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APPLICATION OF CANONICAL REPRESENTATION METHOD TO LINEAR RANDOM VIBRATIONS

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In this paper is presented the canonical representations method for analyzing the random vibrations of a linear system in the cases of two covariance functions commonly used in the study of the random vibrations of mechanical systems.

The results obtained with the canonical method representations are compared with the exact ones obtained by the method of transfer functions.

Keywords: canonical representation, random process, random vibrations of linear system

1. Introduction

The application of the canonical representations method in the study of dynamical systems with random excitations is analogous with the function series expansion method used for deterministic dynamical systems [1].

Using canonical representations, the advantage is that the time dependence is transferred to some deterministic functions, the randomness being preserved by the random variables. In this way, the operations of derivation and integration of random processes are reduced to the usual operations of deterministic functions, which can lead to significant simplifications. On the other hand, from the practical point of view, random processes can be represented only approximately through canonical representations, which converge in mean-square sense in an array of values of the argument or on an argument range of variation; but the rapidity of convergence should be investigated in each case.

Starting from a canonical representation of a random process one can deduce a certain expansion in series (also called canonical) of its covariance function and vice versa. This is of great practical importance because in most practical problems the excitation covariance function is known. An important feature of these representations is the ability to build the canonical representation of a random process and of its covariance function in infinite ways, allowing a convenient choice of the coefficients and of the coordinate functions of the expansion in terms of the rapidity of convergence and volume of calculation

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involved. A big disadvantage of this method is that it generally cannot provide sufficient information on the distribution of the investigated random processes, except for Gaussian random processes. This disadvantage becomes stronger in case of nonlinear dynamical systems for which the output is generally not Gaussian, even when the input is Gaussian.

Also for nonlinear systems the canonical representations method becomes practically available if associated with certain linearization methods, which generally result in loss of information on the type of nonlinearity of the system characteristics.

Canonical representations method is especially useful for evaluating the mean and the covariance function (and hence the spectral density) of the randomly excited dynamical systems response.

2. Canonical representation of random processes

A scalar random process $\{x_t, t \in [t_0, T]\}$ defined by the relation,

$$x_t = X\varphi(t), \tag{2.1}$$

where X is a random variable with $\mathrm{E}\{X\} = 0$, and $\varphi(t)$ is a determinist function defined on $[t_0, T]$, is an elementary random process. Obviously,

$$m_{\mathrm{x}}(t) = \mathrm{E}\{X\varphi(t)\} = \mathrm{E}\{X\}\varphi(t) = 0 \tag{2.2}$$

The covariance function (equal to the correlation function) of the elementary random process $x\{t\}$ is given by

$$c_{\mathrm{xx}}(t,t') = k_{\mathrm{xx}}(t,t') = \mathrm{E}\{|X|^2\}\varphi(t)\bar{\varphi}(t') \tag{2.3}$$

We consider the scalar random process $\{x_t^0, t \in [t_0, T]\}$

$$x_t^0 = \sum_n x_{nt}, \tag{2.4}$$

where $\{x_{nt}, t \in [t_0, T]\}$ are uncorrelated elementary random processes:

$$x_{nt} = X_n \varphi_n(t), \\ \mathrm{E}\{X_n\} = 0; \quad \mathrm{E}\{|X_n|^2\} = D_n; \quad \mathrm{E}\{X_n \bar{X}_m\} = 0,\ m \neq n \tag{2.5}$$

The random process $\{x_t^0\}$ has zero average ($m_{\mathrm{x}^0}(t) = 0$) and its correlation function is given by

$$k_{\mathrm{x}^0\mathrm{x}^0}(t,t') = \sum_n k_{\mathrm{x}_n\mathrm{x}_n}(t,t') = \sum_n D_n \varphi_n(t)\bar{\varphi}_n(t') \tag{2.6}$$

