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EXPERIMENT FOR THE REDISCOVERY OF THE PERFECT PROPORTION OF RECTANGLES

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Lucrarea prezintă un experiment desfăşurat pentru redescoperirea proporţiei perfecte la dreptunghiuri şi pentru evaluarea parametrilor care ar putea influenţa percepţia proporţiei perfecte. Parametrii evaluaţi au fost culoarea, mărimea şi raza de racordare a dreptunghiurilor. Au fost luate în considerare mai multe nivele de variaţie pentru fiecare parametru. A fost analizată, de asemenea, influenţa sexului subiectului asupra percepţiei proporţiei. Rezultatele experimentului au confirmat teoria matematică consacrată a secţiunii de aur.

The paper presents an experiment carried out for the rediscovery of the perfect proportion of rectangles and for evaluation of the parameters that might influence the perception of this perfect proportion. The evaluated parameters were colour, size and connection radius of rectangles. There were considered several levels of variation for each parameter. It was also analysed the influence of the subject's gender upon the perception of proportion. The experiment results confirmed the ancient mathematical theory of golden ratio.

Keywords: visual perception, perfect proportion, golden section

1. Introduction

The perfect proportion is an issue with deep roots in history. During time it was called Golden Section, Golden Ratio or Golden Mean. Sometimes, it was called by enthusiasts the Divine Proportion.

There are two ways to determine the perfect proportion:

- geometrical;
- arithmetical.

The ***geometrical way*** starts with the following problem. Let determine a point C on the segment AB in order to obtain the proportion (see also Figure 1):

$$\frac{AB}{AC} = \frac{AC}{CB} \tag{1}$$

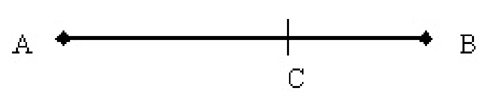

Fig. 1. AC/CB – the perfect proportion

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The ratio φ [2] expresses an irrational number.

$$\varphi = \frac{|AC|}{|CB|} = 1{,}618033 \tag{2}$$

The ancient Greeks considered that this ratio represents the golden section and they used it extensively in their architectural work. Because they were interested in rectangles, they applied the following geometric methodology (Figure 2) for obtaining the rectangle with golden section [1].

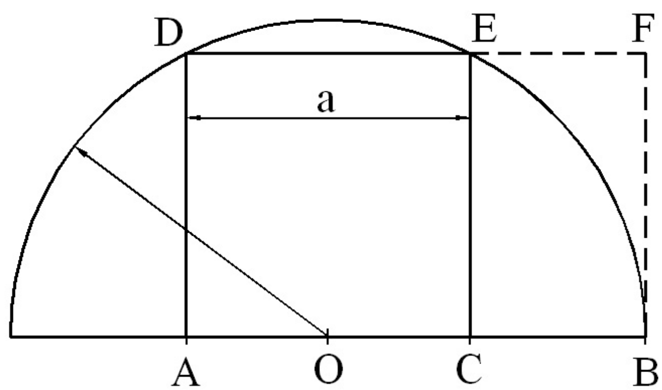

Fig. 2. Construction of the "perfect" rectangle

It is known from the very beginning the length a of the shorter side of the perfect rectangle that is to be constructed. The construction starts with the drawing of square *ADEC* that has the length of the side equal to a. It is marked with O the middle of the lower side of the square. The point O is the centre of a semicircle that reaches points D and E. The semicircle determines point B (at intersection with extended *AC*) and the new rectangle *ADFB* is the perfect rectangle that was sought.

The ***arithmetical way*** is based on Fibonacci series. The Fibonacci series, called also the series of natural growth, has the following formula:

$$x_i = x_{i-2} + x_{i-1} \tag{3}$$

Actually, the numbers in the Fibonacci series are:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233 …

The limit of ratios of Fibonacci numbers was discovered to be number φ:

$$\lim_{i \to \infty} \frac{x_i}{x_{i-1}} = \varphi \tag{4}$$

The φ number has a lot of remarkable properties. Some of them are presented in [2]:

$$\varphi = \frac{1}{\varphi - 1} \tag{5}$$

$$x_i^2 = x_{i-1}\, x_{i+1} + (-1)^i \tag{6}$$

$$\sum_{i=0}^{n} x_i = x_{i-2} + 1 \tag{7}$$

Other properties are indicated by the Romanian mathematician and philosopher **Matila Ghyka** [3]:

$$\varphi = \lim_{i \to \infty} \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}} \tag{8}$$

$$\frac{1}{\varphi} = 1 + \lim_{i \to \infty} \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \ldots}}} \tag{9}$$

