IMPROVEMENT OF THE DYNAMIC MODEL OF POLLUTED INSULATORS UNDER AC VOLTAGE BASED ON THE ELECTRO-THERMAL CHARACTERISTICS

Sihua WANG¹, Wanli XU², Mingxing TIAN³

The flashover dynamic model of polluted insulators can simulate the dynamic change process of the development of discharge, determine the critical voltage that insulators can withstand, and provide a reference for selection of insulators and optimization for structure of insulators. In this paper, a new flashover dynamic model of polluted insulators under ac voltage based on electro-thermal characteristics of insulators is proposed. This model is suitable for the prediction of discharge development under different ambient temperatures. The critical flashover voltages obtained by this model are in good agreement with those in experiments.

**Keywords**: dynamic model; thermal characteristics; insulator flashover; AC arc

1. Introduction

In power systems, insulators are widely used to insulate and connect various electrical components. When the air humidity is high, the dirt, deposited on insulator surface, will be wet, resulting in a reduction in insulators’ electrical characteristics or a flashover. This reduction in electrical performance depends on many parameters, such as the properties of the insulator material and its stability, the shape of the insulator, the type and distribution (i.e. uniform or uneven distribution) of the deposits in the pollution layer and the environmental conditions (fog, dew, rain, snow or ice...), leakage current caused by locally heating and partial arcs. In addition, long distance transmission lines often need to pass through different areas with complex geography climate environments. Therefore, proper selection of insulators, depending on the environmental conditions in different regions, is of great significance in reducing the occurrence of insulator flashover accidents. Predicting the critical flashover voltage by constructing a mathematical model contributes to optimize the structure of insulators and select materials of insulators, and we can use the mathematical model in place of time consuming and repetitive tests.

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Literature [1] proposed a DC dynamic model based on taking potential gradient criterion as a development criterion. Jabbari et al. [2] established a dynamic model based on the finite element method. YANG Qing et al. [3] took the influence of surface branch beneath the arc channel into consideration and proposed an AC dynamic flashover model based on the circuit network. However, there are two inevitable problems. On the one hand, in order to simplify the calculation, the existing dynamic models ignored the thermal characteristics of the arc and the pollution layer, which have an important influence on the flashover mechanism, proved by a large number of experimental studies [4-5]. On the other hand, the majority of literatures for flashover dynamic models use arc parameters (A and n), and both A and n have been considered as invariant parameters in these documents. However, according to Slama et al. [6], the arc parameters are dynamic parameters that depend on the equivalent electrical circuit and thermal characteristics of the discharge.

Therefore, this paper will build a developed dynamic model of polluted insulators under AC voltage based on the electro-thermal characteristics, which considers the effects of arc temperature and ambient temperature. The effectiveness of the improved model was proved by comparing the computer simulation results with other researchers’ experimental results and other existing model simulation results.

2. Background

Obenaus [7] first proposed quantitative analyses of arcs on contaminated surfaces, the model of the partial arc and the unabridged part of the pollution layer (see Fig.1). The electrical equation is given as follows

\[ U = U_E + U_{arc} + IR_p(L - x) \]  

where \( U \) is the applied voltage; \( U_E \) denotes voltage drop of electrode that can be ignored; \( I \) represent leakage current; \( R_p(x) \) means the resistance of the the unabridged part of the pollution layer; \( x \) is the arc length and \( U_{arc} \) is the arc channel voltage. \( U_{arc} \) can be defined as follows [6]

\[ E_{arc} = \frac{U_{arc}}{x} = A \times I^{-n} \]  

Fig. 1 A model of pollution flashover circuit [7]
where $A$ and $n$ are arc parameters that depend on the environmental conditions in which the discharge is burning (i.e., temperature, pressure, etc.). The values of $A$ and $n$ vary from one literature to another due to different environmental conditions (i.e., experimental conditions) [8-12]. Some values of $A$ and $n$ used in the flashover literatures are shown in Tables 1 and 2.

