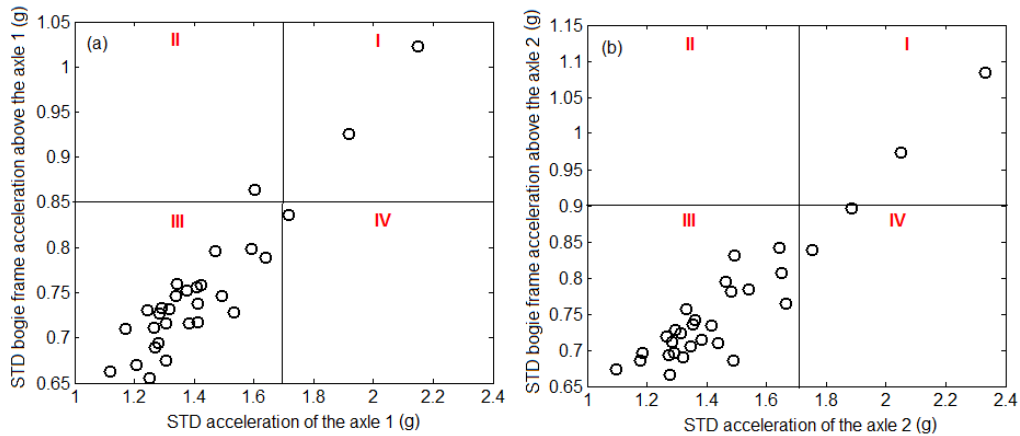


Fig. 4. Scatter plot of STD accelerations: (a) pairs of values ($STDa_{w1,i}$, $STDa_{b1,i}$); (b) pairs of values ($STDa_{w2,i}$, $STDa_{b2,i}$).

In general, the correlation between the data series of two variables has three important characteristics, namely direction, form and association degree.

The direction and the form of the correlation between the STD bogie frame acceleration and the STD wheelset acceleration can be visualized via the scatter plot. Figure 4 shows the scatter plots, in which the value pairs are represented ($STDa_{w1,i}$, $STDa_{b1,i}$) – diagram (a), and ($STDa_{w2,i}$, $STDa_{b2,i}$) – diagram (b), for $i = 1...30$. Depending on the dispersion of the points on the diagrams, the type of correlation between STD wheelset acceleration and STD

bogie frame acceleration can be appreciated. Both the coordination points ($STDa_{w1,i}$, $STDa_{b1,i}$) and the coordination points ($STDa_{w2,i}$, $STDa_{b2,i}$) present a low scatter degree, which indicates that STD wheelset acceleration and STD bogie frame acceleration are correlated. The correlation is positive, since the increase in STD bogie frame acceleration is associated with the increase in STD wheelset acceleration. The placement of the points in the scatter plot can help with the estimation of the type of relation between the two pairs of acceleration series – $STDa_{w1}$ and $STDa_{b1}$, and $STDa_{w2}$ and $STDa_{b2}$, respectively. It is observed that the most of the points are placed in quadrants I and III, and on this basis it can be estimated that there is a linear relation between the acceleration of the wheelset and the acceleration of the bogie frame.

