OPTIMAL POWER FLOW BASED ON DIFFERENTIAL EVOLUTION OPTIMIZATION TECHNIQUE

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This article deals with one of the best evolutionary algorithms that has been used to solve the Optimal Power flow (OPF) problem. It basically consists from three operation steps, mutation, crossover, and selection. The objective of the Optimal Power Flow is to find a steady state operation point which minimize the objective function. In this article, the objective function is the total fuel cost of generating units, the total active power losses in the transmission lines and the total load bus voltage deviation separately for each one. Minimization of objecting function can be satisfied by choosing a suitable optimal control variables while maintaining an acceptable system performance of the state variables in terms of their limits. The control variables that used in this algorithm are the magnitude voltage of the generator, the tap changer of the transformer, the injection reactive power compensative devise and the active power of the generator except the slack generator. The state variables are the reactive power of the generators, the load bus voltages and slack active power. The proposed algorithm has been applied on the IEEE 30 bus system and gives good result when compare with other optimization techniques.

Keywords: Optimal power flow, Differential Evolution (DE), Fuel cost, active power losses, Voltage deviation

1. Introduction

Nowadays, Facilities facing the rapid increase in the demand for the electricity with slow strengthening projects due to financial and political problems. The favorable of operating and planning requires the consideration of various factors such as reducing the generation cost, losses, pollution, etc. [1]. In this regard, Optimal Power Flow (OPF) generally seeks to improve a range of objectives under certain constraints [2]. OPF is a mathematical approach for a particular problem of the global power system optimization which aims to identify

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the least control movements to maintain the power system in the most desirable state while meeting the engineering and economic constraints [3, 4].

In the past, the problem of OPF was solved using many traditional optimization techniques such as linear programming, non-linear programming, quadratic programming, Newton method, etc. [5]. Unfortunately, some difficulties are associated with these techniques in handling non-linear, discrete continuous functions and constraints. Regardless for these traditional techniques have disadvantages of waste of time, converge on the local minimum and mathematical formulation of the problem with more restrictions [6]. To overcoming these deficiencies, many modern stochastic algorithms depend on artificial intelligence are developed to solve the OPF problem like the Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), etc.

R. Storn and K. Price suggested a new heuristic algorithm called Differential Evolution (DE) to meet the requirement of the optimization problem. DE algorithm is aimed to minimize problem which may non-differential, nonlinear and multimodal. DE is simplicity to perform and it has better convergence for optimization [7]. A. Shrivastava and H. M. Siddiqui describes single objective DE to constraint the optimization problem showing that DE algorithm is efficiently working in stressed conditions and also with increasing load condition [8]. H. R. Cai, C. Y. Chung used Differential Evolution of transient stability constraints in optimal power flow (OPF) problems [9].

2. Problem Formulation of Optimal Power Flow

A. Objective Functions

Three different objective functions in this article are considered separately for each one to determine the effectiveness of the proposed algorithm.

- **The fuel cost**

The total fuel cost of thermal generation units can be formulated as:

$$F_c = \sum_{i=1}^{N_g} F_{ci} \tag{1}$$

$$F_{ci} = a_i P_{gi}^2 + b_i P_{gi} + c_i \tag{2}$$

where F_c is the total fuel cost of generating units; F_{ci} is the fuel cost of the i^{th} generator; a_i ,b_i ,c_i are the fuel cost coefficients of i^{th} generator; P_{gi} is the active power of i^{th} generator; N_g is the number of generators including the slack bus [10].

- **The active power losses**

The active power losses of the transmission line can be formulated as:

$$P_{loss} = \sum_{k=1}^{N} g_{(i,j)} (V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)) \quad (3)$$

where N is the number of transmission line; $g_{(i,j)}$ is the line mutual conductance between bus i and j; V_i, V_j are the per unit magnitude voltages of buses i and j respectively; δ_i, δ_j are the phase angles of the voltage V_i and V_j respectively [10].

- **The voltage deviation**

The voltage at load buses can be realized by minimizing the load bus voltage deviation from 1.0 per unit. The load bus voltage deviation can be formulated as:

$$V_d = \sum_{i=1}^{NL} (V_i - 1)^2 \quad (4)$$

where V_d is the total voltage deviation at the load buses; V_i the per unit voltage at load bus i and NL is the number of load buses [10].

