ESTIMATION OF DENSITIES IN A CELL TRANSMISSION BASED MODEL

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The purpose of Information in Transportation Systems (ITS) is to provide the drivers with proper and immediate information about eventually events or incidents on the road. If a driver will be notified through ITS system for a considerable increasing of the density in any part, he will be warned that the congestion has occurred due to any incident on the highway. The parameter of density is not possible to be easily measured in short intervals on short length cells. In this paper is used a cell transmission model known as CTM, to predict the density on short intervals (10 seconds) on any particular part of a road only by having the flow measures from two loop detectors on the entry and the end sections of the highway segment. This model incorporates the traffic flow variability on the two measuring points, in e cell transmission model in order to obtain the diagram of the relationship of flow and density, from which then, can principally be estimated the densities of each cell. Beside the flow, we also must do a judgment if the state of the road segment is subjected to free flow or congested flow condition. On a free flow state all the cells are supposed to be in free flow mode and the densities of each of them are below the critical density of the entire highway segment. On a congested flow state all the cells are in congested mode and their densities are above the critical density of the entire highway segment. Evaluation of the model is done for two CTM model variant and it was concluded that the obtained density results of the free flow mode offer more accurate match with the real conditions of the subjected highway segment. Evaluation of the model is done through a Mat Lab code editor and the obtained results are drawn graphically. A mean absolute percentage error is calculated between the modeled densities and the measured densities, that shows an acceptable range of the error between free flow state densities and measured densities, as presented in the results and conclusion section.

Keywords: Cell, density, congestion, freeway, free flow, congestion.

1. Introduction

Macroscopic models describes traffic flow as continuum fluid flow by main characteristic parameters such as, speed, traffic density and traffic flow or traffic volume. The LWR Lighthill–Whitham–Richards (LWR) [1] model is a macroscopic traffic flow model which has evolved from flow conservation law. It is a combination of a conservation law defined via a partial differential equation

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and the relations of parameters described by fundamental diagram [2]. The calculation of every mentioned parameter of the LWR model requires *complex and hard handling of some differential equations solving*. A discrete model that describes on a simply way the evolution of the all traffic flow in every time and distance increment, can be used to calculate traffic parameters avoiding the tedious calculation of derivative equations. The Cell Transmission Model (CTM) is one of these discrete models, which was originally obtained from the second order model of a traffic flow, was firstly developed by *Daganzo in 1994* [3], [4]. For more, on this paper, the prediction of the traffic density on any particular cell is done without having information of traffic flow volumes of each cell. It is needed only to take into consideration of the entering flow to the first cell (first cell) and the exit flow from downstream (last cell), known as inflow and out flow, respectively, and based on the relationship of these two parameters we can predict the density of any intermediate cell [5], [6].

1.1. Fundamentals of the CTM model

In this paper, as we stated on the abstract section, we aim to predict the density parameters on any particular part of a road segment. *CTM* relies on simplification of the fundamental diagram (Fig.1.1) of the flow-density relationship that on this paper will be referred as *FDR* diagram. In order to be quickly linked with the *FDR* of the *LWR* model from which has evaluated cell transmission model, let we have a brief mention of its parameters. In the Fig.1.1 we can distinguish three parameters: maximal flow-capacity, Q_{cap} [veh/s], density, ρ [veh/m], critical density, ρ_{cr} [veh/m], jam density, ρ_j [veh/m], free flow speed, v_f [km/hr] and backward speed or congestion wave speed, w [km/hr]. The idea was to divide this road in cells with same length and the time in units or time steps and update the number of vehicles on each cell every clock tick. The length of cells must be chosen to fulfill the below condition, that with speed on free conditions, during one clock tick, vehicle will prescribe a whole cell. In the beginning, every cell is filled with a number of vehicles that is equal to the storage capacity of cell, or with other words this value is the production of the length of cell, *l[m]* and the jam density ρ_j [veh/m].