Consider now the random process

$$x_t = \varphi(t) + \sum_n X_n \varphi_n(t) \tag{2.7}$$

where $\varphi(t)$ is a given determinist function.We can write

$$\mathrm{E}\{x_t\} = \varphi(t),\ c_{xx}(t,t') = k_{x^0x^0}(t,t') = \sum_n k_{x_n x_n}(t,t') = \sum_n D_n \varphi_n(t)\overline{\varphi}_n(t') \tag{2.8}$$

A random process representation of the form (2.7), where $\varphi(t) = m_x(t)$, is called the canonical representation of the random process. The covariance function reprezentation $c_{xx}(t,t')$ through the relation (2.8) is called canonical representation of the covariance function. The random variables X_n are called coeficients and the functions $\varphi_n(t)$ are called coordinate functions of the canonical representation.

In many cases canonical representations facilitate the application of certain (especially linear) operators to random processes, as their time dependence is expressed by means of deterministic functions (coordinate functions). Therefore using canonical expansions, operations (such as integration, derivation, solving differential equations, etc.) in which random processes are involved, can be reduced to regular operations with deterministic functions. The fact that the coefficients of the canonical representations are uncorrelated random variables simplifies significantly the description of the correlation functions of random processes.

It can be shown that if the covariance function $c_{xx}(t,t')$ is continuous and $T < \infty$, the canonical representation of the random process $\{x_t\}$, converges in the mean square for all $t \in [t_0, T]$.

Consider now a random stationary process $\{x_t, t \in (-T,T)\}$. Its covariance function $c_{xx}(\tau), \tau \in (-2T, 2T)$ can be represented on the interval $(-2T,2T)$ through a Fourier series of period $4T$:

$$c_{xx}(\tau) = \sum_{n=-\infty}^{\infty} D_n e^{i\omega_n\tau}, \quad \omega_n = \frac{n\pi}{2T}, \tag{2.9}$$

where

$$D_n = \frac{1}{4T}\int_{-2T}^{2T} c_{xx}(\tau) e^{-i\omega_n\tau} d\tau, \ n = 0, \pm 1, \pm 2, ... \tag{2.10}$$

Relation (2.9) can be written as

$$c_{xx}(t-t') = \sum_{n=-\infty}^{\infty} D_n e^{i\omega_n t}\, \overline{e^{i\omega_n t'}}\,. \tag{2.11}$$

Relation (2.11) gives the canonical expansion for the covariance function of the random process $\{x_t\}$. Therefore, the canonical representation of the random process $\{x_t\}$ is

$$x_t = m_x + \sum_{n=-\infty}^{\infty} X_n e^{i\omega_n t}, \tag{2.12}$$

where X_n are uncorrelated random variables, with zero mean and the dispersions D_n, so that all the coefficients D_n are positive.

We have shown that a possible canonical representation for a stationary random process on the finite interval (*-T, T*) is its trigonometric series expansion after the *4T* periodic harmonics (in the sense of mean square convergence). From (2.9) we have

$$\mathrm{E}\left\{\left|x_t - m_x\right|^2\right\}\Big| = c_{xx}\left(0\right) = \sum_{n=-\infty}^{\infty} D_n . \tag{2.13}$$

3. Application of canonical representation to random vibration of linear systems

In this paper is exemplified the application of the canonical representations method in order to determine the r.m.s. response of a linear oscillating system with one freedom degree to a random excitation represented by a stationary random process in the broad sense. The equation of motion of this system is

$$\ddot{x} + 2\omega_p \zeta \dot{x} + \omega_p^2 x = -\ddot{x}_0 = \omega_p^2 x_0 \tag{3.1}$$