Studying the curves in the structure of plants and animals, the English biologist Theodore Cook discovered other properties of the number φ [4]:

$$\varphi = \frac{\sqrt{5} + 1}{2} \tag{10}$$

$$\varphi^2 = \frac{\sqrt{5} + 3}{2} = \varphi + 1 \tag{11}$$

But Cook was not the only one to study nature in search for number φ. Several scientists analysed the biological domain and they discovered that number φ can be found frequently in nature – plants, animals and human body. Some examples are the following [2]. Number φ is the ratio between:

- the number of clockwise curves and counter-clockwise curves of sunflower seeds;
- the vertical distance between horse’s tail and ground and the vertical distance between head and tail.

There are also claims that the rectangle with golden section is the most pleasing rectangle. Gustav Theodor Fechner presented the results of an experiment in a book [5] that has become famous worldwide. 33% of the 500 subjects had chosen the golden section as the most pleasing rectangle.

In another experiment, **it was** used a rectangle that was drawn over two sheets of paper. The subjects were asked to move the sheets in order to produce a rectangle that possesses the perfect proportion. It was discovered that the persons fond of golden section, were very precise in constructing it. [6]

Mike Baxter considers that most of the modern cars obey to the golden section rule. In his book [7], he produced some examples using the front view of cars.

In a challenging article [8], George Markowsky confirms that the mathematical properties of number φ are true, but he puts under question some wide-accepted theories. These theories state that the number φ was used in the construction of the Great Pyramid of Keops and Athens' Parthenon. Also, he considers a misconception the presence of the golden ratio in human body.

In the same article [8], Markowsky opposes the claim that the rectangle with golden section is the most aesthetically pleasing. He reanalyses the results of the Fechner's experiment and draws the conclusion that those results can be hardly seen as strong evidence.

He performed his own experiment, but he does not indicate any information about the sample of subjects, like number or gender. Anyway, the subjects were asked to select the most pleasing rectangle from two matrices of rectangles. In the first matrix, there were randomly arranged 48 rectangles having the same height. The ratio between height and length varied from 0.4 to 2.5. In the second matrix, the same rectangles were orderly arranged. Both sets of 48 rectangles contained 2 rectangles with golden ratio.

Analysing the results of his experiment, Markowsky found out that people did not select the golden rectangle in the first matrix. The most selected rectangles in both matrices were rectangles close to the golden ratio. Actually, the most selected was the rectangle with a ratio of 1.83.

Summarizing all the information presented in introduction, it is certainly that the number φ, that is associated to golden ratio, is a number with remarkable and undeniable mathematical properties. Most of the experts agree that number φ can be found in the structure of living beings. But the theory of golden rectangle is seriously and objectively challenged. There are doubts that the golden rectangle is the most aesthetically pleasing rectangle.

2. Design of experiment

Considering the contradictory results presented in introduction, the author of the present paper designed an experiment with three aims:

- determination of the most aesthetically pleasing ratio for rectangles;
- evaluation of parameters that might influence the perception of perfect proportion;

- evaluation of influence of gender upon the perception of perfect proportion.

It was decided that no golden rectangle will be shown to the subjects of experiment. If the golden section is the most pleasing, it would emerged as so after mathematical calculations.

In order to ease the statistical calculations, there were selected 5 ratios between length and height of rectangles: **1; 1.5; 2; 2.5; 3.** Another decision was that the length of rectangle will be always the larger size. So, all the rectangles will be horizontal-oriented.

The selected parameters that might influence the perception of perfect proportion were the following:

- colour;
- size;
- connection radius of rectangles.

Actually, not the size of connection radius was considered as a parameter, but the ratio between connection radius and height. The parameters had 3 – 5 levels of variation.

Each rectangle received a code consisting of four digits, according to the level of variation of rectangle's ratio and parameters. The significance of each digit is shown in Table 1.