B. System constraints

The minimization of the above objective functions are subjected to a number of equality and inequality constraints

- ***Equality constraints***

These constraints represent the load flow equations:

(i) Active Power balance equation

$$\sum_{i=1}^{NB} P_i = P_{gi} - P_{di} = V_i \sum_{j=1}^{NB} V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (5)$$

(ii) Reactive power balance equation:

$$\sum_{i=1}^{NL} Q_i = Q_{gi} - Q_{di} = V_i \sum_{j=1}^{NL} V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \quad (6)$$

where NB is the total number of buses except the slack bus; NL is the total number of load buses; P_i, Q_i are the active and reactive power injection into i^{th} bus respectively; P_{gi}, Q_{gi} are the active and reactive power generated at bus bar i respectively; P_{di}, Q_{di} are the load active and reactive power at bus bar i respectively. G_{ij} and B_{ij} are the line transfer conductance and susceptance between bus i and bus j, respectively [11].

- ***Inequality constraints***

The inequality constraints of the system have two type *(i) The inequality constraints on control variable*

$$V_{gi}^{min} \leq V_{gi} \leq V_{gi}^{max} \qquad i = 1,2, \dots \dots, NG$$
$$T_{i}^{min} \leq T_{i} \leq T_{i}^{max} \qquad i = 1,2, \dots \dots, NT$$
$$Q_{Ci}^{min} \leq Q_{Ci} \leq Q_{Ci}^{max} \qquad i = 1,2, \dots \dots, NC$$
$$P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max} \qquad i = 1,2, \dots, NG-1$$

where: - V_{Gi}^{min}, V_{Gi}^{max} are minimum and maximum voltage limit of generator *i;*

- P_{Gi}^{min}, P_{Gi}^{max} are minimum and maximum active power limit of generator *i;*
- *i;*limit of transformer tap maximum and minimum are T_{Ti}^{max}, T_{Ti}^{min}
- Q_{Ci}^{min}, Q_{Ci}^{max} are minimum and maximum reactive power compensative devise at load bus *i.*
- *NG* is the number of generators including the slack generator;
- *NT* is the number of Transformers;
- *NC* is the number of reactive power compensation devices.

(ii) The inequality constraints on state variable

$$V_{Li}^{min} \leq V_{Li} \leq V_{Li}^{max} \quad i = 1,2, \dots \dots, NL$$
$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \quad i = 1,2, \dots \dots, NG$$
$$P_{Gs}^{min} \leq P_{Gs} \leq P_{Gs}^{max}$$

Where: - V_{Li}^{min}, V_{Li}^{max} are minimum and maximum voltage of the load bus *i;*

- Q_{Gi}^{min}, Q_{Gi}^{max} are minimum and maximum reactive power of generator *i;*
- P_{Gs}^{min}, P_{Gs}^{max} are minimum and maximum active power of the slack generator [11].

3. Differential evolution algorithm

The Differential Evolution algorithm (DE) is one of the computational evolutionary techniques that introduced by R. Storn and K. Price in 1995. This algorithm is reliable, diversity optimizer technique and flexibly applicable for a global optimization problem. DE is a stochastic optimization search has the ability of handling non-differentiable, non-linear, non-continuous and mult objective functions. It improves a population for candidate solution using the mutation, crossover and selection operators to reach an optimal solution. It shows a characteristic of great convergence with a few needing of control parameters that keep constant throughout the optimization process and require the minimum setting [12, 13]. Differential Evolution use a real-coded variable instead of a binary or a gray representation. DE typically depends on mutation as the search engine and uses selection as guide search towards the potential regions in the possible region. It resolves the problems using a population P search region of size NP floating point-encoded individuals where the individuals are D-dimensional control variables vectors that improve over G generations to reach the optimal solution, i.e.,

$X = [x_1, x_2, \dots\dots\dots, x_D]$, as vector of control variables.
$P = [X_1; X_2; \dots\dots\dots; X_{NP}]$, as population search region.

Before starts with the optimization process of the Differential Evolation algorithm, an initialization step is used to converte a group of control variables into vector X. Number of control variables D is the size of the vector and each vector gives one solution from the solution in the space of the problem defined. Each solution represents a specific value of objective function.

The vectores of control variables are generate into the population search within its minimum and maximum limits using the equation given below

$$X_i = X_i^{min} + rand\left(X_i^{max} - X_i^{min}\right) \quad (7)$$

where X_i^{min} and X_i^{max} are the minimum and maximum limit of control variable *i*; $rand$ is a random number between 0 and 1 [14].