### Table 1

<table>
<thead>
<tr>
<th>Authors</th>
<th>Ref</th>
<th>$A$</th>
<th>$n$</th>
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<tbody>
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<td>Wilkins</td>
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<td>Hampton</td>
<td>[9]</td>
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<td>0.240</td>
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<tr>
<td>Farzaneh</td>
<td>[11]</td>
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</tr>
<tr>
<td>Ghosh</td>
<td>[12]</td>
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<td>0.590</td>
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<tr>
<td>Topalis</td>
<td>[8]</td>
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<td>0.374</td>
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### Table 2

<table>
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<th>Electrolyte</th>
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</tr>
<tr>
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<td>461</td>
<td>0.42</td>
</tr>
<tr>
<td>FeCl2</td>
<td>270</td>
<td>0.66</td>
</tr>
<tr>
<td>CuSO4</td>
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</table>

3. Improvement of the Dynamic Model of Polluted Insulators under AC Voltage

3.1. Improved Dynamic Model for AC Insulators Flashover Based on Electro-thermal Characteristics

#### 3.1.1. Equivalent electrical circuit and calculation of circuit parameters

The equivalent electrical circuit, shown in Fig. 2, simulates a contaminated insulator that has produced a localized arc of length $x$ and a residual layer of length $(L-x)$. The electrical equation of this model is given by

$$U = R_{arc}(x)I + j\omega R_{arc}(x)I + \frac{R_p(x)I}{1 + j\omega R_p(x)C_p(x)}$$

(3)

with
\[ R_{\text{arc}}(x) = r_{\text{arc}} \times x = x \times \frac{\rho_{\text{arc}}}{s_{\text{arc}}} \; ; \; \Gamma_{\text{arc}}(x) = l_{\text{arc}} \times x \; ; \; R_p(x) = r_p \times (L - x) = \frac{\rho_p}{s_p} \times (L - x) \]

\[ C_p(x) = \frac{c_p}{L - x} = \varepsilon_p \times \frac{s_p}{L - x} \]

in which \( R_{\text{arc}} \) and \( \Gamma_{\text{arc}} \) are the resistance and inductance of the arc; \( R_p \) and \( C_p \) mean the resistance and capacitance of the pollution layer respectively; \( \omega = 2\pi f \); \( f \) is the frequency; \( \varepsilon_p \) denotes the relative dielectric Constants; \( r_{\text{arc}}, \rho_{\text{arc}} \) and \( s_{\text{arc}} \) are the linear resistance of the arc channel, resistivity and cross-sectional area respectively. \( L \) denotes the insulator leakage distance. Because the linear inductance \( l_{\text{ac}} \) is considered to have very low value [13], therefore, the electrical equation can be re-expressed by eq. (4).

\[ U = R_{\text{arc}}(x)I + \frac{R_p(x)I}{1 + j \omega R_p(x)C_p(x)} \] (4)

According to Wilkins[14], the resistance of the unabridged part of the pollution layer \( R_p(x) \) is given by eq.(5).

\[ R_p(x) = r_p \times \left( (L - x) + \frac{b}{2\pi} \ln \frac{b^2}{4 \times \pi^2 \times a_{\text{arc}}^2} \right) \] (5)

where \( r_p \) is the linear resistance of the unabridged part of the pollution layer; \( b \) and \( a_{\text{arc}} \) represent width of the pollution layer and arc radius, respectively.

3.1.2. Calculation of \( A \) and \( n \)

Slama et al.[6] analyzed the physical significance of the arc parameters \( A \) and \( n \), and redefined the \( A \) and \( n \) under AC, based on the equivalent electrical circuit and the thermal characteristics of the discharge, as follows

\[ A = r_p \frac{k}{\alpha} \left( \frac{\pi \times \lambda(T_{\text{arc}}) \times T_{\text{arc}}}{r_p} \times \frac{\alpha}{k} \right)^{(n+1)/2} \] (6)

\[ n = \frac{k(k - 1)}{k - 1 + \left( r_p c_p \omega \right)^2} \] (7)

with

\[ k = \frac{\alpha \cdot r_{\text{arc}}}{r_p} \; ; \; \alpha = 1 + \left( \omega \cdot \sigma_p \cdot \varepsilon_p \right)^2 ; \; \omega = 2 \cdot \pi \cdot f \; ; \; c_p = \varepsilon_p \cdot s_p \; ; \; r_p = \frac{R_p(x)}{L - x} \]

