OPTICAL STARK EFFECT IN SEMICONDUCTOR QUANTUM WELLS: A COMPARATIVE STUDY

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The subband energy levels in finite triangular, parabolic and square GaAs-GaAlAs quantum wells under intense laser field are calculated by using a variational method, in the effective mass approximation. The dependence of the absorption coefficient related to the interband transitions on the laser field amplitude and the geometric shape of the quantum wells is discussed.

**Key words:** triangular quantum well, parabolic quantum well, square quantum well, laser field, optical absorption.

1. Introduction

With the development of the molecular-beam epitaxy growth method, quantum wells can be made into different forms. Such structures present desirable optical features for devices design and favorable “Stark shift” characteristics, which can be used to control and modulate the intensity output of the devices. For instance, the parabolic quantum wells (PQWs) have been used as the graded barrier part of the quantum-well lasers to improve the optical confinement factor and enhance the carrier collection into the thin quantum well so as to reduce the threshold current density [1,2]. PQWs are also used to design infrared detectors with low leakage currents [3] and employed in resonant tunneling devices for their potential applications in high-speed circuits [4]. V-shaped quantum wells (VQWs) have been grown and experimentally studied, those structures being potentially interesting for designing millimeter wave communication devices [5], light-emitting devices [6] and MOSFET devices [7]. As a consequence, many theoretical studies of the electronic states in graded composition quantum wells...
(GCQWs) with different profiles have been performed [8-12]. The general picture indicates a substantial dependence of the electronic states on the main features of the well profile.

More recently, these studies have been extended to quantum wells under intense electric fields created by high-intensity THz lasers. Neto et al. [13] have derived the laser-dressed quantum well potential for an electron in a square quantum well (SQW), in the frame of a non-perturbation theory and a variational approach. A simple scheme based on the inclusion of the effect of the laser interaction with the semiconductor through the renormalization of the effective mass has been proposed by Brandi and Jalbert [14]. Ozturk et al [15] have investigated the effect of the laser field on the intersubband optical transitions for a graded QW under external electric field.

In this work, we studied effects of the laser field on the energy spectra in finite VQW, PQW and SQW GaAs quantum wells. The absorption coefficient for the interband transitions is also discussed as a function of the laser parameter and geometric shapes of the wells. To our knowledge, this is the first study of the effects of laser fields on the energy spectra in VQWs and PQWs.

2. Theory

For a particle moving in a quantum well under a high-frequency laser field, the Hamiltonian is given by:

\[ H = \frac{(\mathbf{p} + q \mathbf{A})^2}{2m^*} + V(z). \]  

(1)

Here \( q \) and \( m^* \) are the particle electric charge and the effective mass, respectively, \( \mathbf{A} \) is the vector potential of the laser field and \( V(z) \) is the quantum well potential energy.

We use an isotropic, single-band, effective-mass Hamiltonian for the electron (hole) single-particle states. We ignore the effects of the effective-mass discontinuity at the well interfaces and we use the well masses throughout the entire structure. We assume that the laser field can be represented by a monochromatic plane wave of frequency \( \omega \). For a linear polarization along the \( z \)-direction, the vector potential of the laser field is \( \mathbf{A}(t) = A_0 \cos(\omega t) \hat{u}_z \). For these single-particle Hamiltonians, we find the bound electron and hole states \( \Psi_\sigma(z) \), \( \sigma = e, h \), with energies \( E^\sigma_k \) by solving:

\[
\left[ -\frac{\hbar^2}{2m_\sigma} \frac{d^2}{dz^2} + V_d(z, \alpha_\sigma) \right] \Psi_\sigma^k(z) = E^\sigma_k \Psi_\sigma^k(z),
\]  

(2)
where $V_d^\sigma(z, \alpha_0^\sigma)$ is the laser “dressed” potential:

$$V_d^\sigma(z, \alpha_0^\sigma) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} V^\sigma \left[ z - \alpha_0^\sigma \sin(\omega t) \right] dt,$$

(3)

and $\alpha_0^\sigma = \frac{q A_0}{m^*_\sigma \omega}$ is laser dressing parameter. For a finite quantum well, the carrier confinement potentials are given by:

$$V^\sigma(z) = \begin{cases} V^\sigma_0, & |z| \geq b \\ V^\sigma_0 \left| \frac{z}{b} \right|^n, & |z| < b \end{cases},$$

(4)

where $V^\sigma_0$ is the conduction (valence) band offset, and $b = L / 2$ is the half-width of the quantum well. We study the $n=1, 2, \infty$ cases, corresponding to the V-shaped quantum well (VQW), parabolic quantum well (PQW) and square quantum well (SQW), respectively.