The optimization process of the DE is performed by three main operations: Mutation, Crossover and Selection. These stages can be cleared as follow:

A. Mutation

The objective of mutation is to enable the search diversity in the parameter space as well as to direct the existing vectors with suitable amount of parameter variation in a way that will led to better results at a suitable time. It keeps the search robust and explores new areas in the search domain. There are 4 types of mutation.

$$X_i = X_{r1} + F \times (X_{r2} - X_{r3}) \quad (8)$$

$$X_i = X_{r1} + F \times (X_{r2} - X_{r3}) + F \times (X_{r4} - X_{r5}) \quad (9)$$

$$X_i = X_{best} + F \times (X_{r1} - X_{r2}) \quad (10)$$

$$X_i = X_{best} + F \times (X_{r1} - X_{r2}) + F \times (X_{r3} - X_{r4}) \quad (11)$$

where $r1 \neq r2 \neq r3 \neq r4 \neq r5$ are randomly selected number from the population search; X_i is the target vector; X_{best} is the vector which gives best value of ojective function among all the vectors in the current generation; X_{r1}, X_{r2}, X_{r3}, X_{r4} and X_{r5} are randomly chosen vector in the population of current generation; F is the scaling factor which may have value between 0 and 1. The mutation process is the mainly process in DE where weighted differences of randomly chosen vectors is used to mutate the target vector [3, 15].

B. Crossover

Crossover aims at reinforcing prior successes by generating child individuals out of existing individuals or vectors parameters. Crossover is the process of generating trail vector from mutated and target vector. The crossover constant is used to determine if the newly generated individual is to be recombined. To form the trail vector of the control variable a random number is generated. If this value is less than crossover constant, then mutated vector variable is considered otherwise target vector variable is considered as shown in equation (12).

$$x_{trail}^{(G)} = \begin{cases} x_{mutate}^{(G)} & \rightarrow if\ (rand \leq C_R) \\ x_{target}^{(G)} & \dots\dots otherwise \end{cases} \tag{12}$$

where x is a control variable vector; the superscript G is the generation number; $rand$ is a random number between 0 and 1; C_R is the crossover constant [16, 17].

C. Selection

Fitness function (objective functions) of the trail vector and the target vector are compared and the vector which has minimum fitness function value is selected for the next generation as shown in the equation (8).

$$x_i^{(G+1)} = \begin{cases} x_{trail\_i}^{(G)} & \rightarrow if \dots f(x_{trail_i}^{(G)}) \leq f(x_{target\_i}^{(G)}) \\ x_{target\_i}^{(G)} & \dots\dots\dots\dots\dots\dots \quad otherwise \end{cases} \tag{13}$$

where x is a vector of control variable; G is the generation number; $f(x)$ is the fitness function value for the vector x; i starts from 1 to the number of population size. For the control variables in the target vector and the trail vector. These processes mutation, crossover and selection are repeated for next generation until stopping criterion. A flow chart of the DE process is represented in Fig. (1) [16, 17].

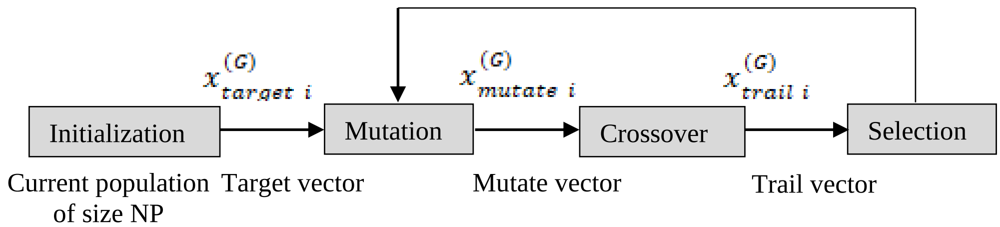

Fig. 1. Differential Evolation Process

4. Results and discussion

The proposed algorithm is carried out on the system of IEEE 30 bus in the Fig. (2) [18]. This system has 24 control variables as follow: 6 generator magnitude voltage; 4 transformers tap changer; 9 injection reactive compensative devise and 5 generator active power except the slack generator. Table 1 provides the values of minimum and maximum of active and reactive power of each generator with its fuel cost coefficients. The lower and upper limits of the load voltages are taken as 0.95 pu and 1.10 pu respectively.

The parameters of Differential Evolution are considered as follow:
Population size NP is 66;
Maximum number of generations G_{max} is 100;
Crossover constant CR is 0.5;
Weighting factor F is 0.8.