in which \( f \) denotes the frequency; \( c_p, \varepsilon_p \) and \( s_p \) are the linear capacitance, dielectric constant and cross-sectional area of the pollution layer, respectively. \( s_p \) can be simplified to the cross-sectional area of the partial arc \( s_{\text{arc}} \). The thermal
Improvement of the dynamic model of polluted insulators under AC voltage based on the (...) 165

conductivity $\lambda(T_{arc})$ depends on the arc temperature $T_{arc}$ and species constituting the arc column [6].

Based on the heat equation and the Mayr arc model, Hadjrioua expressed the instantaneous variation of the arc temperature as follow [15]

$$\frac{dT_{arc}}{dt} = \frac{1}{m \times C_v} \times \left[ \sigma_{arc} \times E_{arc}^2 - r_{arc} \times \left( \frac{\pi \times \lambda(T_{arc}) \times T_{arc}}{r_{arc}} \right)^{(n+1)/2} \right]$$

(8)

where $T_{arc}$ is the arc temperature; $m$ denotes the mass of the added air plasma; $C_v$, $\sigma_{arc}$ and $r_{arc}$ are, the specific heat, the arc conductivity and the arc linear resistance, respectively [15].

By calculating the instantaneous variation of the arc temperature at every time-interval of the dynamic model, the instantaneous value of the arc temperature during the development of the partial arc can be obtained.

3.1.3. The Effect of Ambient Temperature on Contamination Conductivity

The pollution on the surface of the insulator ionizes in solution, by being exposed to moisture, will cause a higher conductivity than purified water, it has a strong temperature dependence, the fundamental cause of which is actually the change in the viscosity of the solution, not the change of the ions itself in the solution [18]. The correction formula for the conductivity of the pollution layer at different ambient temperatures is given as follows [16]

$$\sigma_\theta = \sigma_{25} \left( \frac{\eta_\theta}{\eta_{25}} \right)^{-d}$$

(9)

where $\sigma_\theta$ (S/m) and $\sigma_{25}$ (S/m) denotes the conductivity of the solution when the temperature is $\theta$ (°C) and 25°C, respectively; $\eta_\theta$ (N·s/m²) represents the viscosity of the solution when the temperature is $\theta$ (°C); $\eta_{25}$ (N·s/m²) means the viscosity of the solution at a temperature of 25°C, which is equal to 0.0000083 N·s/m²; $d$ is a dimensionless constant, which is about 0.806 ~0.933, and its average is 0.877. When the temperature is $\theta$ (°C), the viscosity of the solution is described as follows [16]

$$\log \frac{\eta_\theta}{\eta_{25}} = \frac{B(25-\theta) - C(25-\theta)}{\theta + D}$$

(10)

where B, C and D are constants, with the value of 1.1278, 0.001895°C⁻¹ and 88.93°C, respectively. Therefore, according to both (9) and (10), we obtain the functions of solution conductivity as follows

$$\sigma_\theta = \sigma_{25} \times 10^{-d \left[ \frac{B(25-\theta) - C(25-\theta)}{\theta + D} \right]}$$

(11)
On the other hand, when the ambient temperature is 20°C, the surface conductivity of the dirty layer is presented as follows

$$\sigma_{20} = (369.05 \rho_{\text{ESDD}} + 0.42) \times 10^{-6}$$  \hspace{1cm} (12)

where $\rho_{\text{ESDD}}$ is equivalent salt density.

Therefore, by arranging eqs.(11) and (12), we get the function of solution conductivity $\sigma_\theta$ depends on $\theta$ and $\rho_{\text{ESDD}}$

$$\sigma_\theta = (415.0633 \rho_{\text{ESDD}} + 0.4736) \times 10^{-d} \left[ \frac{B(25-\theta) - C(25-\theta)}{\theta + D} \right]^{-6}$$  \hspace{1cm} (13)

From eq.(13), the relation curves that the conductivity $\sigma_\theta$ varying with temperature $\theta$ are obtained, when $\rho_{\text{ESDD}}$ takes different values. As presented in Fig. 3, the influence of temperature on the conductivity is the greater.