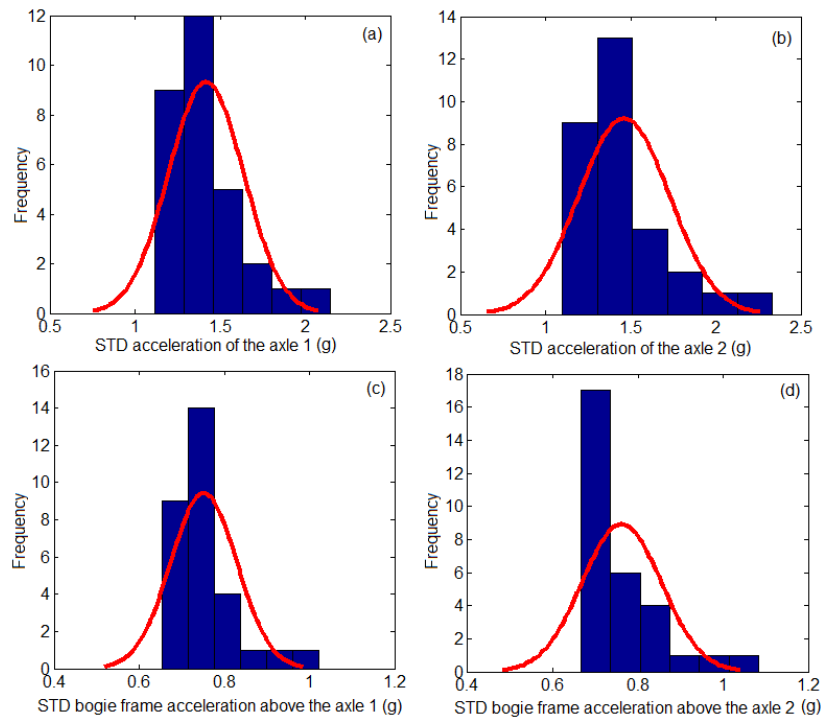


Fig. 5. Verification of hypothesis of normal distribution of the STD accelerations:
(a) for $STDa_{w1}$; (b) for $STDa_{w2}$; (c) for $STDa_{b1}$; (d) for $STDa_{b2}$.

To determine the association degree between the two variables - STD bogie frame acceleration and STD wheelset acceleration, the correlation coefficient needs to be calculated. Since the scatter plot has confirmed that the relation between the two variables has a direction and this relation is linear, the determination of the association degree can be conducted via the Pearson coefficient. First, the hypothesis that at least one of the variables has an approximatively normal distribution has to be verified. As seen in the diagrams in figure 5, this hypothesis is not corroborated for any of the four variables.

In this case, the determination of the association degree between the STD wheelset acceleration and the STD bogie frame acceleration can be conducted through the Spearman coefficient, a non-parametric correlation coefficient. Unlike the Pearson coefficient, in the case of the Spearman coefficient no assumption is made about the normality of the variables. The relation between the two variables does not have to be linear, yet has to have a direction. While Pearson's correlation assesses linear relationships, Spearman's correlation assesses monotonic relationships (whether linear or not).

Spearman correlation coefficient is calculated with the below relation [20]

$$r_s = 1 - \frac{6 \sum_{i=1}^n d_{i,2}^2}{n^3 - n}, \quad (1)$$

where $n = 30$, the number of pairs of values ($STDa_{w1,i}$, $STDa_{b1,i}$) or ($STDa_{w2,i}$, $STDa_{b2,i}$), for $i = 1 \dots n$, and d_i is the difference between the two ranks of each pair of values

$$d_{i1} = \text{rg}(STDa_{w1,i}) - \text{rg}(STDa_{b1,i}), \quad d_{i2} = \text{rg}(STDa_{w2,i}) - \text{rg}(STDa_{b2,i}). \quad (2)$$

The correlation coefficient r_s can take values between -1 and +1. The sign of Spearman correlation coefficient indicates the association direction between the independent variable ($STDa_{w1}$ or $STDa_{w2}$) and the dependent variable ($STDa_{b1}$ or $STDa_{b2}$). For instance, if $STDa_{b1}$ tends to rise when $STDa_{w1}$ increases, Spearman correlation coefficient is positive; if $STDa_{b1}$ tends to decrease when $STDa_{w1}$ goes up, Spearman correlation coefficient is negative. A Spearman correlation coefficient equal to zero means that there is no tendency for $STDa_{b1}$ to increase or decrease when $STDa_{w1}$ goes up. Spearman correlation coefficient rises the more $STDa_{w1}$ and $STDa_{b1}$ become closer to being perfectly monotone functions between them. When $STDa_{w1}$ and $STDa_{b1}$ are entirely monotonously connected, the Spearman correlation coefficient becomes 1. A relation in a perfectly monotonous increase implies that for any two pairs ($STDa_{w1,i}$, $STDa_{b1,i}$) and ($STDa_{w1,j}$, $STDa_{b1,j}$) (for $1 \leq i, j \leq n$ and $i \neq j$), $STDa_{w1,i} - STDa_{w1,j}$ and $STDa_{b1,i} - STDa_{b1,j}$ always have the same sign. A relation in a perfectly monotonous decrease signifies that such differences always have opposite signs.