Considering the canonical representations of the random input process $x_0\left(t\right)$ and output process $x\left(t\right)$ of the oscillating system described by the equation (3.1)we can write

$$x_0\left(t\right) = \sum_{n=-\infty}^{\infty} X_{0n} e^{i\omega_n t},\ x\left(t\right) = \sum_{n=-\infty}^{\infty} X_n e^{i\omega_n t} \tag{3.2}$$

where the dispersions $\mathrm{E}\left\{\left|X_{0n}\right|^2\right\} = D_{0n}$ are known. The random variables X_n will be determined introducing relations (3.2) in (3.1) and identifying the coefficients of the same spectral components. It is considered that the random process $x_0\left(t\right)$ is a stationary random process in the broad sense with zero mean and with the covariance function $c_{x_0x_0}(\tau)$. The dispersion σ_{xe} of the exact solution is compared with the dispersion σ_x of the approximate solution, obtained by the canonical representations method, for different expressions of the covariance function used in practice [2].

3.1. Exponential covariance function

In this case the covariance function, $c_{x_0x_0}(\tau)$, and the one sided spectral density, obtained by applying the Fourier transform of $c_{x_0x_0}(\tau)$ are

$$c_{x_0x_0}(\tau)=\sigma_{x_0}^2 e^{-\alpha|\tau|},\quad G_{x_0}(\omega)=\frac{2\sigma_0^2\alpha}{\pi\left(\omega^2+\alpha^2\right)},\ \omega\geq 0 \tag{3.3}$$

Fig. 1 illustrates typical plots of exponential covariance function and its one sided spectral density:

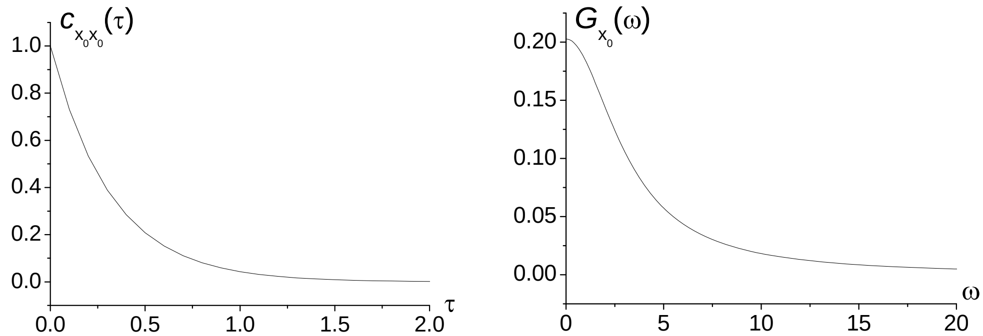

Fig. 1. The exponential covariance function and its one sided spectral density

Considering a unit value for σ_0, with the physical dimension of the random process $x_0(t)$, it can be written

$$c_{x_0x_0}(\tau)=\sum_{n=-\infty}^{\infty} D_{0n}e^{i\omega_n\tau},\ \omega_n=\frac{n\pi}{2T},\ n=0,\pm1,\pm2,... \tag{3.4}$$

The coefficients D_{0n} are given by

$$D_{0n}=\frac{1}{4T}\int_{-2T}^{2T} c_{x_0x_0}(\tau)\mathrm{e}^{-i\omega_n\tau}d\tau=\frac{1}{4T}\int_{-2T}^{2T}\mathrm{e}^{-\alpha|\tau|-i\omega_n\tau}d\tau=\frac{1}{2T}\int_{0}^{2T}e^{-\alpha\tau}\cos(\omega_n\tau)d\tau= \\ =\frac{\alpha}{2T}\frac{1+(-1)^{n+1}\mathrm{e}^{-2T\alpha}}{\alpha^2+\omega_n{}^2} \tag{3.5}$$

Introducing the expansion (3.2) in equation (3.1) it follows

$$\sum_{n=-\infty}^{\infty}\left(-\omega_n^2+2i\omega_\mathrm{p}\omega_n\zeta+\omega_\mathrm{p}^2\right)X_n e^{i\omega_n t}=\omega_\mathrm{p}^2\sum_{n=-\infty}^{\infty}X_{0n}e^{i\omega_n t} \tag{3.6}$$