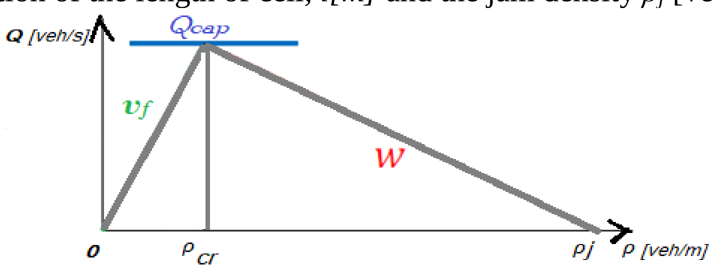

Fig.1.1 Fundamental diagram (FDR)

As it was briefly described on the unit of discretized model, the *CTM* model discretizes the *LWR* model in every time unit choosing a time step Δt and road segment choosing length unit-cells of length *l*[m]where these two parameters are chosen in order to fulfill the condition:

$$l = v_f.T_s \quad and \quad T_s < l/v_f$$

Where:

v_f is the free-flow speed or the average speed that vehicles develop during traveling under free flow conditions, and

T_s is the time step usually in seconds (T≈0.2-15 sec.)

Based on the *CTM* model, the number of vehicles in one cell is described according to a vehicle-conservation equation (1).

$$n_i(k+1) = n_i(k) + y_i(k) - y_{i+1}(k) \quad (1)$$

Where:

$n_i(k+1)$ is the number of vehicles in cell *i* at time step *k+1*

$n_i(k)$ is the number of vehicles in cell *i* at time step *k*,

$y_i(k)$ is the number of vehicles entering from cell *i-1* to *i* during the time *k* and *k+1 and* is the flow that is determined by comparing the sending and receiving flow of cell i-*1* and *i*, respectively. According to *CTM*-first part of the model [3], $y_i(k)$ is assumed to be the smallest of three values listed below:

$n_{i-1}(k)$, the number of vehicles in cell $_{i-1}$ at time *k*,

Q_i, the capacity flow into *i* for time interval *k*,

$N_i(k)$-$n_i(k)$ is the amount of empty space in cell *i* at time *k* (this quantity ensures that the vehicular density on every section of the road remains below density).

As it seems from the above stated conditions, a cell can maximally receive a number of vehicles, which their adding should not exceed the maximal number of vehicles that can be present on it during time *k*, or a number of vehicles equal to the capacity flow or the a number of vehicles that the empty space of cell can accept during time *k*. Now equation for $y_i(k)$ takes the form as in (2):

$$y_i(k) = \min(n_{i-1}(k), Q_i(k), N_i(k) - n_i(k) \quad (2)$$

2. CTM model of a Highway with three cells

2.1.State Space Presentation

If we denote with $\rho_{i,}$(k) the density of a cell (uniform or non-uniform length), instead of the number of vehicles n_i on the a unit length cell, then we can bring equation (1) *to* (3*)* for density $\rho_i(k+1)$ of the cell *i* updated time step in *(k+1)*, where T_s is the discrete time unit in seconds, (Fig.1.1).

$$\rho_i(k+1) = \rho_i(k) + \frac{T_s}{L}(q_i(k) - q_{i+1}(k)) \quad (3)$$

From above equation we see that beside from the value of the density from the previous time step, density of a cell *i* also depends from the inter cell flows on the previous time step [4]. Analyzing a highway partitioned in three cells (for the sake of simply illustration) with an on ramp and an off ramp, by assuming that the belonging cells can be in the free flow either in congested mode the densities on each cell can be written as in equations (4), (5) and (6).

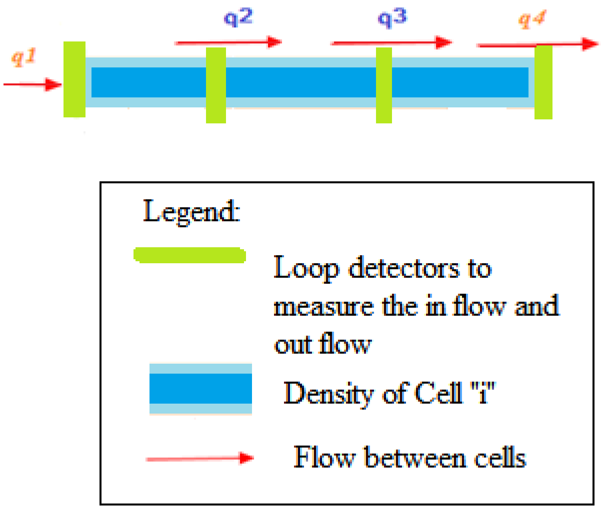

Fig.1.2. Presentation of highway partitioned in cells and inter cell flows

The densities on each cell are:

$$\rho_{i-1}(k+1) = \rho_{i-1}(k) + \frac{T_s}{L}(q_{i-1}(k) - q_i(k) + r(k)) \text{ or}$$
$$\rho_1(k+1) = \rho_1(k) + \frac{T_s}{L}(q_1(k) - q_2(k) + r(k)) \quad (4)$$
$$\rho_i(k+1) = \rho_i(k) + \frac{T_s}{L}(q_i(k) - q_{i+1}(k)) \text{ or}$$
$$\rho_2(k+1) = \rho_2(k) + \frac{T_s}{L}(q_2(k) - q_3(k)) \quad (5)$$
$$\rho_{i+1}(k+1) = \rho_{i+1}(k) + \frac{T_s}{L}(q_{i+1}(k) - q_4(k) - f(k)) \text{ or}$$
$$\rho_3(k+1) = \rho_3(k) + \frac{T_s}{L}(q_3(k) - q_4(k) - f(k)) \quad (6)$$

With the elaboration of the inter-cell flow law [5] can be defined the expressions for the inter cell flows q_1, q_2 and q_3 in the above equations.

Before the inter cell flows elaboration is given, a reasonable description of the congestion must be given further, since as we assumed above, the cells can be in either free or congested mode. Congestion is defined as the state of the traffic with high density rates, or with other words the density of that part of the highway expressed in cell is equal or higher than the critical density based on the fundamental diagram of relationship of flow and density. Referred to the mentioned diagram, can be noticed that the congested flow belongs to higher values of the density, above the critical density values where the flow drops down. That can be described with enormous number of vehicles travelling at low speeds and with short distance spaces between each other.

The common modes of cells, used in analysis of researchers are the fully congested mode when the three cells are congested, denoted with *CCC*, and free

flow mode when the three cells are in free flow mode, denoted with *FFF* mode. The other middle modes that are out of the scope of this paper are those with last one and two cells in congested mode, written by *FCC* and *FFC*, respectively.
Now, for the *FFF* mode, the densities of the cells are lower than the critical density and the inter cell flows are as follows:

$$q_i(k) = \min\left(v_{fi-1}.\rho_{i-1},\ Q_{i-1}\ w_i(\rho_J - \rho_i)\right) or \quad (7)$$

$$q_2(k) = \min(v_{f1}.\rho_1,\ Q_1\ w_2(\rho_J - \rho_2)) = v_{f1}.\rho_1$$

$$q_{i+1}(k) = \min(v_{fi}.\rho_i,\ Q_i\ w_{i+1}(\rho_J - \rho_{i+1}),) \quad (8)$$

$$q_3(k) = \min(\boldsymbol{v_{f2}.\rho_2},\ Q_2\ w_3(\rho_J - \rho_3)) = v_{f2}.\rho_2$$

In *CCC* mode, the densities of the cells are higher that the critical density, and the inter cell flows are:

$$q_i(k) = \min\left(v_{fi-1}.\rho_{i-1},\ Q_{i-1}\ w_i(\rho_J - \rho_i)\right) or \quad (9)$$

$$q_2(k) = \min(v_{f1}.\rho_1,\ Q_1\ w_2(\rho_J - \rho_2)) = w_2(\rho_J - \rho_2)$$

$$q_{i+1}(k) = \min(v_{fi}.\rho_i,\ Q_i\ w_{i+1}(\rho_J - \rho_{i+1}),) \quad (10)$$

$$q_3(k) = \min(v_{f2}.\rho_2,\ Q_2\ w_3(\rho_J - \rho_3)) = w_3(\rho_J - \rho_3)$$

After subtracting the expressions for inter cell flows in the equations of densities for the *FFF* mode, we have:

$$\rho_1(k+1) = \rho_1(k) + \frac{T_s}{L}(q_1(k) - v_{f1}.\rho_1(k) + r(k)) \quad (11)$$

$$\rho_2(k+1) = \rho_2(k) + \frac{T_s}{L}(v_{f1}.\rho_1(k) - v_{f2}.\rho_2(k)) \quad (12)$$

$$\rho_3(k+1) = \rho_3(k) + \frac{T_s}{L}(v_{f2}.\rho_2(k) - q_4(k) - f(k) \quad (13)$$