The calculation of Spearman coefficient has been done for the variables $STDa_{w1}$ and $STDa_{b1}$, and $STDa_{w2}$ and $STDa_{b2}$, respectively, corresponding to the 30 measuring sequences. The results derived are as follows: $r_s = 0.85$, for $STDa_{w1}$ and $STDa_{b1}$, $r_s = 0.82$, for $STDa_{w2}$ and $STDa_{b2}$. These results validate that there is a monotonously increasing relation between the STD wheelset accelerations and the STD bogie frame accelerations.

Once established that the STD wheelset accelerations and the STD bogie frame accelerations are correlated, a further step would be to determine the type of dependence between the two variables. This process means to identify the

regression function between the two variables, and this can be done in the Matlab environment using the Curve Fitting Toolbox.

Starting from the hypothesis that there is a linear connection between the two variables, emphasized in the scatter plot, the linear equation (regression equation) describing this connection is $y = 0.33x + 0.28$, where y is $STDa_{b1}$ and $STDa_{b2}$, while x is $STDa_{w1}$ and $STDa_{w2}$. The corresponding regression lines are shown in the diagrams in figure 6.

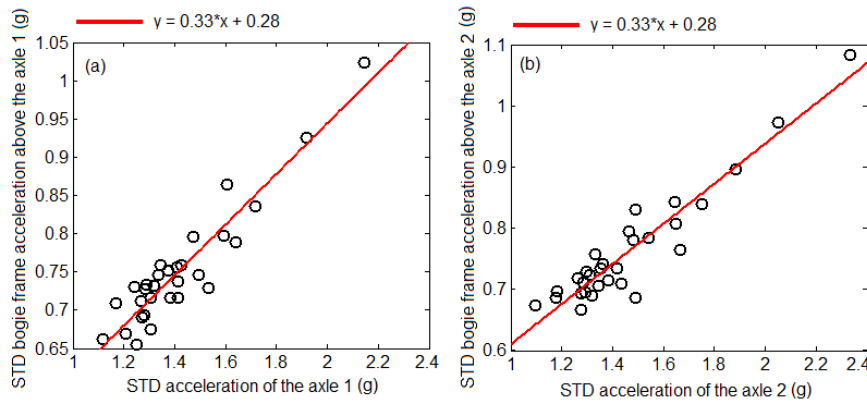


Fig. 6. Scatter plot of STD accelerations and the regression line:
(a) pairs of values ($STDa_{w1,i}$, $STDa_{b1,i}$); (b) pairs of values ($STDa_{w2,i}$, $STDa_{b2,i}$).

However, the use of linear regression involves checking the condition that the residues are normally distributed, with the average of zero value. The scatter plots for residuals are in figure 7, while figure 8 is their histogram on which the curve of the normal scatter has been overlaid. According to diagram (a) in figure 7, residuals are scattered between $-0.06g$ and $0.05g$, with an average of $2.06 \cdot 10^{-16} g$; as seen in diagram (b), residuals are scattered between $-0.08g$ and $0.06g$, with an average of $-8.6 \cdot 10^{-17} g$. The histograms in figure 8 shows that the condition that the residues are normally distributed is not verified.

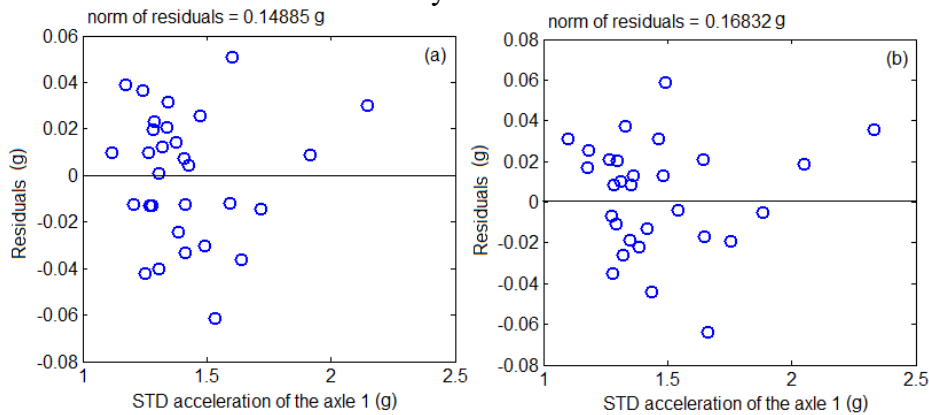


Fig. 7. Scatter plots of residuals for the linear regression:

(a) for the regression line in diagram (a), fig. 6; (b) for the regression line in diagram (b), fig. 6.