![Fig. 3. Predictive value of temperature influence on conductivity](image)

3.1.4. Influence of Ambient Temperature on Reignition Condition of AC Arc

The fundamental difference between AC and DC insulator flashovers is the reignition process of AC arcs. In order to maintain the burning of the arc, the applied peak voltage $U_m$ must be greater than the arc reignition voltage peak $U_{cx}$, which is formulated as follows

$$U_{cx} = U_d x$$  \hspace{1cm} (14)

where $U_d$ is the minimum breakdown voltage of the residual arc.

The mathematical model of the arc reignition is formulated as follows [17]

$$U_d = U_{da} \left[ 1 + \frac{S_0 / S_b - 1}{1 + \frac{4a^*(S_0 - S_b)M_d}{\eta^2}} \right]^{-1/\beta}$$  \hspace{1cm} (15)
where $U_{da}$ is the voltage gradient of air breakdown; $S_0$ denotes the function of heat flux at the initial arc temperature, with the value of 350.8 J/(m·s) at 3000 K; $S_b$ represents the function of heat flux at ambient temperature; $a^*$ means the fitting constant, with the value of $3.78 \times 10^{-6} \text{m}^3/\text{J}$; $\beta$ is the fitting constant, with the value of 1.778; $r_b$ denotes the arc boundary radius; $t_d$ is the time to reach the maximum value of AC. The ambient temperature will affect the heat flux $S_b$, and it can be expressed as follows[18]

$$S_b = 0.0002 \cdot T^\beta$$

(16)

where $T$ is the temperature in K; the unit of $S_b$ is J/(m·s). By substituting corresponding $S_0$, $S_b$ and $U_{da}$ (dependently on temperature) in eq.(16), we can obtain the reignition voltage gradient for different values of ambient temperature. Take, for example, the temperatures are 0°C, 20°C and 40°C, the $U_{da}$ values corresponding to be 5620 V/cm, 5233 V/cm and 4900 V/cm, respectively. Therefore, one can obtain the voltage gradient predictions of reignition. As shown in Fig.4, when increasing the peak value of the arc current at the preceding half cycle of the AC, the minimum breakdown voltage of the residual arc decreases.

![Fig.4 Predictive value of temperature influence on reignition voltage gradient](image)

3.2. Simulation of Dynamic Model for AC Pollution Flashover of Insulators

If the impedance $Z_{eq}$ of the equivalent circuit decreases with the increase of the discharge length $x$, the arc develops, which is described as follows[19]

$$\frac{d|Z_{eq}|^2}{dx} \leq 0$$

(17)

According to Slama et al. [6], eq.(18) can be reformulated as follows

$$0 < k < 1$$

(18)

If the condition of eq.(15) is satisfied, the arc will move forward at velocity $V_d$, and $V_d(t)$ is expressed as[13]
\[ V_d(t) = \frac{I}{q} \] (19)

where \(q\) is the average charge per unit length, which can be considered as a constant [13].

Fig. 5 shows the flow chart of the AC pollution flash dynamic model. When it comes to the arc development part, the arc will extend to \((x_i = x_{i-1} + dx)\), with a speed \(V_d\). Then judging whether the arc reignition conditions of this ambient temperature condition are satisfied. If not, it is considered that when the leakage current crosses zero, the arc will not extinguish but become weaker and shorter. Otherwise, the arc flashover judgment will be performed directly. And if the flashover condition is not satisfied, the program returns to the part of calculation model parameters, and the cycle is repeated until the arc length equals the creepage distance, i.e. the flashover occurs.

![Simulation flowchart of polluted flashover](image)

**4. Simulation results and comparative analysis**

4.1. Analysis of simulation results of Dynamic Model for AC Pollution Flashover of Insulators

According to Fig.5, we performed the dynamic simulation experiment for a plate insulator model with length \(L = 10\) cm and width \(b = 4\) cm, and adopted the conditions consistent with the experiment in literature[20] as parameters for simulation calculations. The ambient temperature is set to 20°C, \(r_p = 5\) kΩ/cm. The
initial arc length is 1/100 of the leakage distance (i.e. the starting arc length $x_0 = 1/100L$).

