CLOSED-LOOP OPTIMAL CONTROL OF A SYSTEM „TROLLEY - PAYLOAD”

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The problem of optimal control of a system „trolley - payload” has been studied in the article. A criterion for the problem was the duration of the trolley acceleration. The problem has been stated in the closed-loop form. It provided the feature of elimination of external disturbances. The control constraints were non-symmetrical. It allowed for utilizing a crane drive in soft-control mode. The problem has been solved for rest initial state of the system. In order to find problem solution modified particle swarm optimization method has been used.

Keywords: optimal control, closed-loop, pendulation oscillation.

1. Introduction

The problem of optimal control of the system „trolley - payload” is very important both for practical and theoretical purposes. The case when the criterion is the duration of a system’s movement is called the time-optimal control problem. The solution of the problem allows for advancing the control systems of overhead, bridge and tower cranes [1-4].

A wide range of methods was used for solving time-optimal control problem: principle maximum [1, 3, 4], variational calculus and dynamical programming [4], controllability function method [5] and others. In the theory context, the time-optimal control problem investigations lead to improvement of optimization methods and their applications.

In the article [2] the non-symmetrical control constraints have been used. It allowed us to obtain the soft-control mode of the system motion. The same constraints will be used in the following research. In addition to that, the problem solution has to be found in the closed-loop form [4]. On the practical level, it provides a significant advantage: closed-loop control eliminates all the impacts of *a priori* unknown external disturbances. Combination of these two characteristics in the problem setting makes it very complicated.

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The used in the article approaches may be applied to other optimal control problems, the theory of stability [6], synthesis of optimal automatics systems [7], etc.

2. Set of the optimal control problem

The system „trolley - payload" is presented in Fig. 1. Such dynamic model is widely known for problems of optimal control of overhead, bridge [1-4] and tower [3] cranes.

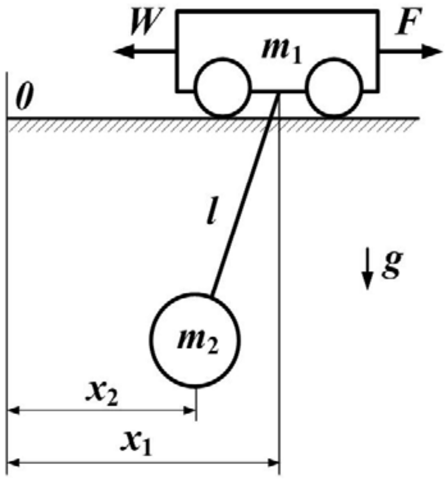


Fig. 1. The dynamic model of the system „trolley - payload"

The equations of the system's motion for the current research are the linearized differential equations:

$$\begin{cases} m_1 \dfrac{d^2 x_1}{dt^2} + m_2 \dfrac{d^2 x_2}{dt^2} = F - W \cdot sign\left(\dfrac{dx_1}{dt}\right); \\ \dfrac{d^2 x_2}{dt^2} + \dfrac{g}{l}(x_2 - x_1) = 0, \end{cases} \qquad (1)$$

where m_1 is the reduced mass of the trolley;
m_2 is the mass of the payload;
x_1 and x_2 are the coordinates of the centers of masses m_1 and m_2, respectively;
g is the acceleration of gravity;
l is the length of the flexible suspension;
F is the driving or braking force acting on the trolley;
W is the reduced force resisting the motion of the trolley.

The reason why we have used linearized differential equations (1) is connected with the angle of the payload deviation. In practice, the real angle of the payload deviation is not bigger than 10…15 degrees. For that range of angles, the error of calculation (in terms of payload position) is equal to 0.5…2.0% (these values are obtained as an expansion of the *sin* and *cos* functions in series in the nonlinear equations of the system (fig. 1) motion).

In the current investigation, the trolley acceleration has been considered. Assume that the velocity of the trolley during that mode does not change its sign. That is why $sign\left(\frac{dx_1}{dt}\right)=1$. Let us denote: x_2-s_T=y_1 and $u=\frac{F-W}{m_1}\cdot\Omega_0^2$. These denotations allow us to rewrite the motion equations (1) in the following form:

$$\begin{cases} \frac{dy_j}{dt}=y_{j+1}; \quad j=\overline{(1,\ 3)}; \\ y_4=u-\Omega^2 y_3, \end{cases} \tag{2}$$

where s_T is the final position of the trolley;
u is a control function (or just control);
y_j is the j-th phase coordinate of the system;
Ω is the natural frequency of the payload about the moving trolley, $\Omega=\Omega_0\sqrt{\frac{(m_1+m_2)}{m_1}}$;
Ω_0 is the natural frequency of the payload as a mathematical pendulum, $\Omega_0=\sqrt{\frac{g}{l}}$.