Table 1

Significance of code digits

Digit	*Ratio*	*Colour*	*Height / Size [mm]*	*Ratio between connection radius and height*
1	1	white	40	0
2	1.5	yellow	70	0.15
3	2	green	100	0.3
4	2.5	blue	-	-
5	3	black	-	-

Fig. 3. Samples used in experiment

Obviously, it resulted 225 of different rectangles. They were made from coloured cardboard (Fig. 3). It was preferred the cardboard because it is light (easy to manoeuvre) and somehow a natural material ("warm" to subjects). It was considered that a computer image will negatively influence the accuracy of determinations.

A methodology was designed for this experiment. It consists from the following steps:

a. The aim of experiment is communicated to subject.

b. The notion of golden section is presented to subjects. (The value of number φ is strictly not mentioned to subjects).

c. The subjects are instructed how to assess the aesthetical value of rectangles. They will give a mark to each rectangle, according to the following evaluation scale:

1 – totally bad proportionate;
2 – bad proportionate;
3 – acceptable proportion;
4 – well proportionate;
5 – the perfect proportion.

d. Each subject receives only one rectangle in a randomly order.

e. Each subject notes her/his mark on a printed form.

f. The last 2 steps are repeated 225 times.

g. The printed and filled forms are collected.

3. Experiment's results

The experiment was run with 57 subjects (23 female and 34 male subjects). All the subjects were students at a technical faculty. All the experiment's sessions were supervised by the author of the present paper.

The experiment's results were gathered in a spreadsheet and statistical calculations were performed. Among other statistical values, there was calculated the mean for each rectangle. The means of rectangles according to ratio of sides are displayed in Table 2.

Table 2

Means according to ratio of sides

Ratio	1	1.5	2	2.5	3
Mean	2.69	**3.08**	2.98	2.55	2.12

It can be easily observed that the highest mean corresponds to ratio 1.5, the nearest to 1.618 – the golden ratio. It can be considered as a confirmation of the theory? Let see how the diagram looks (Fig. 4).

The highest point of the interpolation curve appears to be at the right of ratio 1.5. In order to determine precisely the ratio corresponding to the highest point, the regression curve should be mathematically expressed.

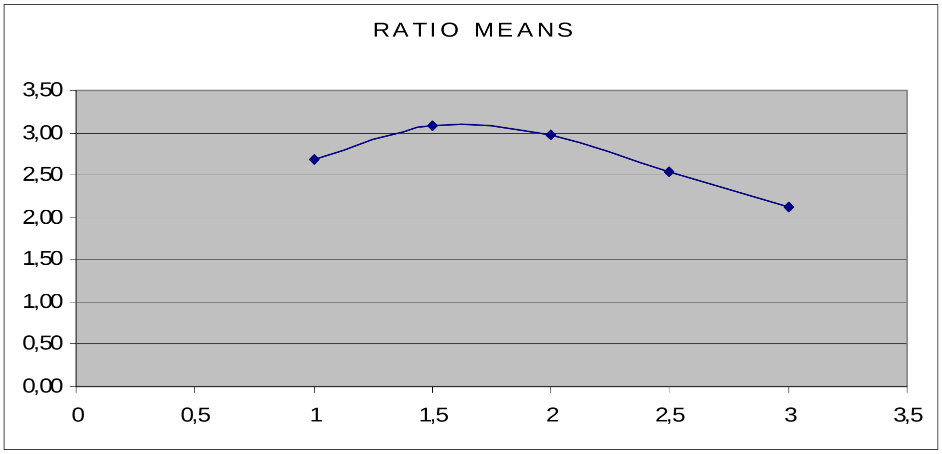

Fig. 4. Diagram of means according to ratio

Using an appropriate computer programme, several regression functions (y) and the associated correlation coefficients (r) were determined. The associated derivative functions were also calculated and the coordinates of the highest point were established. x_{max} corresponds to the function's peak. The most significant regression functions were:

- quadratic function:

$$y = 1.382 + 1.917x - 0.563x^2 \quad (12)$$

$r = 0{,}991$

$x_{max} = 1.702$

- third level function:

$$y = -0.003 + 4.408x - 1.921x^2 + 0.228x^3 \quad (13)$$

$r = 0{,}99982$

$x_{max} = 1.607$

- forth level function:

$$y = 0.0001 + 3.948x - 1.17x^2 - 0.15x^3 + 0.06x^4 \quad (13)$$

$r = 0{,}999993$

$x_{max} = 1.618$

The forth level function has the highest correlation coefficient and can be considered as the function that describes in the highest degree the relationship between the ratio of rectangle's sides and the rectangle's aesthetic perception. Its highest point corresponds to $x = 1.618$, so the experiment's results can be considered as a ***confirmation*** of the wide-accepted theory. But, it should be noted that the corresponding mean mark is just a little bit above 3 – which indicates an acceptable proportion. Concluding, this is a confirmation, but not a significant confirmation.