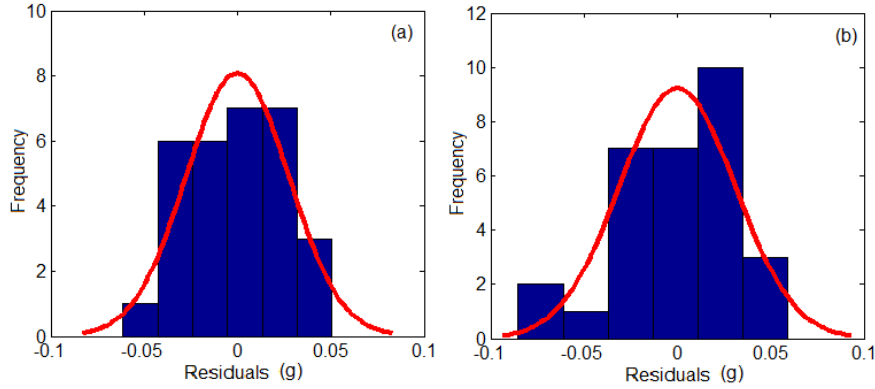


Fig. 8. Verification of hypothesis of normal distribution of the residuals for linear regression: (a) for the regression line in diagram (a), fig. 6; (b) for the regression line in diagram (b), fig. 6.

Further on, a non-linear connection it is considered between the STD wheelset accelerations and the STD bogie frame accelerations, hence the regression curves best describing this relation will be determined. To this purpose, the curve corresponding to the second order polynomial regression equation and the curve for the third order polynomial regression equation will be considered.

The diagrams in figure 9 feature the regression curves described through second degree polynomial equations. For the pair of data series $STDa_{w1}$ - $STDa_{b1}$, the polynomial equation describing the regression curve is $y = 0.13x^2 - 0.083x + 0.6$ (diagram a), where y is for $STDa_{b1}$, and x for $STDa_{w1}$. For the pair of data series $STDa_{w2}$ - $STDa_{b2}$, on the other hand, the polynomial equation describing the regression curve is $y = 0.13x^2 - 0.089x + 0.62$ (diagram b), where y stands for $STDa_{b2}$, and x for $STDa_{w2}$.

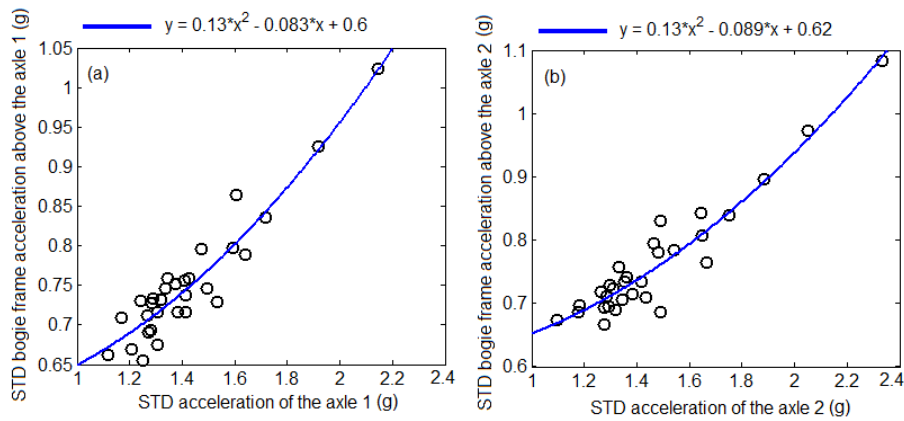


Fig. 9. Scatter plot of STD accelerations and the 2th degree polynomial regression curve: (a) pairs of values $(STDa_{w1,i}, STDa_{b1,i})$; (b) pairs of values $(STDa_{w2,i}, STDa_{b2,i})$.

