LASER FIELD EFFECTS ON THE INTERBAND TRANSITIONS IN DIFFERENTLY SHAPED QUANTUM WELLS

Ecaterina Cornelia NICULESCU[1], Nicoleta ESEANU[2]*, Ana SPANDONIDE[3]

The intense laser field effects on the interband transitions in GaAs–$Ga_{1-x}Al_xAs$ quantum wells having power-exponential confinement potentials are investigated within the effective mass and parabolic band approximations. Tthe energy levels, electron and hole wave functions as well as their corresponding overlap integrals and oscillator strength have been calculated. It is shown that the energy levels and radiative decay time associated with the electron interband transitions can be tuned to a large extent by a proper choice of the nanostructure design as well as by varying the laser field intensity. The noticeable increase of the radiative lifetime with the laser parameter can be attributed to the laser field-induced separation of the confined electron-hole pair.

Keywords: quantum well, laser field, oscillator strength, radiative decay time.

1. Introduction

Linear and nonlinear optical properties of the low-dimensional semiconductor heterostructures, such as quantum wells (QWs), quantum-well wires (QWWs), and quantum dots (QDs) are very important from both basic and technological points of view. Special interest has been focused on GaAs/$Al_xGa_{1-x}As$ heterostructures due to their direct band gap for Al concentration x below 0.40–0.45. The studies on quantum heterostructures open a new field in fundamental physics, and also offer a wide range of potential applications for optoelectronic devices [1-6].

In addition, recent advances in modern material growth techniques (molecular-beam epitaxy – MBE, metal organic chemical vapor deposition - MOCVD) and nanofabrication technology have made it possible to design low-dimensional semiconductor nanostructures with any desired potential shape. By varying the confining profile of a semiconductor QW, both the subband state energies and their wave functions changes, and so do different optical properties depending on them. Motivated by a large variety of technological applications, a lot of works on the QW structures under external perturbations, such as

[1] Prof., Department of Physics, University POLITEHNICA of Bucharest, Romania, e-mail: niculescu@physics.pub.ro
[2] Conf., Department of Physics, University POLITEHNICA of Bucharest, Romania, e-mail: eseanu@physics.pub.ro
[3] PhD student, Department of Physics, University POLITEHNICA of Bucharest, Romania

hydrostatic pressure, magnetic, electric and intense laser fields and also with special doping processes were performed [7-17].

It is well established that a high-frequency intense laser field (ILF) leads to major modifications in the shape of the confining potential of the QW nanostructures [10,18, 19].

Therefore, we consider that it is worth to investigate the effect of ILF on the energy levels and optical properties of a QW with different power-exponential confining potential. To the best of our knowledge this is the first study on such heterostructures under laser field. We found that the interband transition energy and corresponding emission wavelength in the electron-hole recombination process are strongly modified by the laser intensity. They also depend on the confinement profile.

The paper is organized as follows: in Section 2 the theoretical model for the intense laser field effects on the energy levels and optical properties of a QW with power-exponential confining potential is described. In Section 3 we discuss the numerical results on the intraband transition energy, oscillator strength and radiative life-time. Finally, our conclusions are summarized in Section 4.

2. Theory

The "dressing" of the electronic states in a quantum structure means a modification of both the energies and wave functions of the confined electrons under the action of a non-resonant intense laser radiation [19, 20].

We assume an electron (e) or a heavy hole (h) subjected to the confining potential of a quantum well (QW) having a power-exponential form:

$$V_j^{confin}(z) = -V_{0,j}\exp\left[-(2z/L)^p\right], \tag{1}$$

where $j = e,h$ (for electron and hole, respectively), $V_{0,j}$ is the conduction (or valence) band off-set, L is QW width, and z is the QW growth direction (Fig. 1). Such graded profiles describe the diffusion processes [22] which is used in order to realize a desired configuration of the energy levels (for example, a group of equidistant levels). A change in the parameter p leads to an adjustment of the confinement potential shape from the Gaussian well for $p = 2$ to the square well (SQW) for $p \to \infty$. Thus, the flexibility of the present approach in treating the confined carriers in a QW is guaranteed [23].