The initial and the final conditions of the system motion are:

$$\begin{cases} y_1(0)=-s_T;\ y_2(0)=y_3(0)=y_4(0)=0; \\ y_1(T)=0;\ y_2(T)=v_T;\ y_3(T)=y_4(T)=0, \end{cases} \tag{3}$$

where v_T is the final velocity of the trolley (steady velocity of the trolley);
T is the duration of the system's acceleration which is unknown.

Initial conditions (3) mean the state of rest; the final conditions (3) allow for elimination payload oscillations at the moment T.

In the frame of the current study the criterion to minimize is the duration of the trolley acceleration:

$$\int_0^T dt=T\to\min. \tag{4}$$

Minimizing of the time T provides increasing of crane productivity, which is desirable for the tower and overhead cranes in the sea and river ports.

Practical necessity demands to take into account the control u constraints, which is connected with the torque capacity of the trolley drive. Thus, the optimal control u must satisfy the following constraints:

$$u_{\min} \le u \le u_{\max};$$
$$u_{\max} = \frac{F_{\max} - W}{m_1}\Omega_0^2; \tag{5}$$
$$u_{\min} = \frac{-W}{m_1}\Omega_0^2,$$

where $u_{\max}$ and $u_{\min}$ are the upper and the lower boundary of the admissible set of control u;
$F_{\max}$ is the maximum drive force acting on the mass m_1 during acceleration.

It should be noted that the duration of the acceleration T will be found by solving the optimal control problem (2), (3), (5), (6).

3. Soling of the optimal control problem

In order to solve the problem (2), (3), (5), (6) the general form of the solution must be developed. Since the closed-loop control problem is under consideration, the appropriate form of u is as follows:

$$u = u(y,\ A), \tag{6}$$

where y is the vector-function ($y=(y_1, y_2, y_3, y_4)^T$);
A is a vector of some parameters.

From the previous investigation [1-4] it is known that the time-optimal control u switches between $u_{\max}$ and $u_{\min}$. This information allows us to specify the function (6). We may suggest that the time-optimal control u is described with the following formula:

$$u = \begin{cases} u_{\max} \ \ if \ \ \sum_{i=1}^{4} A_i y_i > 0; \\ u_{\min} \ \ if \ \ \sum_{i=1}^{4} A_i y_i \le 0, \end{cases} \tag{7}$$

where A_i is the i-th element of the vector A.

The final position of the trolley s_T may be set by practical reasons. But in the research, we considered parameter s_T as so far unknown argument. That value should minimize criterion (4) as well.

Thus, the problem (2), (3), (5), (6) has been reduced to the finding of the vector A and the parameter s_T. All the calculations were carried out for the parameters of the system that are set in Table 1.

Table 1

Parameters of the system „trolley - payload”

Parameter	Unit	Value
Reduced mass of the trolley, m_1	kg	300
Reduced mass of the payload, m_2	kg	500
Length of the flexible suspension, l	m	5
Steady-state velocity of the trolley, v_T	m/s	0.96
Reduced force resisting the motion of the trolley, W	N	156
Maximum drive force, $F_{\max}$	N	1180

In order to meet the final conditions (3) the terminal criterion was developed:

$$Ter = \|\Delta\| = \sqrt{y_1(T)^2 + (y_2(T) - v_T)^2 + y_3(T)^2 + y_4(T)^2}, \tag{8}$$

where Δ is a vector of phase coordinates deviation from their final (desirable) values (3).

Hence, the complex criterion to minimize is as follows:

$$Cr = \psi \cdot Ter + T \to \min, \tag{9}$$

where ψ is the weight coefficient which reflects the necessity to meet the final conditions (3). In the conducted calculations $\psi=5\cdot10^5$. Such a big value of ψ, which has been established empirically, allows us to find control u that transmits the system to the final state (3) very accurately. It means that the system phase coordinates at the end of the acceleration will be almost equal to its final values (3). In other words, the value of ψ is the good compromise between the accuracy of final conditions (3) satisfaction and the requirement of criterion (4) minimization.

Now we may consider the system as MISO (multiple input, single output) system. The inputs are elements of the vector A and the value of s_T; the output is a value of criterion (9).

Note, that there are only one set of numerical values of A_1, A_2, A_3, A_4, and s_T which minimize criterion (9). Let us suppose that we have found the optimal set of these values. In that case $Cr=T$ since $Ter=0$. Indeed, criterion Ter has a global minimum which is equal to zero. It is achieved when the final condition (3) are completely met. As the final position of the trolley $x_2(T)$ verges towards optimal trolley position the value of criterion (9) reduces. That is why the optimal value of s_T corresponds to the stable movement of the system. Otherwise, it causes the criterion (9) increasing.