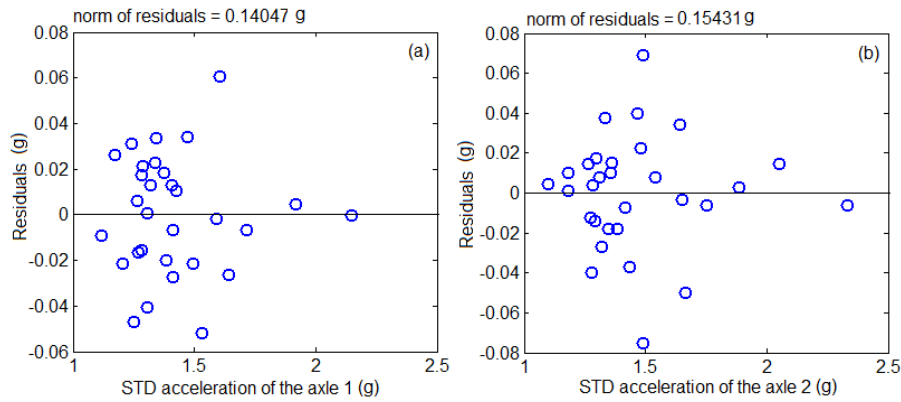


Fig 10. Scatter plots of residuals for the 2th degree polynomial regression: (a) for the regression curve in diagram (a), fig. 9; (b) for the regression curve in diagram (b), fig. 9.

The scatter plot for residuals shows in figure 10. In the diagram (a), residuals can be seen scattered between $-0.05g$ and $0.06g$ with an average of $-6.04 \cdot 10^{-17}g$, while in diagram (b), residuals are scattered between $-0.07g$ and $0.08g$, with an average of $2.12 \cdot 10^{-16}g$.

The diagrams in figure 11 show the regression curves described through the third degree polynomial equations as in $y = 0.035x^3 - 0.039x^2 + 0.18x + 0.47$, for the pair of data series $STDa_{w1}$ - $STDa_{b1}$ (diagram a), and $y = 0.049x^3 + 0.37x^2 - 0.5x + 0.83$, for the pair of data series $STDa_{w2}$ - $STDa_{b2}$ (diagram b), just to mention that y notes $STDa_{b1}$, and $STDa_{b2}$, while x stands for $STDa_{w1}$, and $STDa_{w2}$, respectively. In this case, residuals are scattered between $-0.05g$ and $0.06g$ with an average of $-1.44 \cdot 10^{-17}g$ (see fig. 12, diagram a), and between $-0.07g$ and $0.07g$, with an average of $-1.89 \cdot 10^{-16}g$ (see fig. 12, diagram b).

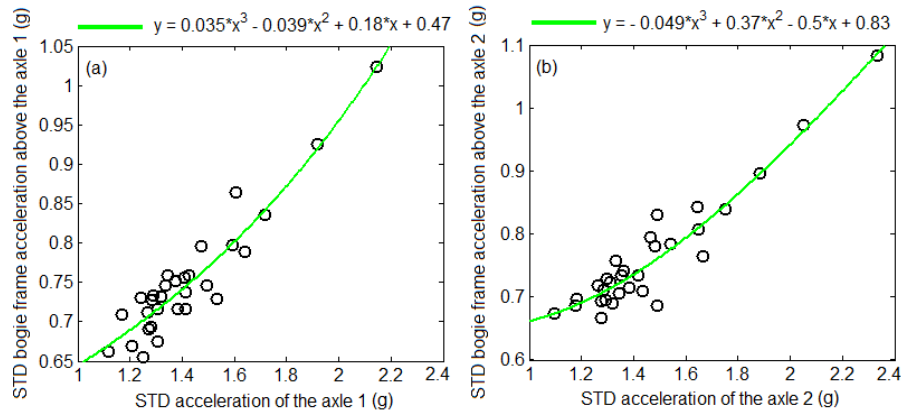


Fig. 11. Scatter plot of STD accelerations and the 3th degree polynomial regression curve:

(a) pairs of values ($STDa_{w1,i}$, $STDa_{b1,i}$); (b) pairs of values ($STDa_{w2,i}$, $STDa_{b2,i}$).

