THE OPTIMIZATION OF A RAIL INDUCTION HEATING INSTALLATION USING THE SIMPLEX METHOD

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The paper presents a method of optimization applied to the 3D finite element model of an induction heating installation. The finite element model is controlled during the optimization process by using Python scripts. The optimized installation is compared with the original one.

Keywords: rail induction heating, optimization, simplex method

1. Introduction

The heating is a stage of the hardening process [1] in which the rail mechanical properties are improved. The hardening process can be applied [1] during or after the manufacturing process of the rail – but it is usually applied during the process due to economical reasons. Fig. 1 contains the schematic representation of a high speed (1m/s) production line in which the hardening of the rails occur also.
The installation represented in Fig. 1 is used to produce the so called premium rails which are hardened with respect to the common ones. The produced rails length can be 72, 90 or 120 m, making them suitable for high speed trains. Regarding the production line, a preheated metal piece is introduced in the rolling mill. After several passes through the rolling mill the shaped rail exits the rolling mill with a temperature value of 700 °C. Due to high lengths of the produced rails, high speeds through the installation and high cooling speeds of the rail surface (1 °C/s) when the entire rail is out of the rolling mill, an important temperature difference appears between its ends. This difference is reduced by the “Heating Inductor” component of the installation [2]. The rail is placed on a conveyor and transported to the next part of the installation. The “Hardening Inductor” heats the rail head up to a temperature called the austenitisation temperature. This type of installation is studied in this paper. In the “Quenching” installation the rail is cooled with a critical cooling speed in order to achieve the martensite microstructure. The final product is delivered after the “Quenching” installation.

2. The 3D finite element model

The finite element model of the “Hardening Inductor” is implemented using the commercial finite element package Flux 3D made by the French company Cedrat [3]. The model computation domain is given in Fig. 2.

The installation represented in Fig. 2 is composed by two inductors [4]. The two inductors design gives the possibility of tuning the heating pattern nonuniformity on the rail during the prototype testing stage. Regarding the rail, only the rail head was kept into the computation domain. The model application can be reduced to a magnetoharmonic type if a criterion for heating pattern nonuniformity can be found. The material of the magnetic cores was considered with a constant relative magnetic permeability and of nonconducting type – no eddy currents can occur in the magnetic core. The material of the rail was
considered to have a nonlinear magnetic permeability. Also the rail region is considered to be a solid conductor region with a constant value of the resistivity. The rail region is thus the most complex region from the computation domain and it has a big influence on the amount of computational resources allocated in the solving process of the finite element model.

3. The simplex method

An optimization method aims to determine the extremum of the so called objective function. The objective function is a mathematical expression used to describe an installation or a physical process. The extremum of objective function represents the best case scenario for the physical process or installation. The objective function depends on one or several parameters which are defined as quantities of interest for the installation or physical process parameters. One of the available optimization methods is the simplex method. The simplex method is based on the concept of simplex which is an n dimensional polytope with n + 1 vertices. The simplex method will be presented in this section for the particular case of n = 2 in which the simplex represents an equilateral triangle. The vertices of the triangle represent combination of the parameters of interest for the optimization problem. The values of the objective function defined in the vertices represent variations of installation or physical process. The objective function is defined on an interval which is created by imposing restrictions on the function parameters. These restrictions are necessary because the parameters are usually restricted by the geometrical configuration of the installation or by the physical constants and laws of physics in the physical process.

The optimization process starts with a randomly generated initial simplex on the definition interval of the objective function and ends usually with a more area reduced simplex on the same interval. The optimization process stops when some criteria regarding the simplex area or the number of iterations are reached. A series of operation can be performed between the initial and final simplex. These are the reflexion, expansion, partial contraction and total contraction operations and are represented in Figs. 3 a) – d).

![Fig. 3. The operations of the simplex method](image)
In a very simplified presentation of the simplex method specific operations it can be said that these operations will move or shrink the initial simplex based on the best and worst case scenarios determined from the objective function values in the vertices of the simplex. The purpose is to obtain a smaller simplex which would mean that all the three values of the objective function defined in the vertices are very close to a local extremum.

The simplex operations are presented next. Let’s assume that the minimum of the objective function is searched. An initial simplex is chosen by random selection in the objective function definition interval. The initial simplex vertices are labeled as P1, P2 and P3 while the objective function values on those points are V1, V2 and V3. Let’s assume that these values are ordered as V1 < V2 < V3 (if not the indexes of P1, P2 and P3 can be permuted accordingly). So V3 would be the most undesirable value of three while the V1 solution would be the one from the point P3, Fig. 3 a), which is computed by determining the reflection of P3 with respect to the P1-P2 axis. The value V5 is tested with respect to the initial values V1, V2 and V3.

The next step depends on the relationship of the newly found value V5 with respect to the old ones V1, V2 and V3. If the new value V5 is better (smaller) than V1 than the expansion operation is tried. This operation consists in checking along the P4-P5 defined direction for a point P6 with a better value than V5. If a better value (V6) is found then P6 is set as a vertex of the simplex. The points P1 and P2 are translated accordingly on parallel axis in order to preserve the equilateral triangle structure of the simplex. It may be that the value V5 is higher than V1 but lower than V3. In this case the expansion operation cannot be applied. However the V5 value is kept and a new simplex (P1-P2-P5) is created since V5 is a better choice than V3 thus the operation of reflection is completed. The case in which V5 is higher than V3 is possible also and as a consequence the reflection operation fails. This means that an local optimum should be searched inside the triangle.