And after subtracting the expressions for inter cell flows in the equations of densities for the *CCC* mode, we have:

$$\rho_1(k+1) = \rho_1(k) + \frac{T_s}{L}(q_1(k) - w_2\left(\rho_J - \rho_2(k)\right) + r(k)) \quad (14)$$

$$\rho_2(k+1) = \rho_2(k) + \frac{T_s}{L}(w_2(\rho_J - \rho_2(k)) - w_3(\rho_J - \rho_3(k)) \quad (15)$$

$$\rho_3(k+1) = \rho_3(k) + \frac{T_s}{L}(w_3(\rho_J - \rho_3(k)) - q_4(k) - f(k)) \quad (16)$$

3. A numerical example of the CTM model-

3.1. Calibration of Fundamental Diagram

For the purpose of the demonstration of the CTM model, in this paper is performed a numerical example which is described below. For the sake of simplicity, are chosen the same freeway segment characteristics as that in earlier sections in order to do an interconnection with the laid state space model of traffic density. The system of performance measurements of the traffic road networks of the Californian state (*PEMS)* [7] is used for traffic data and is considered a freeway link for on the street "Broadway Avenue", Stockton/San Francisco. The freeway is consisted from three cells with different lengths with one on-ramp on the first cell (Fig. 1.3).

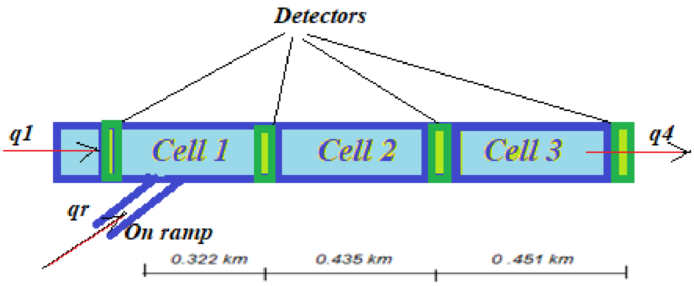

Fig.1.3.Freeway segment with three cells

System of PEMS offers traffic measurements as flow, occupancy, speed, etc. that are collected by detectors every five minute intervals. The choice of the measurements for the applied link is done based on the demands that derive from the *CTM* model. Since we need to involve in the *CTM* model the free flow speed $v_{f,}$ [km/h], maximal flow or capacity Q_M *[veh/h]*, critical density ρ_{cr} [veh/km] and jam density ρ_J *[veh/km]*, beside gathering the primary measurements, we are also pushed to do an calibration of the fundamental diagram to obtain the above mentioned measurements for every cell. In the frame of the calibration procedure of this seminar paper, the first step is to obtain the maximal amount of flow Q_M*[veh/h]*, which provides the highest pint on the fundamental diagram with flow-density relationship. By applying the law of fundamental diagram for the relationship of the flow and density, we can obtain the value of the critical density by projecting the maximal point of the flow Q_M, to the horizontal axe 'x', which from the following fig. we can see that is equal to *59.9* (veh/km). Speed as important parameter of in the frame of calibration, presents the slope of the line drowned to the scattered plots, By the relationship for the flow and speed, $Q=\rho\cdot v$, we can calculate the free flow speed v_f corresponds to the critical density, as $vf= Q_M/\rho_{cr}$, which is equal to *86,2* (km/hr) (17).

$$v_f = \frac{Q_M}{\rho_{cr}} = \frac{5172{,}6}{59{,}9} = 86{,}2\ \left(\frac{km}{hr}\right) \tag{17}$$

As an important part of the calibration is considered the estimation of the characteristic parameters that belong to the right side of the diagram or the congestion part, which are the jam density or the maximal density ρ_J *[veh/km]* and the backward speed *w [km/hr]*(18).

The jam density is determined by finding the outer point from the right side among the scattered plot

From which can be seen the value of the critical density ρ_J*=248* (veh/km).

The backward speed provides the rate at which the flow decreases while the density exceeds its critical value ρ_{cr}, which analogy as the free flow speed presents the slope of the line to the set of points of the right side diagram-congestion flow part. It has been calculated by the formula:

$$w = \frac{Q_M}{\rho_J - \rho_{cr}} \left[\frac{km}{hr}\right] = \frac{517,2}{248-59,9} = 27,5\left(\frac{km}{hr}\right) \quad (18)$$

The values of the calibrated parameters are presented in the *Table 1* and are graphically presented in Fig. 1.4.