The evaluation of the goodness of regression curve fitting can be conducted via the goodness-of-fit statistics with the help of the Curve Fitting Toolbox [21]:

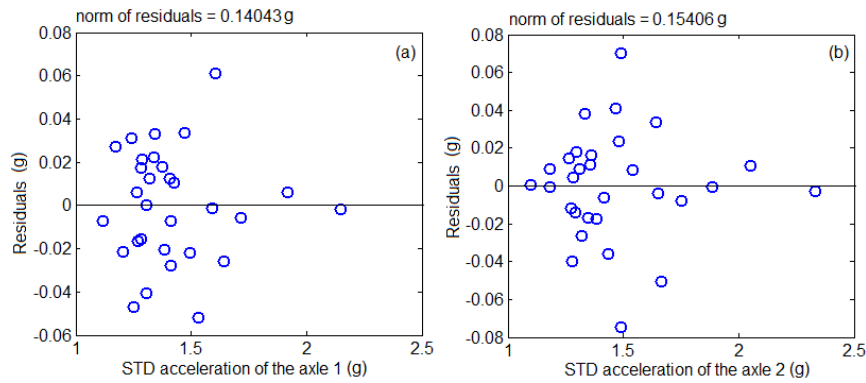


Fig. 12. Scatter plots of residuals for the 3th degree polynomial regression:

(a) for the regression curve in diagram (a), fig. 11;

(b) for the regression curve in diagram (b), fig. 11.

- Sum of Squares due to Error (SSE). This statistic measures the total deviation of the response values from the fit to the response values. A value closer to 0 indicates that the model has a smaller random error component, and that the fit will be more useful for prediction.

- Degrees of Freedom Adjusted R-Square (R-square). This statistic measures how successful the fit is in explaining the variation of the data. R-square can take on any value between 0 and 1, with a value closer to 1 indicating that a greater proportion of variance is accounted for by the model.

- Adjusted R-square. This statistic uses the R-square statistic defined above, and adjusts it based on the residual degrees of freedom. The adjusted R-square statistic can take on any value less than or equal to 1, with a value closer to 1 indicating a better fit. Negative values can occur when the model contains terms that do not help to predict the response.

- Root Mean Squared Error (RMSE). It is an estimate of the standard deviation of the random component in the data. An RMSE value closer to 0 indicates a fit that is more useful for prediction.

Table 1

Values of the statistical parameters for evaluating the goodness of regression curve fitting

	Second degree polynomial regression curve		Third degree polynomial regression curve	
	data series $STDa_{w1}-STDa_{b1}$	data series $STDa_{w2}-STDa_{b2}$	data series $STDa_{w1}-STDa_{b1}$	data series $STDa_{w2}-STDa_{b2}$
SSE [g^2]	0.0197	0.0238	0.0197	0.0237
R-square	0.8880	0.9053	0.8880	0.9056
Adjusted R-square	0.8797	0.8982	0.8751	0.8947
RMSE [g]	0.0270	0.0297	0.0275	0.0302

The values of the statistical parameters defined above, calculated for the second and third order polynomial regression curves are included in Table 1. The values of these parameters are noticed to be basically equal for both polynomial regression curves and that the values for SSE and RMSE are very close to 0, while R-square and Adjusted R-square close to 1. In these circumstances, it can be inferred that the relationship between the STD wheelset accelerations and STD bogie frame accelerations can be described through the second degree polynomial regression curve.

6. Conclusions

This paper investigates the correlation between the vibrations in the bogie frame and the vibrations in wheelset during the railway vehicle circulation at a constant velocity, on a track section in alignment and cross level. To this end, two pairs of experimental data series are used, including the STD accelerations measured in the axles boxes and the STD accelerations measured on the bogie frame against the two wheelsets.

The former stage of the analysis relied on the scatter plot for the pairs of values in the STD wheelset acceleration - STD bogie frame acceleration, corresponding to each pair of experimental data series. It was thus established that STD wheelset acceleration and STD bogie frame acceleration are correlated, and the correlation is positive. Also, a linear relationship between the two accelerations has been estimated.

The association degree between the two variables - STD bogie frame acceleration and STD wheelset acceleration, has been determined on the basis of Spearman correlation coefficient. The values of this coefficient and its sign have pointed to the fact that there is a monotonously increasing relation between the STD wheelset accelerations and STD bogie frame accelerations.

The second stage of the analysis aimed to identify the type of dependence between the two variables. The conclusion to be reached was that the relationship between the STD wheelset accelerations and STD bogie frame accelerations can be described through a second degree polynomial regression curve.

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