A nonresonant laser field of frequency Ω, linearly polarized on the z-axis and described by the vector potential $A_z(t) = A_0\cos(\Omega t)$ also acts on the carriers. The electric charge and the effective mass of the electron (hole) are denoted by $\mp e$ and m_j^*, respectively. The interaction dynamics is described by the time-dependent Schrödinger equation:

$$\left\{-\frac{\hbar^2}{2m_j^*}\left[\frac{\partial}{\partial z}\pm i\frac{eA_z(t)}{\hbar}\right]^2+V_j^{confin}(z)\right\}\psi(z,t)=i\hbar\frac{\partial}{\partial t}\psi(z,t). \qquad (2)$$

The temporal dependence may be transferred from kinetic energy operator to the potential energy one [24] by using the translations $z\to z\pm e\,A_z(t)/(m_j^*\Omega)$ in Eq. (2). So, we find a more convenient equation:

$$\left\{-\frac{\hbar^2}{2m_j^*}\frac{\partial^2}{\partial z^2}+V_j^{confin}\left(z\pm\alpha_j^{(0)}\sin(\Omega t)\right)\right\}\psi(z,t)=i\hbar\frac{\partial}{\partial t}\psi(z,t), \qquad (3)$$

where

$$\alpha_j^{(0)}=e\,A_0/(m_j^*\Omega) \qquad (4)$$

is the laser-dressing parameter for the electron (hole). In the high-frequency limit [25, 26] the laser-dressed eigenstates are solutions of the time-independent Schrödinger equation:

$$\left[-\frac{\hbar^2}{2m_j^*}\frac{\partial^2}{\partial z^2}+V_j^d(z)\right]\phi_j(z)=E\phi_j(z), \qquad (5)$$

where $V_j^d(z)$ is the laser-dressed confining potential with a general form [6, 16]:

$$V_j^d(z)=\frac{\Omega}{2\pi}\int_0^{2\pi}V_j\left(z\pm\alpha_j^{(0)}\sin\varphi\right)d\varphi. \qquad (6)$$

For some simple geometric confining QWs (square-SQW, parabolic-PQW, V-shaped QW) it was shown [6, 16, 27] that the integral in Eq. (6) can be analytically calculated. For more complicated profiles the numerical methods must be exploited. By using a finite difference method (FDM) we have solved Eq. (5) with the laser-dressed potential given by Eq. (6) and obtained the energies of the lowest electron (hole) subbands in the power-exponential QW, denoted by $E_1^{e(h)}$. These energy levels depend on the laser dressing parameter. Note that: $\alpha_h^{(0)}=\alpha_e^{(0)}m_e^*/m_h^*$ (see Eq. (4)) and consequently, we may use a single, more convenient dependence on $\alpha_e^{(0)}$, for all laser-dressing dependent quantities. For simplicity, we will further use the notation α_0 instead $\alpha_e^{(0)}$. Therefore, the interband transition energy, E_{tr}, will be written as:

$$E_{tr}(\alpha_0)=E_g^i+E_1^e(\alpha_0)+E_1^h(\alpha_0), \qquad (7)$$

where E_g^i is the bandgap of the QW.

It is well established that the laser-dressing of the QW confinement potential induces a delocalization of the carriers [28]. Thus, the probability of finding the electron (hole) inside the QW, for the ground state, being defined by:

$$P_j(\alpha_0) = \int_{-L/2}^{+L/2} \left|\phi_j^{(1)}(z)\right|^2 dz \tag{8}$$

will depend on the laser parameter. Here, $\phi_j^{(1)}(z)$ is the first subband wave function of the carrier, $j = e, h$.

In order to clarify the ILF effects on the optical properties of the carriers confined in QW nanostructures it is worth to calculate some basic quantities such as: the overlap integral of the electron-hole pair, oscillator strength of the exciton ground-state, and radiative decay time of the interband transition.

