MODEL ANALYSIS FOR SINUSOIDAL POWER FACTOR CORRECTOR

Dan OLARU¹, Dan FLORICĂU²


The power factor compensation is important for the quality of power delivery. This paper is focused on a differential amplitude control mode, with a single sensor, suitable for sinusoidal operation.

A quantitative analysis is proceeded in order to define error estimation, stability criteria and filtering strategies. The study uses analogical models. For operation and stability characterisation specific indicators are derived and computed. These allow us to obtain significant results despite the nonlinear character of the structure. Numerical simulations and case studies prove the validity and relevance of the model. This quantitative analogical approach makes the design easier, simpler than a digital implementation.

Keywords: power factor correction, control loop, stability limit

1. Introduction

The quality of the power delivery depends on harmonic protection and power factor correction possibilities. Although theoretically, the active power filters may cover all these problems, separate devices used for power factor correction are sometimes advantageous. Sinusoidal operation simplifies the working methods, and gives higher performances. Different parallel or serial configurations have been reported, as in [1], [2], [3], [4]. There are many possibilities for analogical or digital implementations of the signal controller. The digital ones are sometimes preferable because of their flexibility and

¹ Prof., Faculty of Electrical Engineering, University POLITEHNICA of Bucharest. Romania, e-mail: dan.olaru@upb.ro
² Prof., Faculty of Electrical Engineering, University POLITEHNICA of Bucharest. Romania
programmability. However, the analogical solutions are simpler and enable a direct correspondence with the functional blocks, which is preferable for analysis purposes, especially when didactical purposes are taken into account. Relative dependences and sensibilities are easier to study on an analogical scheme. Approximation and stability problems specific to the numerical approach are eliminated. For parallel active power factor correction, a classical method uses a quadrature injected current, as a function of the error voltage, measured on a low value inductor. For three phase configuration, this is described in [1], where are presented numerical algorithms necessary for DSP controllers, based on a finite difference method. Stability is ensured by PI regulators. However, no qualitative and quantitative aspects on the operation and stability performance are given.

The method presented here is based on similar error and control parameters, but the signals are processed by analogical circuits. The paper presents different analysis aspects, focused on error, dynamic characteristics and stability problems. The goal is to identify the control parameters, the critical values of certain components and the optimization possibilities. The analogical approach is advantageous because it enables to separate the continuous operation characteristics from the discrete ones. It is well known that the discrete operation generates additional and different problems concerning the compensation error and stability limits.

In the following are presented, for the first, a basic model scheme, which enables the study of relevant phenomena, and for the second, an enriched scheme with additional circuits, for signal processing and measuring purposes, in order to analyze operation and stability characteristics. PSPICE symbols and numerical simulations are used both for validation and justification purposes. The objectives of the paper are the following: identification of the simplest compensator analogical model and its main parameters; study of the optimisation possibilities, determination of the error and stability limits, derivation of a stability indicator and a sufficient stability criterion.

2. Compensation principle

The functional scheme (Fig.1) uses, as in [1], an error signal given by a small serial sensor inductance. Instead of the switching capacitor technique and finite difference control, here the quadrature injected current is generated by a continuous modulation device. Of course, a real implementation must contain a switching mode equivalent current generator. But, this approach is better for our control and stability analysis. For compensation purposes, a correspondence between electrical and phase parameters is used, with no need to compute signal phase difference.
First, the compensation principle may be considered from an intuitively qualitative point of view. Consider that, for a given non resistive load, we have a controlled source, which injects a pure reactive compensation current to the load connection point. Thus, the total supply current may be lowered. As a consequence, the voltage, across the inductive sensor will be lowered. It results that a minimal voltage on the inductive sensor indicates a minimal reactive load. The quadrature current generator must be controlled in order to minimise the voltage amplitude on the sensor inductance. Thus, the reactive compensation process becomes equivalent to a minimisation process. The error signal represents the magnitude of a difference phasor equal to the voltage amplitude on the inductive sensor. This difference phasor is orthogonal on the current phasor because it represents a reactive component. The quadrature controlled source gives a positive or negative reactive current component. This compensates the reactance of the load and has no direct influence on the active component.