The partial contraction operation is tried in which the value V7 in the new point P7, inside the triangle, is determined. If V7 is smaller than V3 than the partial contraction operation has succeeded and the P7 point is considered a vertex of the newly created simplex. In order to create an equilateral triangle the P1 and P2 vertices are translated inwards along the P1-P2 direction. If the partial contraction operation fails, i.e. V7 < V3, than the total contraction operation is applied. This means that the simplex is restrained to a smaller one P1-P4-P8. The last iterations of an optimization process using the simplex method are done using total or partial contraction operations.

The criteria for stopping the optimization process can be based upon the area of the simplex, the edge length of the simplex or the number of iterations.
The area and edge length of the simplex usually decreases with the number of iterations due to total and partial contraction operations. The value of the optimum is considered to be the best value from ones computed in the final simplex vertices. The optimum value is thus an approximation while the real value is usually contained within the area of the triangle representing the simplex.

The optimum found is to be considered local. The simplex method is a deterministic method which whose output may not be the global optimum. In order to find the global optimum multiple optimization processes can be launched starting from different initial simplexes.

4. The optimization process

The practical implementation of the simplex method applied on the inductor installation from Fig. 2 is described here. The purpose is to obtain a reduced non-uniformity of the heating profile on the rail. The installation is optimized by changing two of its geometrical dimensions. These dimensions are marked P1 and P2 and are shown in Fig. 4, being essentially the length of the longer inductor and the coil opening of the shorter inductor. The variation of these parameters acts directly on the heating pattern on the rail by weakening or strengthening the magnetic field in the appropriate areas.

![Fig. 4. The objective function parameters](image)

The P1 and P2 parameters combination can take values in [20...40] x [400,...,900] mm² bidimensional interval which is in fact the objective function interval definition. This interval is further translated into a grid by discretisation.

The objective function cannot be defined directly as an mathematical expression since the objective function value is determined by solving a finite element model of the type described in section 2 for each (P1, P2) combination of parameters. The optimum of the objective function is achieved when non-uniformity of the railhead heating is minimum. The non-uniformity of the heating pattern of the rail head for the non-optimized configuration of the installation is given in Fig. 5:
The temperature color shade result from Fig. 5 is obtained with a magnetoharmonic – transient thermal finite element model which considers also the rail passage through inductors. This type of model requires more computation and time resources than the actual model (magnetoharmonic type) described in this paper which is used for optimization.

It is hard to appreciate the nonuniformity of the heating based on the colour shade results from Fig. 5 and with that respect to appreciate solely based on the colour shade if a type of inductor would give better results than another. A different approach was proposed to check the heating nonuniformity by considering the source of heating (the rail induced active power volumic density) at the surface of the rail. The integral of the volumic power density along longitudinal lines on the rail surface is used instead. These longitudinal lines are distributed along the surface of the rail. Each integral result corresponds to a point on the rail surface transversal contour. Plotting such a dependence will give the graphic from Fig. 6 which contains on Ox axis the distance on the transversal contour of the rail surface (with respect to the point of origin placed on top of the rail surface on the symmetry line) and on Oy axis an estimation of the heating source value at the rail surface (W/m²):

A norm was defined in order to characterize and directly compare two profiles like the one from Fig. 6.

\[ N_{P1,P2} = \sum_{k} \left( \frac{y_k - y_m}{y_m} \right)^2 \]  (1)
The optimization of a rail induction heating installation using the simplex method

Fig. 6. The non uniform distribution of the heating source on the rail surface

where $y_k$ represent the values from the Oy axis (Fig. 6) and $y_m$ represent the mean value of the $y_k$ set.

The goal of the optimization is to obtain a finite element model for which the norm $N$, defined for a pair of parameters $P_1$ and $P_2$, is minimum. The optimization process was started from an initial simplex $(P_1(28, 550), P_2(28, 550 + 10/\sqrt{3}), P_3(33, 550 + 5/\sqrt{3}))$ with the values vertices being $N_1 = 0.02959$, $N_2 = 0.030746$, $N_3 = 0.04271$. The local optimum was found in the point $P_{optimum} = (23, 547.11)$, with $N_{optimum} = 0.022396$ after seven iterations. Another optimization process was launched from a different initial simplex $(P_1(30, 780), P_2(37, 780), P_3(33.5, 780 + 3.5 \cdot \sqrt{3}))$ but found a higher value than 0.022396 and thus was not recorded. Also the $N_{optimum}$ value is obtained with a shorter and narrower installation ($P_1$ and $P_2$ parameters are smaller than any of the values of the initial simplex). For comparison, the initial and optimized nonuniformity profiles are presented in a superimposed manner in Fig. 7:

Fig. 7. Superimposed induced active power profiles
The optimized profile is slightly smaller meaning that the induced active power in the rail has decreased due to the smaller sizes of the optimized inductor.

The optimization process was successful, leading to an improvement of the overall heating pattern nonuniformity in the rail head with 25 % (according to computed norms initial N<sub>1</sub> and final N<sub>optimum</sub>) but also to smaller values for the parameters P1 and P2 which can be directly translated into the economical cost.

The issue of using more than two parameters for the objective function was never approached due to the important computation and time resources required by optimization process.

6. Conclusions

An implementation of a classical optimization method using finite element models to extract the value of the objective function was proposed. In order to implement the optimization method, several things had to be accomplished – like setting a satisfactory model from the point of view of computational resources, identifying the geometrical parameters relevant for optimization and establishing the expression of the objective function whose extremum gives the optimal installation.

The optimization output led to the improvement of the nonuniformity of the heating pattern and to a cheaper cost of the heating installation. It can be assumed the further reduction of the heating nonuniformity due to thermal conduction from the hot areas to the colder one from the rail head.

REFERENCES