The overlap integral of the electron and hole ground state wave functions will also be an implicit function of α_0:

$$I_{e-h}(\alpha_0) = \int_{-\infty}^{+\infty} \phi_e^{(1)}(z)\phi_h^{(1)}(z)dz\,. \tag{9}$$

By using the envelope-function approximation the oscillator strength f_{eh} for the exciton ground state is given [29] as:

$$f_{eh} = \frac{E_K}{E_{tr}}\left|\int_{-\infty}^{+\infty} \phi_e^{(1)}(z)\phi_h^{(1)}(z)dz\right|^2 \tag{10}$$

where the Kane energy of GaAs is E_K = 25.7 eV.

The oscillator strength f_{eh} not only describes the strength of absorption lines but also relates to the radiative decay time τ defined [30] as:

$$\tau = \frac{2\pi\varepsilon_0 m_0 c^3 \hbar^2}{n e^2 E_{tr}^2 f_{eh}} \tag{11}$$

where n is the refractive index of GaAs, and ε_0, m_0, $\hbar$, e are the fundamental physical constants with their usual meanings.

3. Numerical results and discussion

For numerical calculations of the subband energy levels and wave functions in the QW under study, the following values of the involved parameters have been used: L = 5 nm, $m_e^* = (0.0665 + 0.0835x)m_0$ [27], $m_h^* = (0.3497 + 0.122x)m_0$ [31] (where x denotes the concentration of Al in the structure), $V_{0,e}$ = 130 meV, and $V_{0,h}$ = 118 meV [32]. The bandgap of $Al_xGa_{1-x}As$ at room temperature $T = 293$ K is taken as a function of Al concentration [33]:

$E_g = (1.424 + 1.247x)$ eV and $n = 3.5$. The laser parameter was varied in the range [0; 4 nm], and the values of the shape parameter p were taken as: 4; 8; 10 and $p \to \infty$ (SQW).

Laser-dressed confinement potential

If the semiconductor heterostructure is irradiated with a THz non-resonant intense laser field (ILF), it is well-known that the electrons and holes will "see" different time-averaged laser-dressed confinement potentials [34]. Figure 1 (a, b) displays the confinement potential for electrons, modified by the ILF, as a function of the z-coordinate (given in terms of an effective Bohr radius $a_B^* \cong 10$ nm). Figure 1 (a, b) displays the confinement potential for electrons, modified by the ILF, as a function of the z-coordinate (given in terms of an effective Bohr radius $a_B^* \cong 10$ nm).

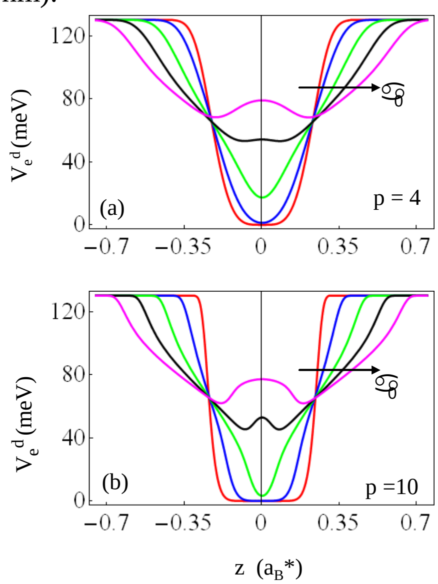


Fig. 1. (Color online) Confinement potential for electron, modified by the laser-dressing parameter α_0 = 0 (red); 1 (blue); 2 (green); 3 (black); 4 nm (pink); a) for p = 4 and b) for p = 10 (see Eq. (1)); the z-coordinate is taken as a multiple of the effective Bohr radius $a_B^* \cong 10$ nm).

The values of the laser parameter are: α_0 = 0; 1; 2; 3; 4 nm. The shape parameter p of the Eq. (1) is 4 and 10, respectively. It is seen that α_0 dramatically modifies the confining potential shape for electrons. Up to $\alpha_0 < L/2$ two effects are noticeable: (i) while the effective “dressed” well width (i.e. the lower part of the QW) decreases with the laser intensity, the width of the upper part of this "dressed" potential increases; (ii) a decrease of the effective well height. For $\alpha_0 > L/2$ (see the pink curve) a supplementary barrier having a "hill"-form appears into the well region and, as a consequence, the formation of a double quantum well (DQW) potential is predicted. Similar laser-induced phenomena were reported for differently shaped QWs [10, 12]. Comparing the Figs. 1a and 1b one can observe that the lower part of the QW is larger when p increases and the laser-induced "hill"-form is favored.