![Fig.1. Compensation principle diagram](image)

For steady state operation, a phasor representation may be used. The capacitive behaviour is showed in Fig.2a and the inductive behaviour in Fig.2b.

![Fig.2. Phasor diagram for capacitive and inductive load](image)

With no concerns of ambiguity, conventionally, same symbols are used both for geometrical segments, phasors, signals or amplitudes. For simplicity, single letters denote magnitudes corresponding to supply voltage (a), load voltage (b), inductor voltage (Δ), voltage-current angle (φ) and x for the segment continuing Δ. Thus, a and b correspond to the voltages $U_A$ and $U_B$ at the terminals of inductive sensor. The control loop acts in order to minimize the difference between the voltage amplitudes on sensor inductor terminals ($a-b$ difference). Because the inductor
voltage is always orthogonal on the current, when the difference between \( a \) and \( b \) phasors is minimised, the phase difference angle \( \phi \) tends to zero, independent of \( \Delta \). On the other hand, the loop is based on a quadrature current that acts only on the reactive component, independent of the active one. Thus, with the compensation process the total current is changed and \( \Delta \) value is modified too. In transient regime conditions, interactions appear between compensation errors, dynamical response and stability proprieties corresponding to circuit parameters like time constants, loop amplification and frequency band characteristics. These depend on circuit implementation and are discussed below.

3. Error analysis

The quality of the compensation process may be measured as an approximation error between the compensated supply current and a pure resistive current (with a unitary power factor). Asymptotical error analysis may be done using, the geometrical diagram of Fig.2, for example. Geometrically, the phasor diagram contains two adjacent triangles, whose angles will be denoted by the corresponding pair of edges. Thus, the \((a,\Delta)\) angle may be computed by Pythagoras generalized theorem in the triangle having edges \( a, b, \Delta \):

\[
\cos (a, \Delta) = \frac{a^2 + \Delta^2 - b^2}{2a\Delta}
\]

Considering the right triangles and the supplementary angles, the phase difference \( \phi \) angle may be represented as:

\[
\phi = \frac{\pi}{2} - (x, a) = \frac{\pi}{2} - [\pi - (\Delta, a)] = (\Delta, a) - \frac{\pi}{2}
\]

Thus:

\[
\sin \phi = -\cos (a, \Delta) = \frac{b^2 - a^2 - \Delta^2}{2a\Delta} = \frac{(b + a)(b - a) - \Delta^2}{2a\Delta} = \frac{2a(b - a) - \Delta^2}{2a\Delta}
\]

It results that:

\[
\sin \phi = \frac{2a(b - a) - \Delta^2}{2a\Delta}
\]

For low values of the sensor inductance, the voltage \( \Delta \) is low, the values \( a, b \) are near, and \( b + a \approx 2a \). Also, for low values, the sine function approximates the argument, so \( \sin \phi \approx \phi \).

It results that the voltage-current phase difference may be estimated as follows:

\[
\phi \approx \frac{2a(b - a) - \Delta^2}{2a\Delta}
\]
The control loop is conceived in order to minimize the \((b-a)\) voltage. It can be seen that the significance of signals \(a\) and \(b\) can be observed in the symbolic feedback diagram as in Fig.3. As reference signal, we take the amplitude of the supply voltage \(a\), and as feedback signal, the load voltage amplitude \(b\), both obtained from a differential peak-detector. Here, the error amplifier corresponds to a transconductance amplifier, equivalent with a quadrature current controlled source, acting only on the reactive component of the load current. For a sufficient gain \(K\), the difference between amplitudes \(a\) and \(b\) is reduced until the phase difference \(\phi\) rests positive and becomes near zero, according to formula (5). It may be shown that, for higher values of \(K\), the phasors \(a\) and \(b\) become symmetrical, on one side and the other of the horizontal axis, giving a slightly nonzero value for \(\phi\). But, for a low value of \(\Delta\), this is negligible.