In order to minimize the criterion (9) the modification of particle swarm optimization method (ME-PSO) has been used. That optimization technique has been developed and investigated in the article [8]. The parameters of the used method are set in Table 2 (the numerical values of parameters are crucial for algorithm performance. That is why we have shown them in Table 2).

Table 2

Parameters of the used optimization technique (ME-PSO)

Parameter	Value
Number of particles in a swarm	30
Acceleration constants $c_1 = c_2$	1.19
Acceptable rate (*AR*)	0.001
Number of iterations	500

By algorithm performance, we mean optimal problem (9) solution accuracy and duration of calculation. In the study, we have used tested values of ME-PSO parameters (Table 2), which are related to the high algorithm performance.

The components of the vector *A* and the value s_T have been obtained as a result of the optimization problem (9) solving. With ME-PSO algorithm we have calculated such values of A_1, A_2, A_3, A_4, and s_T which minimize the value of criterion (9).

All the results are set in Table 3.

Table 3

Values of the elements of the vector *A* and the value s_T

Parameter	Value
A_1	-4122
A_2	1005
A_3	-4953
A_4	-197
s_T	1.00

The duration of the system's acceleration under time-optimal control equals 2.1 s. Thus, closed-loop time-optimal control problem is solved.

Note, that the problem has been solved in the numerical form. Variations of the system parameters lead to the necessity of the vector *A* and the parameter s_T correction. It requires a new solution to the problem (9). Finding the problem (9) solution does not require much calculation resources. In practice, it may be found by mean of a crane control system (microcontroller with custom-built software).

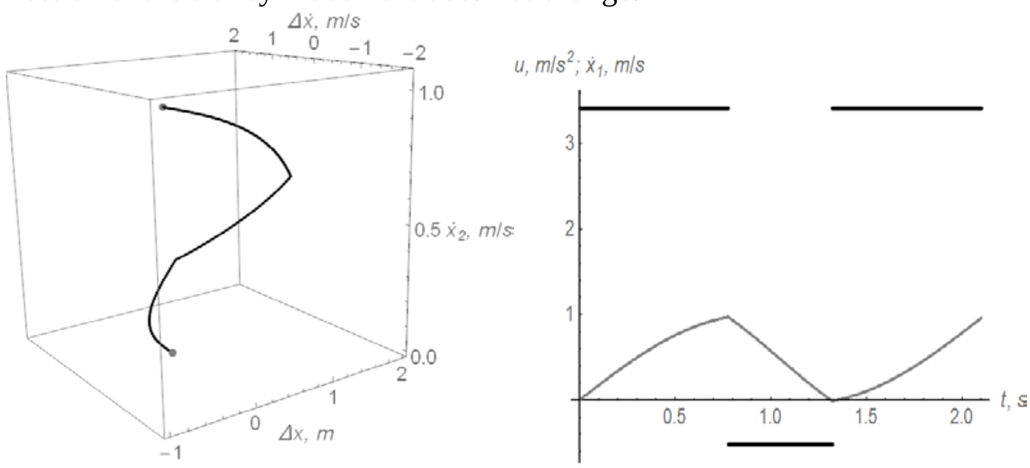
4. Brief results analysis

In order to investigate obtained results, the graphs have been plotted (fig. 2). The curve in the fig. 2 (f) has been built as a parametric plot [9] in 3D-space. It allows for observation of the main system phase coordinates. In fig. 2 (a) gray points denote the initial and final states of the system. Plots in fig. 2 show that all the boundary conditions (3) are satisfied. Hence, further system movement will continue with no payload oscillations. The maximum deviation of coordinates x_1

and x_2 equals 0.5 m, the maximum deviation of their velocities is equal to 0.76 m/s.

A similar effect may be achieved for the deceleration mode of the trolley. Obtained result collectionwise may be exploited for increasing crane productivity. Indeed, there is no need to control the system’s movement in manual mode. It reduces the crane operator utilization and provides the opportunity to design the completely automated crane.

A curve of the trolley velocity is denoted by a gray line in the Fig. 2 (b). It shows that at the end of the acceleration the trolley velocity equals to v_T (0.96 m/s). It also confirms that previous assumption about the constancy of the trolley velocity sign is right: at the moment t=1.3 s the trolley stops but the direction of the trolley movement does not change.



a) b)

Fig. 2. The graphical interpretations of the problem (2), (3), (5), (6) solution: a) 3D phase portrait of the system; b) control function and the trolley velocity

A curve of the control function u (Fig. 2, b) has a switching form. It leads to some undesirable consequences, for instance, high-frequency oscillation of crane metal construction. Another negative factor is high energy losses in the drive. In order to avoid these undesirable features, the constraints to control derivative must be taken into consideration.

Root-mean-square value of the driving force is equal to 882 N or 74.7% of full drive load. It reveals that during trolley acceleration the drive mechanism does not work with full power.