The first parameter that was analysed was the ***colour***. The means of rectangles according to ratio of sides and colour are displayed in Table 3. The diagram of means according to ratio of sides and colour is presented in Figure 5.

Table 3

Means according to ratio of sides and colour

	Ratio				
Colour	1	1.5	2	2.5	3
White	2.68	3.06	2.87	2.50	2.03
Yellow	2.66	3.10	2.97	2.50	2.06
Green	2.68	3.06	2.97	2.55	2.14
Blue	2.71	3.06	3.05	2.57	2.15
Black	2.73	3.12	3.04	2.61	2.20

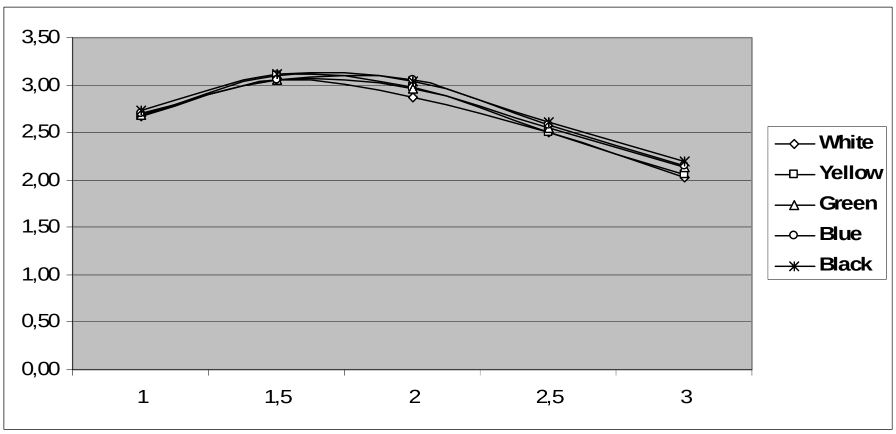

Fig. 5. Diagram of means according to ratio and colour

The results, whether they are presented as a table or as a diagram, are not conclusively. The means in the table are very near and, also, are the curves in the diagram. The only two interesting observations are that the black rectangles scored slightly better and the curves have similar shapes.

The second studied parameter was the ***size***. The means of rectangles according to ratio of sides and size are displayed in Table 4. The diagram of means according to ratio of sides and size is presented in Fig. 6.

Table 4

Means according to ratio of sides and size

	Ratio				
Size [mm]	1	1.5	2	2.5	3
40	2.82	3.15	3.00	2.57	2.24
70	2.69	3.10	3.03	2.55	2.18
100	2.57	2.98	2.91	2.52	1.94

The mean values are also near, but they are distinctively. The curves are similar and have about the same *x* coordinate for the peak. It can be observed that the scores for the greater size (100 mm) are lower than the others. It can be concluded that size influence the perception in a lower degree. Greater sizes conduct to a lower perception of perfect proportion.