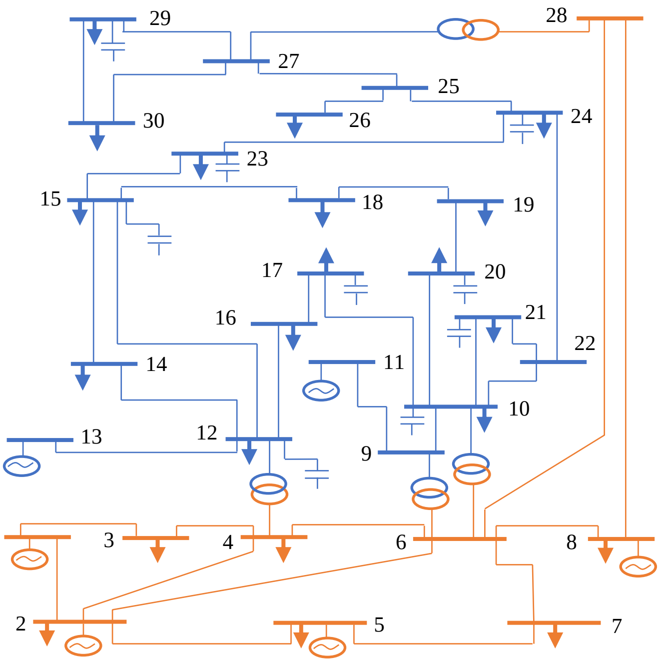

Fig. 2. One Line Diagram of IEEE-30 Bus System

Table 1

Power generation limits and cost coefficients for IEEE 30 bus

Unit Number	$P_{g_i (min)}$ (MW)	$P_{g_i (max)}$ (MW)	$Q_{g_i (min)}$ (MVAr)	$Q_{g_i (max)}$ (MVAr)	a $/h	b $/MWh	c $/MW^{2}h
1	50	200	-20	200	0.00	2	0.0037
2	20	80	-20	100	0.00	1.75	0.0175

5	15	50	-15	80	0.00	1	0.0625
8	10	35	-15	60	0.00	3.25	0.0083
11	10	30	-10	50	0.00	3	0.0250
13	12	40	-15	60	0.00	3	0.0250

In order to choose the best type of mutation for the DE algorithm, a comparison between the four type of mutation are considered according to the minimum objective functions of the fuel cost, active power losses and voltage deviation separately for each one as shown in table 2. Table 2 shows that type 3 of mutataion in equation (10) is the best type which used to get the final result as shown in table 3. Minimum objective function can be achived by choosen an optimal control variables. Figs. 2, 3 and 4 represent the fuel cost, losses and voltage deviation respectively with respect the number of iteration based on DE algorithm.

Table 2

Optimal solutions for different objective functions for the 4 type of mutation

	Objective Functions		
Types of mutation	Fuel cost ($/h)	Losses (MW)	Voltage deviation (pu)
Type 1	800.2477	3.2218	0.0019
Type 2	800.3136	3.4131	0.0018
Type 3	799.365	2.9748	0.0012
Type 4	799.8727	3.1568	0.0015