Considering X_{0n}, X_n real random variables and identifying the absolute values of the same spectral components in the above relation, yields

$$X_n=\frac{\omega_\mathrm{p}^2 X_{0n}}{\sqrt{(\omega_n^2-\omega_\mathrm{p}^2)^2+4\omega_n^2\omega_\mathrm{p}^2\zeta^2}} \tag{3.7}$$

Therefore X_n are uncorrelated random variables with zero mean and with the dispersions

$$D_n = \frac{\omega_p^4 D_{0n}}{(\omega_n^2-\omega_p^2)^2+4\omega_n^2\omega_p^2\zeta^2} \tag{3.8}$$

From (2.9), (3.2), (3.5) and (3.8) results

$$\sigma_x^2 = c_{xx}(0) = \sum_{n=-\infty}^{\infty} D_n = \frac{\alpha\omega_p^4}{T}\lim_{N\to\infty}\sum_{n=0}^{N}\frac{1+(-1)^{n+1}e^{-2T\alpha}}{(\alpha^2+\omega_n{}^2)[(\omega_n^2-\omega_p^2)^2+4\omega_n^2\omega_p^2\zeta^2]} \tag{3.9}$$

It is known that the mean square of the stationary random process $x(t)$, which satisfies the stochastic differential equation (3.1) can be obtained from the relation ([3],[4])

$$\sigma_{xe}^2 = \int_0^\infty G_x(\omega)d\omega = \int_0^\infty \left|H_{xx_0}(\omega)\right|^2 G_{x_0}(\omega)d\omega \tag{3.10}$$

where $H_{xx_0}(\omega)$ is the frequency response function corresponding to the input $x_0(t)$ and output $x(t)$ of the oscillatory system described by the equation (3.1), given by

$$H_{xx_0}(\omega) = \frac{\omega_p^2}{\omega_p^2-\omega^2+2i\zeta\omega_p\omega} \tag{3.11}$$

Using (3.3) and (3.11) in (3.10), yields

$$\sigma_{xe}^2 = \frac{2\alpha\omega_p^4}{\pi}\int_0^\infty \frac{d\omega}{(\omega^2+\alpha^2)[(\omega^2-\omega_p^2)^2+4\zeta^2\omega_p^2\omega^2]} \tag{3.12}$$

Introducing the notations

$$\mu = \frac{\alpha}{\omega_p},\quad \xi = \frac{\omega}{\omega_p},\quad \xi_n = \frac{\omega_n}{\omega_p} = \frac{n\pi}{2\omega_p T},\quad n = 0,1,2,.. \tag{3.13}$$

(3.9) and (3.12) become

$$\sigma_{xe}^2 = \frac{2\mu}{\pi}\lim_{N\to\infty}\eta_x(\zeta,\mu,N),\quad \sigma_x^2 = \frac{\mu}{\omega_p T}\lim_{N\to\infty}\lambda_x(\zeta,\mu,N,T) \tag{3.14}$$

where

$$\lim_{N\to\infty}\eta_x(\zeta,N,\mu) = \lim_{N\to\infty}\int_0^N \frac{d\xi}{(\mu^2+\xi^2)[(1-\xi^2)^2+4\xi^2\zeta^2]} = \frac{\pi}{2[1+2\mu^2(1-2\zeta^2)+\mu^4]}\left[\frac{1}{\mu}+\frac{1+\mu^2-4\zeta^2}{2\zeta}\right] \tag{3.15}$$

and

$$\lambda_x(\zeta,\mu,N,T) = \sum_{n=0}^{N}\frac{1+(-1)^{n+1}e^{-2T\alpha}}{(\mu^2+\xi_n^2)[(\xi_n^2-1)^2+4\xi_n^2\zeta^2]} \tag{3.16}$$