Figure 2 (a, b) illustrates the confinement potential for holes under the same conditions as for the electrons. In contrast with the confinement profile for electrons, the laser parameter augmentation has little effect on the heavy-hole potential because $\alpha_h^{(0)} = \alpha_e^{(0)} m_e^* / m_h^*$ (see Eq. (4)).

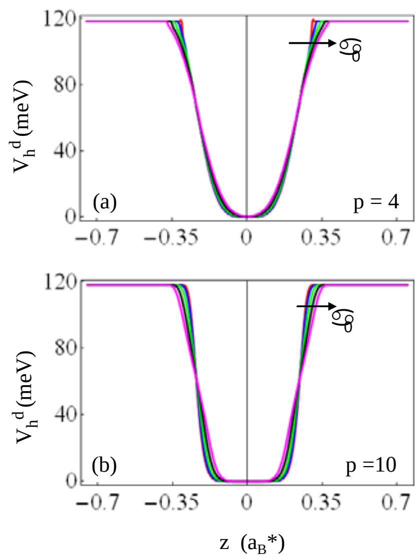
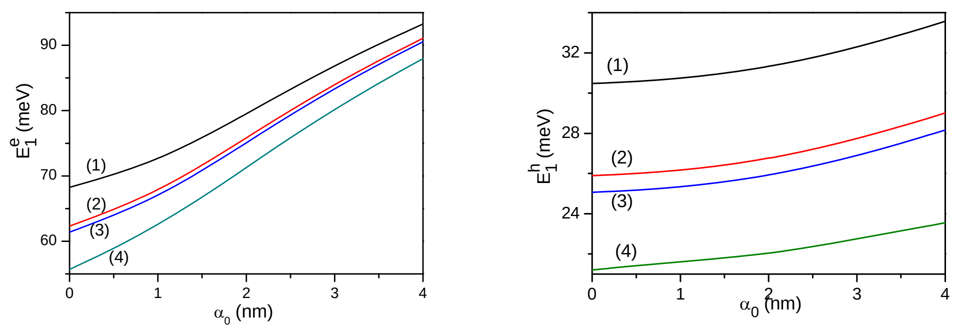


Fig. 2. (Color online) Same as Fig. 1 but for holes.

Laser-dressed energies and localization probabilities

By using the one-dimensional FDM for different values of the laser-dressing parameter Eq. (5) was numerically solved. The method allows us to obtain all the eigenvalues and corresponding eigenfunctions for the electron and hole energy. Here we investigate only the interband transition from the ground electron level to the ground hole level. In Figs. 3a and 3b the ground-state electron and hole energies, respectively, are plotted as functions of the laser parameter α_0 for three finite values of the shape parameter p = 4; 8; 10, and for SQW ($p \to \infty$).

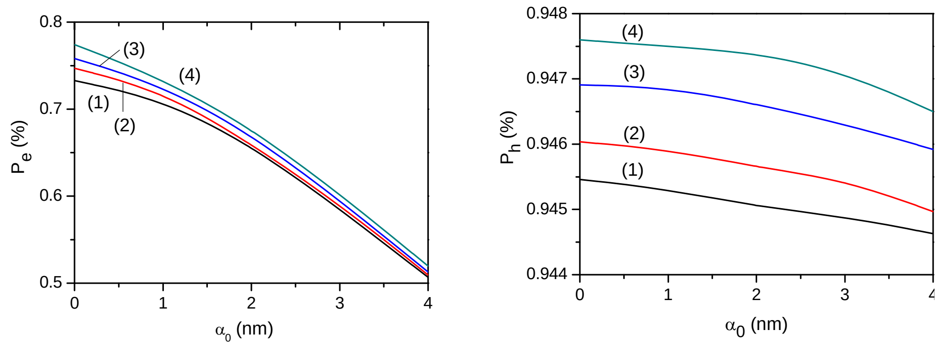


(a) (b)

Fig. 3. (Color online) Electron (a) and hole (b) ground-state energy vs. laser parameter for p = 4 (black, (1)); 8 (red, (2)); 10 (blue, (3)), and for SQW ($p \to \infty$) (green, (4)).