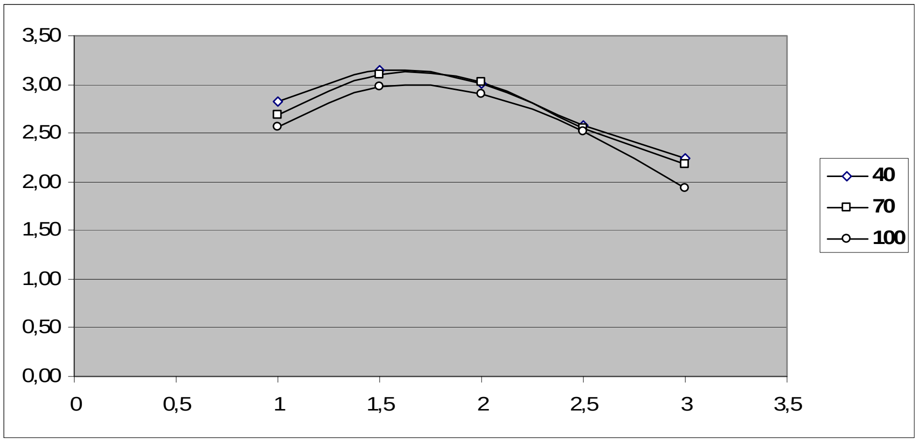

Fig. 6. Diagram of means according to ratio and size

The last parameter that was considered was the ***connection radius***. The means of rectangles according to ratio of sides and connection radius are displayed in Table 5. The diagram of means according to ratio of sides and connection radius is presented in Figure 7.

Table 5

Means according to ratio of sides and connection radius

	Ratio				
Ratio between connection radius and height	1	1.5	2	2.5	3
0	2.94	3.28	3.16	2.66	2.21
0.15	2.84	3.21	3.10	2.59	2.15
0.30	2.30	2.74	2.68	2.38	1.99

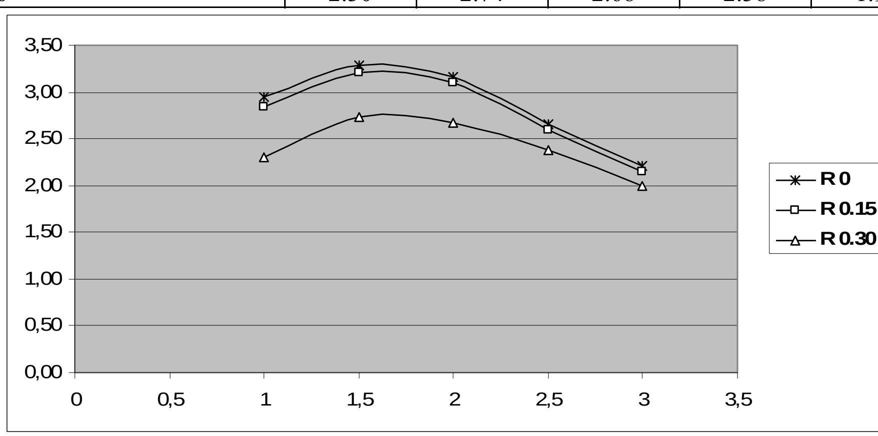

Fig. 7. Diagram of means according to ratio and connection radius

The connection radius is a more significant parameter. As the ratio between connection radius and height increases the perception of the perfect proportion is negatively influenced.

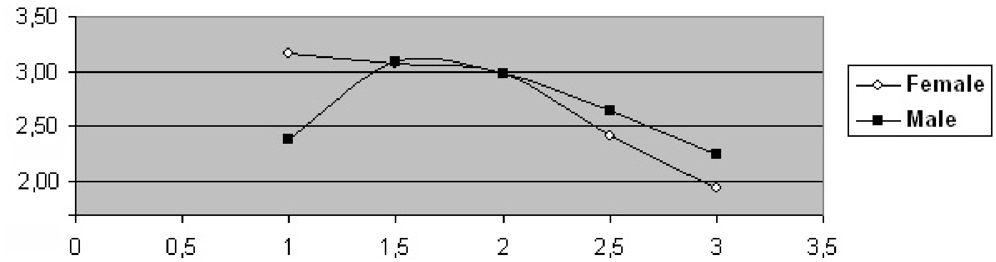

Fig. 8. Diagram of means according to ratio and gender

The influence of gender upon the perception of perfect proportion is displayed in Figure 8. The “male” curve is similar to the other curves analysed above and has the same peak. The “female” curve is obviously not similar to other curves previously presented and has a different highest point. It is not expected that women has a different perception about perfect proportion. Probably the female sample (23) was too small.

4. Conclusions

An experiment was designed to rediscover the perfect proportion and to evaluate the parameters that might influence the perception of perfect proportion. The experiment’s results indicated that:

- the golden ratio is 1.618, confirming the well-established theory;
- the golden ratio was perceived as just an acceptable proportion, obtaining a mean mark of 3.1 from a range of 1 to 5;
- parameters colour and size do not influence significantly the perception of perfect proportion;
- parameter ratio between connection radius and height influences the perception of perfect proportion in an inverse mode;
- the gender influence on perception was inconclusive.

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