From $(3.14)_1$ and (3.15) it follows

$$\sigma_{xe}^2 = \frac{\mu}{1+2\mu^2(1-2\zeta^2)+\mu^4}\left[\frac{1}{\mu}+\frac{1+\mu^2-4\zeta^2}{2\zeta}\right] \tag{3.17}$$

The number of terms, N, considered in the canonical expansion of the process $x(t)$ can be determined imposing the condition that the relative error, e_r, between the exact value σ_{xe} given by (3.17) and the approximate one σ_x obtained from $(3.14)_2$

$$\sigma_x(N,T) = \sqrt{\frac{\mu}{\omega_p T}\lambda_x(\zeta,\mu,N,T)} \tag{3.18}$$

to be less than a given value ε:

$$e_r = \frac{|\sigma_{xe} - \sigma_x(N,T)|}{\sigma_{xe}} < \varepsilon \ll 1 \tag{3.19}$$

In order to satisfy this approximation condition the value of T can be taken of the form $T = N/\omega_p$. In this case, $\sigma_x(N,T)$ can be calculated from the relation

$$\sigma_x(N) = \sqrt{\frac{\mu}{N}\lambda_x(\zeta,\mu,N)} \tag{3.20}$$

where

$$\begin{gathered}\lambda_x(\zeta,\mu,N) = \sum_{n=0}^{N}\frac{1+(-1)^{n+1}e^{-2\mu N}}{(\mu^2+\xi_n^2)[(\xi_n^2-1)^2+4\xi_n^2\zeta^2]}\\ \xi_n = \frac{\omega_n}{\omega_p} = \frac{n\pi}{2N},\ n = 1,2,3..,N\end{gathered} \tag{3.21}$$

In order to calculate $\lim\limits_{N\to\infty}\frac{1}{N}\lambda_x(\zeta,\mu,N)$, it can be written

$$\begin{aligned}&\lim_{N\to\infty}\frac{1}{N}\lambda_x(\zeta,\mu,N) =\\ &= \lim_{N\to\infty}\left[\frac{1}{N}\sum_{n=0}^{N}\frac{1}{(\mu^2+\xi_n{}^2)[(\xi_n^2-1)^2+4\xi_n^2\zeta^2]}\right]+\\ &+ \lim_{N\to\infty}\left[\frac{e^{-2\mu N}}{N}\sum_{n=0}^{N}\frac{(-1)^{n+1}}{(\mu^2+\xi_n{}^2)[(\xi_n^2-1)^2+4\xi_n^2\zeta^2]}\right]\end{aligned}$$

The second limit being null and since $\xi_{n+1}-\xi_n = \frac{\pi}{2N}$, the first limit can be considered as the Riemann series of the integral:

$$\lim_{N\to\infty}\frac{1}{N}\lambda_x(\zeta,\mu,N)=\frac{2}{\pi}\int_0^{\pi/2}\frac{d\xi}{(\mu^2+\xi^2)[(\xi^2-1)^2+4\xi^2\zeta]^2} \tag{3.21'}$$

The equations (3.20) and(3.21)' yield

$$\sigma_x^2=\frac{2\mu}{\pi[1+2\mu^2(1-2\zeta^2)+\mu^4]}\left[\frac{1}{\mu}\cdot arctg\frac{\pi}{2\mu}-\right.$$
$$\left.-\frac{3-4\zeta^2+\mu^2}{8\sqrt{1-\zeta^2}}\ln\frac{\pi^2-4\pi\sqrt{1-\zeta^2}+4}{\pi^2+4\pi\sqrt{1-\zeta^2}+4}+\frac{1+\mu^2-4\zeta^2}{4\zeta}\tan^{-1}\frac{\pi\zeta}{1-\pi^2/4}\right] \tag{3.22}$$

where

$$\tan^{-1}\frac{2\zeta\xi}{1-\xi^2}=\begin{cases}\arctan\frac{2\zeta\xi}{1-\xi^2}, & 0\le\xi<1\\ \frac{\pi}{2}, & \xi=1\\ \pi+\arctan\frac{2\zeta\xi}{1-\xi^2}, & \xi>1\end{cases}$$

Taking for example the values $\alpha=\pi[s^{-1}]$, $\omega_p=2\pi[s^{-1}]$, $N=100$, for the excitation parameters, figure 2 shows the plot of the relative error e_r between σ_{xe} and σ_x, for various situations μ and ζ.