It takes the time $t=0$ as the starting point of arc development to calculate the initial value of each circuit parameter in Fig.2. The starting voltage $U_0 = 100$ V and the time step is set to 0.1 ms. At this point, the initial value of the circuit parameters is solved. Then it is judged according to the arc development criterion-eq.(18) whether the arc develops or not. Raising the applied voltage until the criterion is met. The applied voltage rises by 10 V each time, as shown in Fig.5. After the arc development criterion is satisfied, the arc advances at the speed calculated from eq.(19), resulting in an increase in arc length of the local arc. Equations (14) - (16) is used to calculate the reignition peak of the arc at this ambient temperature. According to the arc re-ignition criterion, it can be judged whether the arc can reignite after the current zero crosses, or not. If the criterion is met, the arc length remains the original length. Otherwise the arc will be shortened. Finally, whether the arc is flashover depends on whether the arc length reaches the leakage distance.

Fig. 6 Simulation result of AC dynamic flashover model

Fig. 7 Simulation result of Analytic parameters A
Fig. 6-8 show the results of a typical simulation experiment. Fig. 6 presents the change of leakage current over time, which can be divided into three phases. The first phase is the beginning of discharge, and the current rapidly rises; The second phase is the arc elongation phase, in which the current slowly increases, and lasts longer; The third phase is the flashover phase, when the arc extends to the critical arc length, the leakage current starts to increase sharply from the critical value. Then flashover occurs when the arc length exceeds the creepage distance. The above process is in line with the actual arc development process. Fig. 7 and Fig. 8 show the variation of the values of arc parameters $A$ and $n$ with time, respectively, indicating that $A$ and $n$ are not a fixed value in the arc development process, but are dynamic parameters affected by the leakage current and the environmental conditions.

4.2. Comparison of simulation results

4.2.1. Comparative Analysis of Static Parameters Model and Dynamic Parameter Model

In order to verify the effectiveness of this dynamic model, we compared the results of actual test[20] with the results of model calculation, and the predictor curves that treats the arc parameter as a constant. The value of the static arc parameters adopts the values in the literature [8, 11, 12], respectively.

It can be seen from Fig. 9 that the critical flashover voltage rises as the linear resistance of the pollution layer increase. The dynamic model prediction values provided in this paper are closer to the experimental values, which confirm that this model is effective.
4.2.2. Comparison of Simulation Results and Experimental Values under Different Ambient Temperatures

In order to verify that this model can be used for flashover voltage prediction at different ambient temperatures, we adopted the conditions consistent with the experiment in literature [21] as parameters for simulation calculations, and compared the results of actual test [21] with the results of model calculation with temperatures between $0^\circ$C and $40^\circ$C. As is shown in Fig.10, the critical flashover voltage gradually decreases as the ambient temperature increases. The curve of predicted value is linked to the experimental data, confirming that the model can reflect the influence of ambient temperature on the AC flashover voltage, to a certain extent.

![Fig. 10 Relationship between critical voltage and temperature](image)

5. Conclusions

(1) Ambient temperature affects the conductivity of the unabridged part of the pollution layer and the reignition voltage of AC arc, and thus affects the flashover voltage. Therefore, it should be considered when building the flashover dynamic model of polluted insulators.

(2) Based on the dynamic value of arc temperature, it is effective to use arc dynamic parameters ($A$ and $n$) in the flashover dynamic model under ac voltage.

(3) In this paper, a dynamic model of polluted insulator under ac voltage is proposed, in which the influence of the dynamic change of arc parameters, arc temperature and ambient temperature on the insulator flashover are considered. It provides a new idea for prediction of the critical voltage.

(4) The simulation results of this model still have some difference from the results of actual experiments, for the fact that the arc shape is considered as cylindrical plasma, and some other deficiencies of our proposed model, it still should be studied to further validate, refine, and improve the model in the future.

Acknowledgments

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