It is significant to emphasize that the approach (3) of the laser-dressed potential is valid for the high-frequency regime, e.g., $\omega \tau >> 1$, where $\tau$ is the transit time of the electron in the quantum structure [13]. For a GaAs-Ga$_{0.7}$Al$_{0.3}$As SQW with a width of about 100 Å, subjected to a monochromatic CO$_2$ laser beam ($\omega \approx 10^{14}$ s$^{-1}$), this condition is very well satisfied [13]. Since, for a given barrier height $V^\sigma_0$, the subband energy levels increase with the decreasing of $n$, one may expect that the high-frequency condition will be even better verified for PQWs and VQWs. For $n = 1$ and $n = 2$ the laser-dressed potential $V_d^\sigma(z, \alpha_0^\sigma)$, as defined in Eq. (3), is given by the following expressions:

$$V_d^\sigma(z, \alpha_0^\sigma)_QW = \begin{cases} \frac{V^\sigma_0 \alpha_0^\sigma}{\pi b} \left[ \frac{|z|}{\alpha_0^\sigma} - 2 \frac{|z|}{\alpha_0^\sigma} \arccos \chi(z) + 2 \sqrt{1 - \chi^2(z)} \right], & |z| \in D_1; \\ \frac{V^\sigma_0 \alpha_0^\sigma}{\pi b} \left[ -2 \frac{|z|}{\alpha_0^\sigma} \arccos \chi(z) + 2 \sqrt{1 - \chi^2(z)} \right] + \frac{|z| - b}{\alpha_0^\sigma} \arccos \frac{|z| - b}{\alpha_0^\sigma} - \frac{1}{2} \left( \frac{|z| - b}{\alpha_0^\sigma} \right)^2, & |z| \in D_2; \\ V^\sigma_0, & |z| \in D_3, \end{cases}$$

(5)
where $\chi(z) = \min\left(\frac{|z|}{\alpha_0^\sigma}, 1\right)$, $D_1 = [0, b - \alpha_0^\sigma)$, $D_2 = [b - \alpha_0^\sigma, b + \alpha_0^\sigma]$, and $D_3 = [b + \alpha_0^\sigma, +\infty)$.

For the SQW, we obtain the same functional form of “dressed potential” as in Ref. [16]:

$$V_d^\sigma(z, \alpha_0^\sigma)_{\text{SQW}} = \begin{cases} 
V_0^\sigma, & |z| \in D_3, \\
V_0^\sigma + \frac{V_0^\sigma}{2\pi b^2} \left(\frac{2z^2 + \alpha_0^\sigma}{2b^2} - \frac{\arccos \frac{|z|}{\alpha_0^\sigma} - \arccos \frac{|z| - b}{\alpha_0^\sigma}}{1 - \left(\frac{|z| - b}{\alpha_0^\sigma}\right)^2}\right), & |z| \in D_2, \\
\frac{V_0^\sigma}{\pi} \arccos \frac{b - |z|}{\alpha_0^\sigma}, & |z| \in D_1.
\end{cases}$$

(7)

The laser field determines a mixing of the eigenstates, so that in Eq. (2) the subband wave functions are taken as linear combinations of the eigenfunctions $\phi_k^\sigma$ of the QW [13]:

$$\Psi_k^\sigma(z) = \sum_j c_j^k \phi_j^\sigma(z),$$

(8)

where $j$ denotes all bound states in the quantum well.