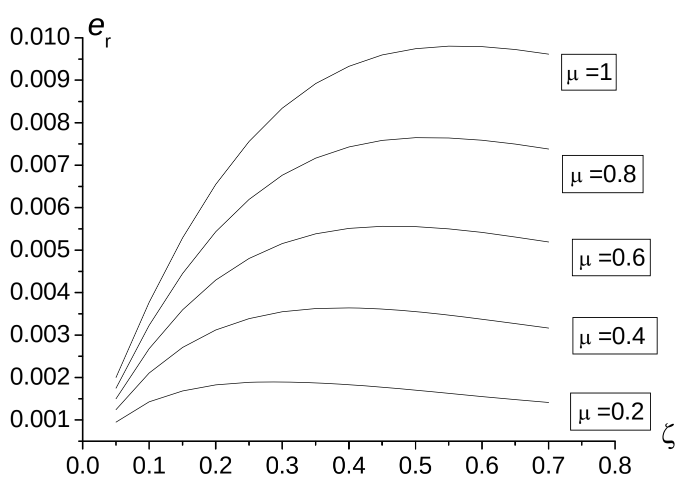

Fig. 2. Relative error between σ_{xe} and σ_x

3.2. Exponential cosine covariance function

Consider now the input covariance function and its one sided spectral density of the form

$$c_{x_0x_0}(\tau)=e^{-\alpha|\tau|}\cos(\beta\tau),\;\; G_{x_0}(\omega)=\frac{\alpha}{\pi}\left[\frac{1}{\alpha^2+(\omega+\beta)^2}+\frac{1}{\alpha^2+(\omega-\beta)^2}\right],\omega\geq 0 \quad (3.23)$$

Their typical plots are given in figure 3.

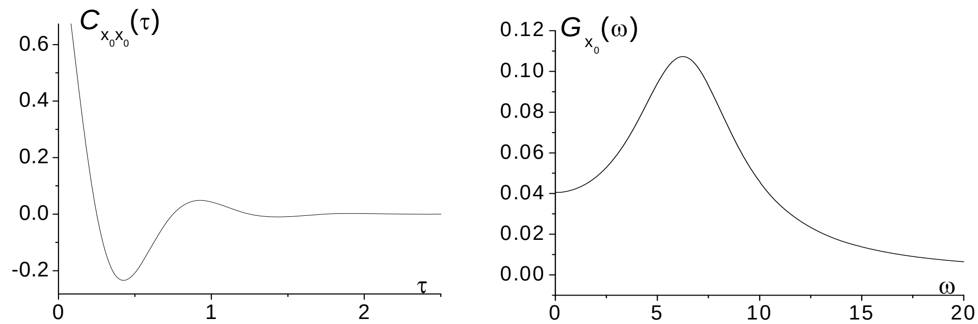

Fig. 3. The exponential cosine covariance function and its one sided spectral density

In this case we obtain for the expansion in (3.4) the coefficients:

$$D_{0n}=\frac{1}{4T}\int_{-2T}^{2T}e^{-\alpha|\tau|}\cos(\beta\tau)e^{-i\omega_n\tau}d\tau=$$

$$=\frac{1}{4T}\Big[\frac{\alpha+(-1)^n e^{-2T\alpha}[-\alpha\cos(2\beta T)+(\beta-\omega_n)\sin(2\beta T)]}{\alpha^2+(\beta-\omega_n)^2}+ \quad (3.24)$$

$$+\frac{\alpha+(-1)^n e^{-2T\alpha}[-\alpha\cos(2\beta T)+(\beta+\omega_n)\sin(2\beta T)]}{\alpha^2+(\beta+\omega_n)^2}\Big]$$