The second period of acceleration (from 0.8 to 1.3 s) the trolley moves with turned off the motor (Fig. 2, b). During that period the motor does not consume power (Fig. 3).

Observing the plot which is shown in fig. 3, one can note two peaks of power. Values of these maximums are approximately equal to each other (1140 W

– at the end of the first period and 1133 W – at the end of the third one). The drive maximum power is 7.5 times as steady-state power. Power overload acts during very short periods of time and does not dramatically affect the motor work.

In order to illustrate one of the advantages of the calculated optimal closed-loop control, we have considered the external stochastic disturbance: a wind rush. It influences both the trolley and the payload, but for the latter, the impact is much bigger. In the calculation, we have used the model of the wind rush, which includes: the speed of the wind, middle transverse section of the payload and air density. The wind rush model (formula) was inserted into the equations (1).

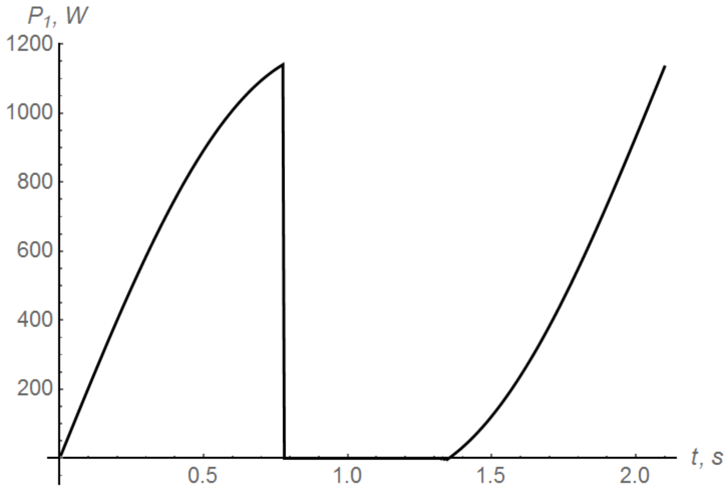


Fig. 3. The curve of the consumed power of the trolley drive

Results of numerical integration of the modified mathematical model we have presented in fig. 4 (it should be noted, in the current section of the article we investigate the optimal control we obtained in the previous one. It was received with no consideration of the external forces).

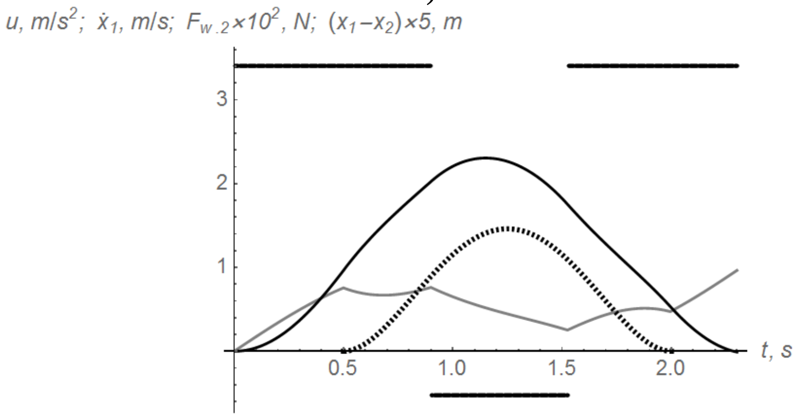


Fig. 4. The curves of the system’s dynamics under optimal control and the wind rush

In Fig. 4 dashed curve represents the wind force which acts to the payload (it is denoted as $F_{w.2}$). Black full line curve relates to the deviation in positions of the payload and the trolley. At the end of the trolley's acceleration that value equals to zero. Hence, further movement of the payload will be with no oscillations.

The trolley velocity at the end of the acceleration equals to v_T. Thus, all the final conditions (3) are met. Such a result has been achieved with variation in the duration of the trolley's drive on-off periods.

Showed example supports the statement about invariant (to the external disturbances) property of the closed-loop optimal control.

5. Conclusions

In the article, the closed-loop problem of optimal control has been solved. Obtained optimal control of the system „trolley - payload” allows for minimizing the duration of the acceleration mode by taking into account the non-symmetrical control constraints.

A novelty of the developed approach is in reducing the initial problem to the non-linear programming problem and the using for its solving advanced PSO-based technique.

Obtained closed-loop optimal control shows the invariant property to the external disturbances (for instance, a wind rush).

Developed in the research approach may be used for calculation of the system „trolley - payload” deceleration.

Further investigation is connected with the generalization of the developed methodology for arbitrary order systems. Important direction to research is the methodology's invariance to the variety of the optimization criteria, control constraints, boundary conditions, etc.

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