The dressed bound-state energies $E_k^\sigma$ and the coefficients $c_j^k$ are determined using the system of linear equations:

$$\left(H^\sigma - E^\sigma S^\sigma\right)C^\sigma = 0,$$

(9)

where $H^\sigma$ and $S^\sigma$ represent, respectively, the matrices of the particle energy in the laser field, and the norm with respect to the basis functions $\left\{\phi_j^\sigma(z)\right\}$. $C^\sigma$ denotes the column matrix of the coefficients $c_j^k$. 
Once all bound state energy levels are determined, the absorption coefficient corresponding to interband transitions \( \{h_j \rightarrow e_j\} \) between the states of heavy-hole and electron can be evaluated. Neglecting the population effects for both initial and final states, the absorption coefficient reads [13]:

\[
A(E) = A_0 \sum_j \left| \left\langle \Phi^h_k \right| \Phi^e_k \right|^2 \Theta(E - E^i_j).
\] (10)

Here \( A_0 = \frac{\pi e^2 E_p}{n_0 c \hbar m^*_e m^*_h}, E_p \) is the Kane matrix element, \( n_0 \) is the refractive index of the QW, \( l \) is the total thickness of the structure, and \( E^i_j \) is the interband transition energy. By choosing \( A_0 \) as the “natural” unit for \( A(E) \), Eq. (10) describes the usual ladder profile for the absorption coefficient, as expected from the joint density of states of the 2D subbands.

3. Results and discussion

In the above calculations, we will consider an \( L=100 \) Å GaAs-Ga\(_{1-x}\)Al\(_x\)As quantum well. As in Ref [13] we take \( m^*_e = 0.0665m_0, \; m^*_h = 0.36m_0 \) and an aluminum concentration in the barrier material \( x = 0.30 \). For the electron (hole) states the barrier height \( \sigma \) is obtained from the 60% (40%) rule of the band gap discontinuity, \( \Delta E_g = 1247x \) (meV).

In order to show the effects of the potential-shape, \( n \), the laser dressing parameter, \( \alpha_0 \), on single-particle spectra in QWs, we have calculated energy levels of the electron and heavy-hole in GaAs-Al\(_x\)Ga\(_{1-x}\)A QWs as a function of \( n \) and \( \alpha \).

Figs. 1(a)-3(a) present the laser-dressed potential \( V^\sigma_d \) as a function of the \( z \) position, for different values of the laser parameter, and for \( n = 1, \; n = 2 \) and \( n = \infty \). The corresponding bound-state energy levels versus the laser-dressing parameter are shown in Figs. 1(b)-3(b). The numbers on the curves represent the indices of the electron (hole) subbands. The zero point of energy is situated at the edge of the conduction (valence) band in the finite barrier GaAs QW. In our calculations, the energy of the hole is considered to be a positive value.

It is seen from the figures that the ground state energy in the conduction band presents an increase with the laser intensity. Note that, at a given value of the laser parameter, the energy shift decreases with the increase of \( n \), as a result of a weaker localization of the electron wave function in the structure.
Fig. 1. VQW: a) Laser-dressed potential versus \( z \) position, for different values of the laser parameter; b) Bound-state energy dependence on the laser field parameter.

Fig. 2. The same as Fig. 1, for PQW.

For the SQW, in agreement with Ref. [13], the energy levels are more laser-sensitive as the subband index increases. For the PQW and VQW, where the geometrical confinement overwhelms the laser effect, \( E^e_2 \) is less dependent on the \( \alpha_0 \). Furthermore, in the VQW case, \( E^e_2 \) is slowly decreasing with the increase of \( \alpha_0 \). The laser field less affects the energy of a heavy-hole than the energy of an electron, this behavior being a common feature of the quantum structures [13].
We have also investigated the changes of the absorption coefficient $A(E)$ by the laser field. The staircase lineshape of the absorption coefficient in the GaAs-Ga$_{0.7}$Al$_{0.3}$As VQW, PQW and SQW with $L = 100$ Å and $\alpha_0^e = 0, 10, 20$ and $30$ Å is shown in Fig. 4. As seen in this figure, the laser field induced blue shift of the fundamental absorption edge for VQW is greater than for PQW and SQW.

The optical Stark effect on the excited energy levels is stronger for the SQW and thus the shift induced by the laser field on the $2e-2h$ interband transition is greater than for the PQW and VQW.
4. Conclusions

We have studied the effects of the high-frequency laser field on the intersubband transitions in different shaped quantum wells (VQW, PQW and SQW). The dependence of the absorption coefficient on the laser parameter, geometric shape of the wells was discussed. The results we obtained for SQW are in agreement with previous results. To our knowledge, this is the first study of the effects of laser fields on the energy spectra in VQW and PQW. We conclude that for the quantum wells the laser field amplitude have a significant effect on the electronic and optical properties. This gives a new degree of freedom in various device applications based on the interband transition of electrons.

REFERENCES