Using (3.8) we have:

$$\sigma_x^2=c_{xx}(0)=\sum_{n=-\infty}^{\infty}D_n=\frac{\omega_p^4}{2T}\sum_{n=0}^{\infty}\frac{R_n}{[(\omega_n^2-\omega_p^2)^2+4\omega_n^2\omega_p^2\zeta^2]},$$

$$R_n=\frac{\alpha+(-1)^n e^{-2T\alpha}[-\alpha\cos(2\beta T)+(\beta-\omega_n)\sin(2\beta T)]}{\alpha^2+(\beta-\omega_n)^2}+ \quad (3.25)$$

$$+\frac{\alpha+(-1)^n e^{-2T\alpha}[-\alpha\cos(2\beta T)+(\beta+\omega_n)\sin(2\beta T)]}{\alpha^2+(\beta+\omega_n)^2}$$

Using (3.11) and (3.23)in (3.10) yields

$$\sigma_{xe}^2 = \int_0^\infty G_x(\omega)\mathrm{d}\omega = \frac{2\alpha\omega_p^4}{\pi}\int_0^\infty \frac{(\omega^2+\alpha^2+\beta^2)d\omega}{[(\omega^2-\alpha^2-\beta^2)^2+4\alpha^2\omega^2][(\omega_p^2-\omega^2)^2+4\zeta^2\omega_p^2\omega^2]} \tag{3.26}$$

Introducing the notations

$$\mu = \frac{\alpha}{\omega_p},\ \xi = \frac{\omega}{\omega_p},\ \lambda = \frac{\beta}{\alpha} = \frac{\beta}{\mu\omega_p} \tag{3.27}$$

it follows

$$\sigma_{xe}^2 = \frac{2\mu}{\pi}\int_0^\infty \frac{[\xi^2+\mu^2(\lambda^2+1)]d\xi}{\{[(\xi^2-\mu^2(\lambda^2+1)]^2+4\mu^2\xi^2\}[(1-\xi^2)^2+4\zeta^2\xi^2]} \tag{3.28}$$

On the other hand

$$\begin{aligned}\sigma_x^2 = \lim_{N\to\infty}\frac{1}{2N}\sum_{n=0}^{N}\Bigg\{&\frac{1}{(\xi_n^2-1)^2+4\xi_n^2\zeta^2}\cdot\\ &\cdot\Bigg[\frac{\mu+(-1)^n e^{-2N\mu}[(\beta/\omega_p-\xi_n)\sin(2\beta T)-\mu\cos(2\beta T)]}{\mu^2+(\beta/\omega_p-\xi_n)^2}+\\ &+\frac{\mu+(-1)^n e^{-2N\mu}[(\beta/\omega_p+\xi_n)\sin(2\beta T)-\mu\cos(2\beta T)]}{\mu^2+(\beta/\omega_p+\xi_n)^2}\Bigg]\Bigg\}\cong\\ \cong \lim_{N\to\infty}\frac{\mu}{2N}\sum_{n=0}^{N}\Bigg\{&\frac{1}{(\xi_n^2-1)^2+4\xi_n^2\zeta^2}\left[\frac{1}{\mu^2+(\lambda\mu-\xi_n)^2}+\frac{1}{\mu^2+(\lambda\mu+\xi_n)^2}\right]\Bigg\}\end{aligned} \tag{3.29}$$

Taking the values

$$\alpha = \pi[s^{-1}],\ \omega_p = \beta = 2\pi[\mathrm{rad}/s], N = 100,\ \mu = 0.5,\ \zeta = 0.25 \tag{3.30}$$

and using Origin software, one obtains from (3.29) and (3.30) $\sigma_x^2 = 1.575$ respectively $\sigma_{xe}^2 = 1.595$ so that the relative error is $e_r < 0.01$.