Table 1

Summary of calibrated parameters

FF Conditions	Maximal Flow Q_M *[veh/hr]*	Free Flow Speed V_f *[km/hr]*	Critical Density ρ_{cr} *[veh/km]*	Jam Density ρ_J *[veh/km]*	Backward Speed *w [km/hr]*
Cell 1	5580	84.8	65.8	248	30.6
Cell 2	4176	96.8	43.1	248	20.3
Cell 3	4268	106.7	40.0	248	20.5

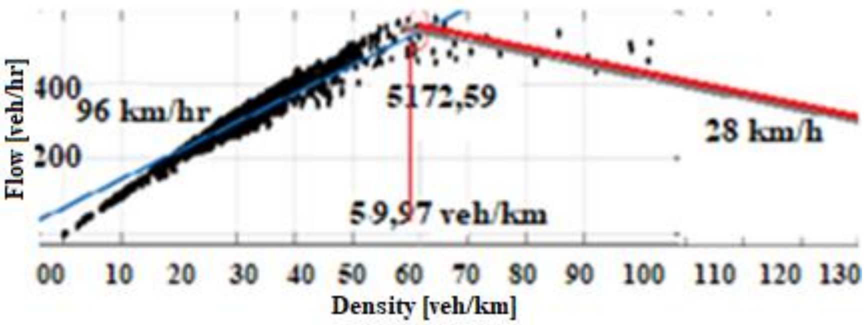

Fig.1.4. Diagram for flow-density calibrated parameters

3.2. Algorithm for FFF model

Table 2

	Pseudo Code of CTM model in Matlab editor
L1=0.322; %Length of Cell 1 [km] *L2=0.435; %Length of Cell 2 [km]* *L3=0.451; %Length of Cell 3 [km]* *rJ1=230; %Jam Density 1 [veh/km]* *rJ2=230; %Jam Density 2 [veh/km]* *rJ3=230; %Jam Density 3 [veh/km]* *rcr1=65.8; %Critical Density 1* *rcr2=43.1; %Critical Density 2* *rrcr3=200.0; %Critical Density 3*	*T=10/3600; % Time interval, 10 sec* *q1=q1; % Inflow in Cell 1-* *q4; % Outflow from Cell 3-* *QR; %Flow from On-ramp* *TT=1:1007; % Number of intervals* *% Reallocate the densities* *r1pred=zeros (length (q1), 1); %* *r2pred=zeros (length (q1), 1); %*

vf1=84.8; %Free flow speed 1 [km/h] *vf2=96.8; %Free flow speed 2 [km/h]* *vf3=106.7; %Free flow speed 3* *w1=30.6; %Backward (congestion)* *w2=20.3; %Backward speed 2 [km/h]* *w3=20.5; %Backward speed 3 [km/h]*	*r3pred=zeros (length (q1), 1); % Density of Cell 3 Vector* *Xhat = [25;23;21];* *% Initialization- Initial value of Matrix Density* *A= [1-vf1*T/L1 0 0; vf1*T/L2 1-vf2*T/L2 0; 0 vf2*T/L3 1];* *Bu= [T/L1 0 0; 0 T/L2 0; 0 0 -T/L3];* *Br= [T/L1 0 0; 0 T/L2 0; 0 0 T/L3];* *Qu= [q1 (ii, :); 0; q4 (ii, :)];* *Qr= [qr (ii); 0; 0];* *% Begin CTM Algorithm* *for ii=1:1007* *Xhat=A*xhat+Bu*qu+Br*QR;* *% store the predictions* *r1pred (ii) =xhat (1);* *r2pred (ii) =xhat (2);* *r3pred (ii) =xhat (3); end*

4. Results and Conclusions

Evaluation of the density values of cell is performed with discrete time intervals of $T_s=10$ seconds, adapting to the basic condition of the relation: $T<L/v_f$, for proper work with system matrices, otherwise there will be obtained negative values of density parameters. The initial values of the densities $\rho_0 = [\rho_{10}, \rho_{20}, \rho_{30}]^T$ and estimated covariance matrix P_o are assumed (as described in the above pseudo-algorithm in Mat Lab).

For the purpose of the results evaluation, measured traffic densities for five minute intervals are used for comparison with the estimated densities with *CTM* model. The performance of the model was quantified by calculating the Mean Absolute Percentage Error (*MAPE) [8]* given in (*19).*

$$\mathrm{MAPE} = [\frac{1}{n} \cdot \sum_{k=1}^{n} \left| \frac{\rho_{mod}(k)-\rho_{meas}(k)}{\rho_{meas}(k)}] \cdot 100 \right| \tag{19}$$

where, $\rho_{mod}(k)$ and $\rho_{meas}(k)$ are the estimated by CTM model and measured values of density, of each cell during the k^{th} discrete time interval and n is the number of observations.