Table 3

Final simulation results according to DE algorithm

<table>
<tr><th colspan="2" rowspan="2">Control Variables</th><th colspan="2">Limit</th><th rowspan="2">Initial value</th><th colspan="3">Optimal value when the objective function is</th></tr>
<tr><th>Min</th><th>Max.</th><th>Fuel cost</th><th>Losses</th><th>Voltage deviation</th></tr>
<tr><td rowspan="6">Generators Output (MW)</td><td>P_2</td><td>20</td><td>80</td><td>80</td><td>49.4324</td><td>79.2353</td><td>24.8360</td></tr>
<tr><td>P_5</td><td>15</td><td>50</td><td>50</td><td>21.0294</td><td>49.8854</td><td>17.4310</td></tr>
<tr><td>P_8</td><td>10</td><td>35</td><td>20</td><td>20.4390</td><td>34.9794</td><td>22.5212</td></tr>
<tr><td>P_{11}</td><td>10</td><td>30</td><td>20</td><td>11.4407</td><td>29.8693</td><td>26.5815</td></tr>
<tr><td>P_{13}</td><td>12</td><td>40</td><td>20</td><td>12.1244</td><td>39.9372</td><td>15.8961</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="6">Generators Voltage (p.u)</td><td>V_1</td><td>0.95</td><td>1.10</td><td>1.05</td><td>1.1000</td><td>1.0984</td><td>1.0236</td></tr>
<tr><td>V_2</td><td>0.95</td><td>1.10</td><td>1.04</td><td>1.0877</td><td>1.0969</td><td>1.0076</td></tr>
<tr><td>V_5</td><td>0.95</td><td>1.10</td><td>1.01</td><td>1.0597</td><td>1.0788</td><td>1.0100</td></tr>
<tr><td>V_8</td><td>0.95</td><td>1.10</td><td>1.01</td><td>1.0717</td><td>1.0874</td><td>1.0082</td></tr>
<tr><td>V_{11}</td><td>0.95</td><td>1.10</td><td>1.05</td><td>1.0981</td><td>1.0936</td><td>1.0718</td></tr>
<tr><td>V_{13}</td><td>0.95</td><td>1.10</td><td>1.05</td><td>1.0982</td><td>1.0861</td><td>1.0314</td></tr>
<tr><td rowspan="4">Tap Position</td><td>T_{6-9}</td><td>0.9</td><td>1.10</td><td>1.078</td><td>1.0263</td><td>1.0302</td><td>1.0128</td></tr>
<tr><td>T_{6-10}</td><td>0.9</td><td>1.10</td><td>1.069</td><td>1.0353</td><td>1.0799</td><td>1.0639</td></tr>
<tr><td>T_{4-12}</td><td>0.9</td><td>1.10</td><td>1.032</td><td>0.9534</td><td>0.9622</td><td>0.962</td></tr>
<tr><td>T_{27-28}</td><td>0.9</td><td>1.10</td><td>1.068</td><td>0.9904</td><td>1.0129</td><td>0.9667</td></tr>
</table>

Injuction reactive power compesative devise (MVAr)	$Q_{c_{10}}$	0.0	0	0	3.4696	4.0694	4.8252
	$Q_{c_{12}}$	0.0	5.0	0	4.8539	4.7537	4.4397
	$Q_{c_{15}}$	0.0	5.0	0	4.0618	3.6173	4.7161
	$Q_{c_{17}}$	0.0	5.0	0	4.5515	3.8820	4.7843
	$Q_{c_{20}}$	0.0	5.0	0	3.9622	1.5845	3.9687
	$Q_{c_{21}}$	0.0	5.0	0	3.6786	1.2629	2.5953
	$Q_{c_{23}}$	0.0	5.0	0	4.7318	3.0301	1.3279
	$Q_{c_{24}}$	0.0	5.0	0	4.845	4.8127	4.4503
	$Q_{c_{29}}$	0.0	5.0	0	4.4291	3.7028	2.1101
Fuel cost ($/h)				901.96	799.365	964.7443	822.4529
Losses (MW)				5.83	8.7635	2.9748	10.1994
Voltage deviation (pu)				0.0779	0.0952	0.0785	0.0012