4. Spectral analysis

In what follows, the discret spectral densities components, estimated for canonical representations of system input and output (3.2), are given by [2]

$$G^*_{x_{0n}} = 2\frac{D_{0n}}{\Delta\omega},\ G^*_{x_n} = 2\frac{D_n}{\Delta\omega},\ \Delta\omega = \omega_{n+1}-\omega_n = \frac{\pi}{2T},\ n = 1,2,\dots,N \tag{4.1}$$

In Figs. 4 and 5 are shown comparatively the values (4.1) obtained by canonical representations and the plots of spectral densities $G_{x_0}(\omega)$ and $G_x(\omega)$ calculated for $\alpha = \pi[\mathrm{s}^{-1}]$, $\omega_p = 2\pi\,\mathrm{rad/s}$, $N = 100$, $\mu = 0.5$ and $\zeta = 0.3$ (exponential correlation function) and for values (3.30) (exponential cosine correlation function).

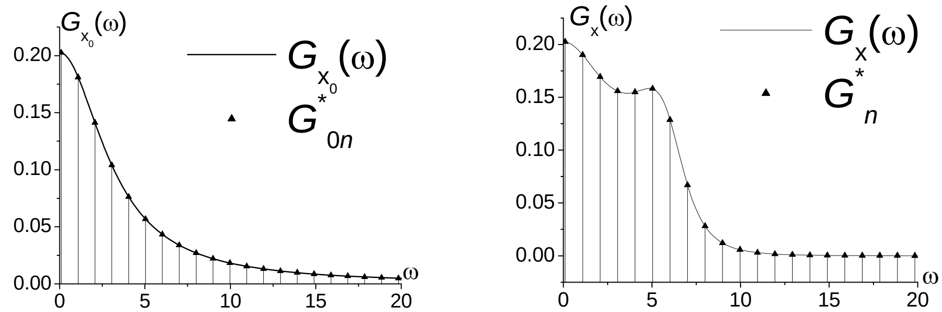

Fig. 4. One sided spectral densities for the exponential covariance function

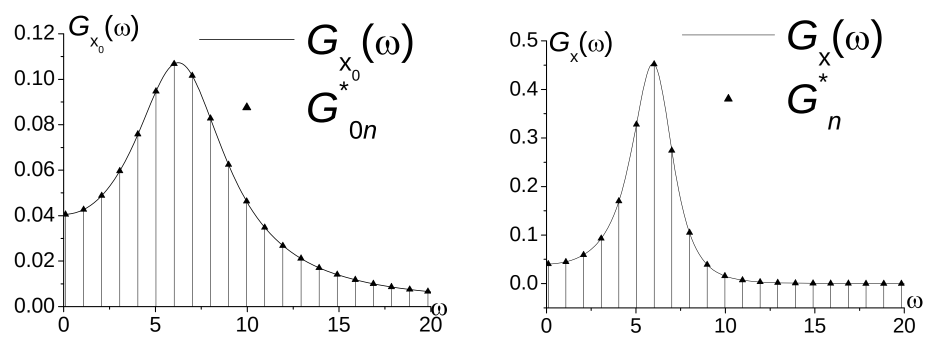

Fig. 5. One sided spectral densities for the exponential cosine covariance function

5. Conclusions

The results obtained in this paper demonstrate the applicability of canonical representations method to analysis of linear random vibrations. The relative errors between the exact r.m.s. output and its approximation obtained by the canonical representation method were less than 1% for both types of the considered input correlation functions. Moreover, the discrete spectral frequency components of the canonical representation of the input and output random processes are practically equal with the exact values obtained by Fourier transform of input correlation function and by transfer function method for output spectral density, respectively.

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