Evaluation of results is done for both traffic state conditions: free flow (FFF) mode and congestion (CCC) which are the two most extreme cases that resemble the real traffic flow on highway.

Table 3

Performance Measure of Model (MAPE)

MAPE [%]	Cell 1	Cell 2	Cell 3
CTM (CCC)	40	39.9	40
CTM (FFF)	20.33	12.59	20

As we can see from *Table 3*, for free flow conditions *(FFF)* model, the MAPE results for *Cell 1*, *Cell 2* and *Cell3* are 20, 3 %, 12.6 % and 20 % respectively, and for congested conditions an average value 44% for the three cells are obtained. Estimation of densities within CTM –FFF model conditions has proved to be the model that offers more accurate results with measured field density densities, so it is considered promising in the application of many traffic control strategies algorithms. The results of measured field densities are given on figure 1.5 while those from the estimation are graphically presented on the below figure *1.6.* The model of the CTM density estimation can be further enriched for accuracy by using different filters and be applied on different traffic control algorithms that use density as output, such as on ramps and off ramps metering logics. It is worth mentioning that the possibility of applying such a model should not be excluded even in segments of complex complexity and with significant interruptions of traffic flows such as segments between the signaled crossings. New aspects should be considered in such mentioned cases in order to obtain a suitable model for forecasting traffic parameters in general.

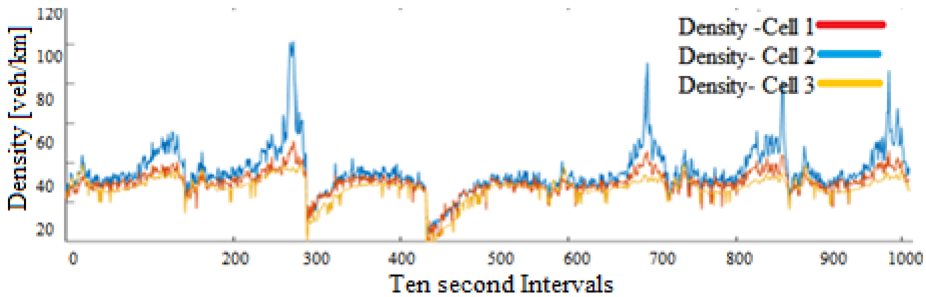

Fig.1.5. Graphical presentation of field measures densities

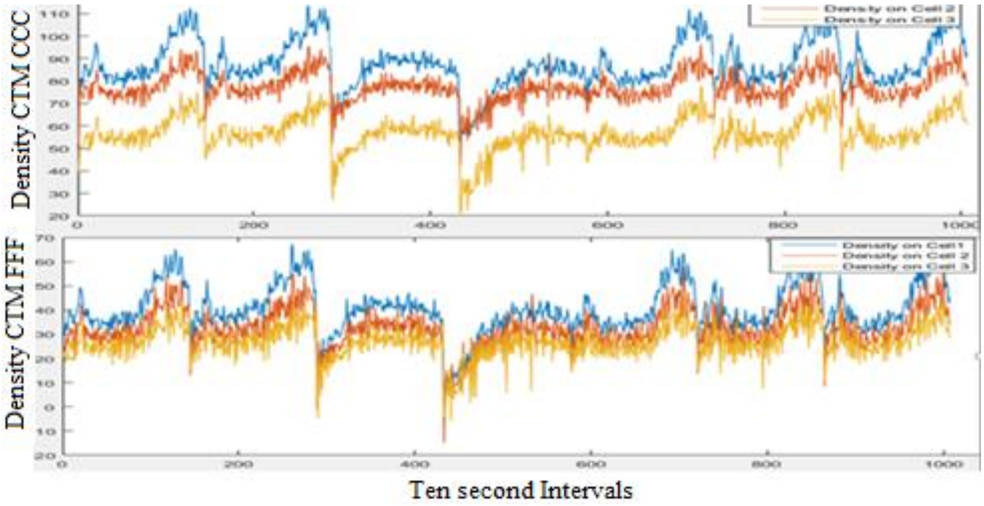

Fig.1.6. Graphical presentation of estimated densities for CTM-CCC model and for CTM-FFF model

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