From Fig. 3a we notice that a characteristic blue-shift of the electronic ground state energy occurs, as expected [10, 12, 35]. This is a consequence of the laser-dressing effect on the confinement profile (Fig. 1 a, b). The energy raising is more pronounced for $\alpha_0 < L/2$, and for further large laser intensities the electronic level becomes less sensitive to the ILF, i. e. a coalescence of the bound states seems to occur. A similar phenomenon was reported in Refs. [10, 12, 36]. The laser-induced push-up effect also depends on the shape parameter (p). The ground energy level is reduced by higher values of p due to the weaker geometric confinement. An analogous but much weaker behaviour is visible for the ground-state hole energy (Fig.3b).

In Figs. 4a and 4b the probability of finding the carrier (electron and hole, respectively) inside the power-potential QW is plotted as a function of the laser parameter α_0, in the same conditions as in Figs. 3a and 4a. We found that the localization probability of the electron (P_e) is strongly diminished by the laser field intensity in agreement with the ILF-induced push-up effect in Fig. 3a. This P_e also depends on the shape parameter p, namely it is reduced by higher values of p due to the weaker geometric confinement. The effect is more pronounced for small α_0. The localization probability of the hole (P_h) is softly influenced by the laser field. The explanation is directly related to the unaffected confinement profile for holes according to Figs. 2a and 2b.



(a) (b)

Fig. 4. (Color online) Probability of finding the carrier: (a) electron and (b) hole, inside the QW, vs. laser parameter; same conditions as in Fig. 3.

Optical properties

By summing-up the results above, Fig. 5 displays the grouped data plot of the transition energy and corresponding emission wavelength in the electron-hole recombination process $\lambda = \frac{hc}{E_{tr}}$ (*h* and *c* are the well-known fundamental physical constants) as functions of the laser parameter.

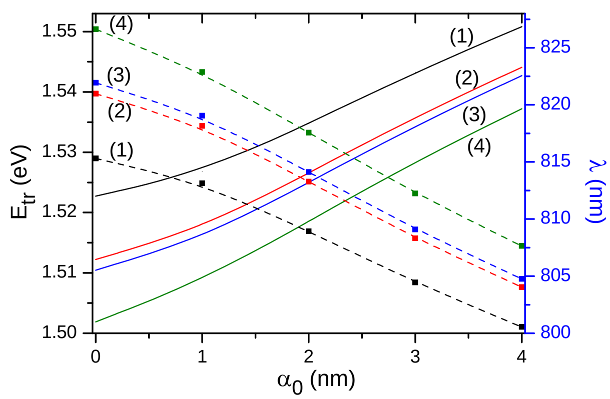


Fig. 5. Grouped data plot of the transition energy (solid curves) and corresponding emission wavelength (dashed curves) in the electron-hole recombination process vs. laser parameter; same conditions as in Fig. 3.

We found a global increase of almost 30 meV in the transition energy (E_{tr}) as α_0 raises from zero up to 4 nm. This augmentation will establish the wavelength range of the QW nanostructure's tunability: $\lambda \in (800\,nm, 815\,nm)$ for p = 4 and $\lambda \in (807\,nm, 827\,nm)$ for SQW ($p \to \infty$). The transition energy also depends on the shape parameter (p). It is reduced by increasing p due to unlike behaviour of the energy levels E_e and E_h (see Figs. 3a and 3 b).

To better understand the ILF effects on the optical properties of the QW nanostructures under study we have calculated the overlap integral of the electron-hole pair, oscillator strength of the exciton ground-state, and radiative decay time of the interband transition. Fig. 6 shows the squared overlapping integral (I^2_{e-h}) of the electron and hole as a function of the laser parameter.