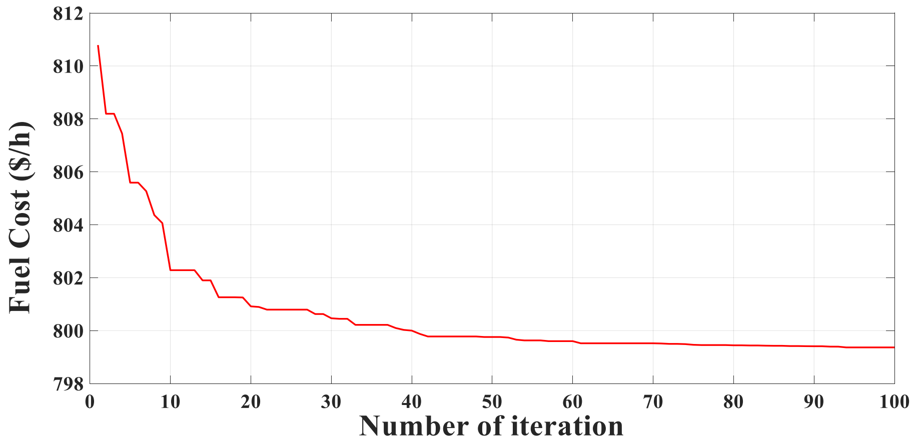

Fig. 3. The fuel Cost based on DE for IEEE 30 bus

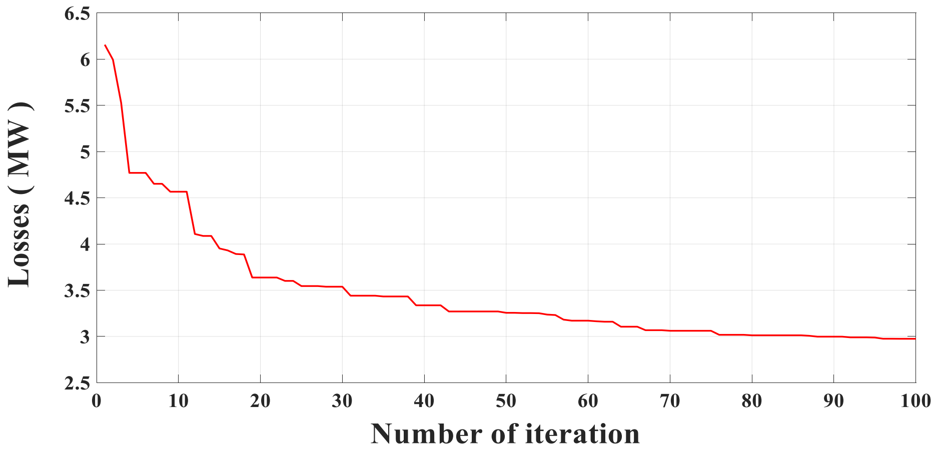

Fig. 4. The active power losses based on DE for IEEE 30 bus

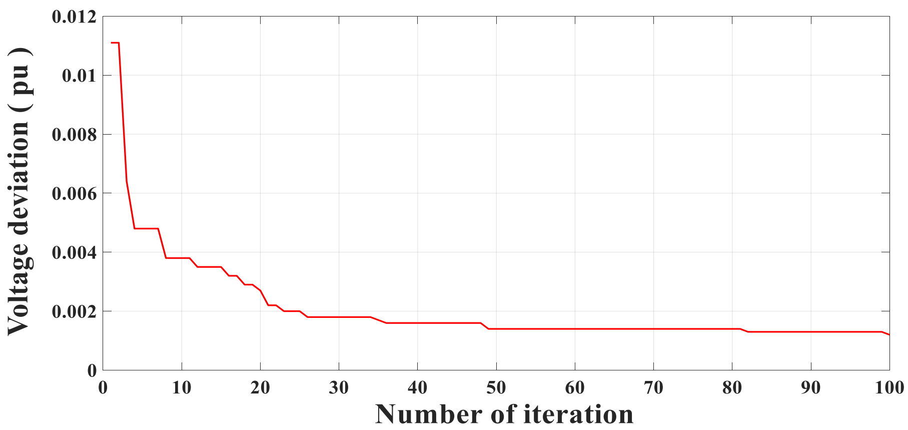

Fig. 5. The voltage deviation based on DE for IEEE 30

In this article, the Differential Evolution optimization minimize the three objective functions of each of the fuel cost of generation units ($/h), the active power losses in the transmission lines (MW) and the voltage deviation at the load buses from the initial values of 901.96 $/h, 5.83 MW and 0.0779 pu to the optimal values of 799.365 $/h, 2.9784 MW and 0.0012 pu respectively with reduction of 11.36% in fuel cost, 48.49% in active power losses and 98.45% in voltage deviation as shown in table 4. This table shows that's the proposed algorithm of the Differential Evolution gives good result when compare with other references.

Table 4

Comparison between the initial, different references and the proposed algorithm of Differential Evolution algorithm according to the fuel cost, active power losses and voltage deviation of IEEE 30 bus

References	Fuel cost ($/h)	Active power losses (MW)	Voltage deviation (pu)
Initial	901.96	5.830	0.0779
[10]	-	-	0.0019
[11]	801.66	3.032	
[15]	802.23	-	-
[16]	800.56	3.240	
[17]	801.84	-	-
Proposed algorithm of DE	799.365	2.974	0.0012
	(reduction of 11.36%)	(reduction of 48.49 %)	(reduction of 98.45 %)

4. Conclusion

The Differential Evolution optimization technique has been used to solve the Optimal Power Flow problem. This technique consists of three steps of mutation, crossover and selection. The proposed algorithm tested four types of mutation and choose the best one according to the minimum objective function. Three objective functions have been tested separately for each one. These objective functions are the fuel cost of the thermal units, the active power losses in the transmission lines and the voltage deviation at the load buses. The effectiveness of the proposed algorithm is tested on the system of the IEEE 30 bus and gives good result when compared with other optimization techniques. The proposed algorithm of the Differential Evolution method introduces an accuracy as well as convergence speed and simplicity.

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