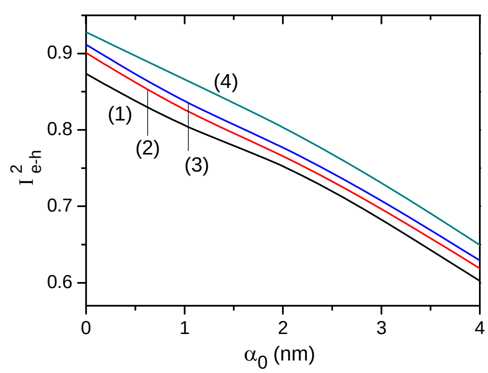


Fig. 6. Squared overlapping integral vs. laser parameter; same conditions as in Fig. 3.

It is obvious that the laser-induced effects reduce this quantity by a factor 1.5. Such a behaviour is mainly due to the spreading of the electron wave function (WF) over the enlarged well, whereas the corresponding WF of the hole remains almost unaffected.

On the other hand, the increasing shape parameter p leads to an enhanced squared overlapping integral for the whole range of laser intensity. This effect is connected with geometrical confinement of the carriers.

The oscillator strength f_{eh} of the exciton ground state is plotted as a function of the laser parameter (Fig. 7). This quantity strongly decreases (almost 1.5 times) as α_0 raises from zero up to 4 nm. We explain this effect as follows: the squared overlapping integral decreases (Fig. 6) and E_{tr} increases (Fig. 5) with laser parameter; so, according to Eq. (10) f_{eh} is diminuted by the ILF.

Also, the variations of E_{tr} and I^2_{e-h} with p imply a modification in f_{eh} so that it increases with p (see Eq. (10) and Figs. 5, 6).

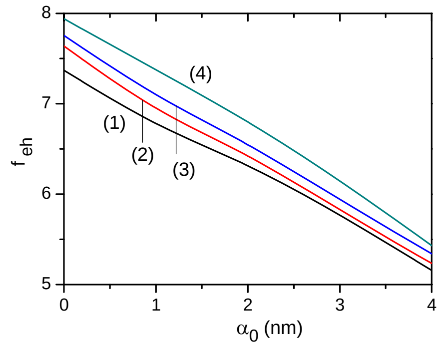


Fig. 7. Oscillator strength of the exciton ground state vs. laser parameter; same conditions as in Fig. 3.

The radiative decay time (τ) of the interband transition also depends on the laser field intensity and on the shape parameter p (Fig. 8).

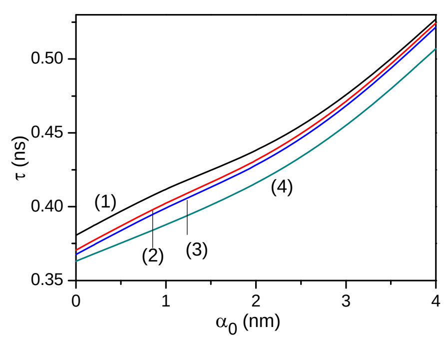


Fig. 8. Radiative decay time of the interband transition vs. laser parameter; same conditions as in Fig. 3.

The ILF causes a significant enhancement of the radiative lifetime due to the decreasing of the oscillator strength. In fact, when the laser parameter increase, there is a competition between the quickly decreasing of f_{eh} and a slowly increasing of the E_{tr} (see Eq. (11)). Thus, the rising α_0 leads to an augmentation of the radiative decay time in a non-linear manner with a change of slope which is mainly generated by the variation of f_{eh}.

4. Conclusions

We have studied the effects of both confinement potential shape and intense laser field on the intraband transitions in GaAs–$Ga_{1-x}Al_xAs$ quantum wells having power-exponential confinement potentials. Theoretical calculations were performed in the framework of the effective-mass and non-degenerate parabolic band approximation, by using a finite difference method. It has been shown that the energy levels and the localization of the carriers within the well is strongly affected by the shape of the power-exponential potentials. Under laser fields the wave function of the electron is spreaded in the lateral AlGaAs barriers leading to a delocalization of the particle, whereas the hole localization is weakly affected by the radiation. Consequently, the electron-hole separation is an increasing function of the laser parameter α_0, an effect which is associated with a reduction in the oscillator strength. We also find a noticeable increase of the radiative decay time of the interband transition with the laser parameter. This phenomenon could be attributed to the laser-induced polarizability of the confined electron-hole pair.

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