![Feedback symbolic diagram](image)

**Fig.3. Feedback symbolic diagram**

### 4. Model implementation

The compensation process is based on a voltage-measured, current-controlled loop, having as error signal the difference between voltage amplitudes on inductor sensor terminals. The basic scheme (Fig.4) represents, with PSPICE symbols, a simple model circuit operating according to the principle presented above. The circuit components are the following: supply source (V1), reactive load (L2,R5), inductive sensor (L1); differential peak detector (D1,R1,C1,R2,D2,R3,C2,R4), differential error amplifier (EA); amplitude modulator (multiplication operator); quadrature voltage source (V2); controlled current source (transconductance unitary operator). In this scheme, the differential peak detector is chosen as a simple solution. The advantage is a fast and stable operation. Each branch is defined by two time constants, for example, corresponding to the capacitor C1 and resistors R1, R2. The first defines the rising time response and the second, the descending time response, determining implicitly the distortion degree. For a reasonable time response, a low distortion degree must be accepted. On the other hand the quality of the compensation process is dependent on the control loop gain, defined by the error amplifier EA. For a high gain, the distortion component is amplified too. It results a compromise between compensation quality and distortion degree.
5. Dynamical analysis

In order to obtain a better solution, solving simultaneously the compensation and distortion problem, an additional low pass filter must be introduced, as in Fig.6. For analysis purposes, several other circuits and components are introduced as well. The high order low pass filter gives a good attenuation for peak detector distortion, enabling a high value for the control loop gain. On the other hand, the time constant of the filter and phase delay degrade the dynamical characteristics. Thus, the filter parameters must be chosen for a good compromise between compensation quality and stability characteristics. Quality compensator parameters, phase difference and distortion degree are estimated using several additional simple electronic devices. These are plotted, as rms values, during numerical simulations.

The phase difference is estimated as rms output value of an exclusive-or operator. The first input is driven by the squared supply voltage. The second input is driven by the load current squared signal. Both input signals are obtained by a limit-amplification-triggering operation. The distortion degree is estimated by the
harmonic content, measured with a high pass filter, having as input the voltage signal from the current sensor (R5). Fig.5 and Fig.7 show respectively the evolution of the supply parameters and of the quality parameters defined above. By numerical experiments, it may be observed that without the additional filter, a short and smoother transient regime occurs, with a distortion level greater than the phase difference error. When the filter is present (Fig.7), a longer oscillating transient regime occurs and the distortion level is lower than the phase error.

![Enhanced scheme of compensator](image)

**Fig.6.** Enhanced scheme of compensator

![Supply voltage and current evolution with the enhanced compensator](image)

**Fig.7.** Supply voltage and current evolution with the enhanced compensator

### 6. Stability analysis

The compensation control loop implies reactive elements, active devices (control loop amplifier, controlled quadrature generator) and nonlinear elements (peak detectors and amplitude modulator). Because of the high gain (necessary to obtain a low compensation error) and the phase delay (due to the reactive and
inertial elements), both combined with the phase step (due to the quadrature generator), stability problems arise. The electric circuit is strongly nonlinear because of the peak detector and amplitude modulator. Therefore a frequency domain analysis is impossible. In order to study the circuit behaviour and to obtain a sufficient stability condition, a special model scheme was designed (Fig.8). The output level (R6 voltage) is used as an *aposteriori* stability indicator. The scheme contains two parts. The first part is an equivalent scheme of the initial control circuit from Fig.4, added with a perturbation stimulus voltage source (V1), applying an external test perturbation. Because we are interested only in the AC response to the stimulus, diodes D1 and D2 are short-circuited. For the stability analysis only the topology of the closed control loop is important. The entry point for the stimulus may be chosen anywhere. For simplicity, this was chosen at the modulator control input. The output was considered as the sensor inductance voltage. The loop differential amplifier was placed on the feedback branch.

The second part represents a signal processing scheme used only for stability evaluation. At its output a scalar indicator is obtained. The presented method is based on the following principle. If the control is stable, the negative feedback signal is opposite to the stimulus. When control is unstable, there are moments when the (normally) negative reaction becomes positive. In this situation, for certain frequency components, the polarity and the direction of variation of the feedback is the same as for the stimulus. The effect is stronger when the fundamental is affected in the same way. For the unstable case, when the feedback signal exceeds the stimulus signal and their polarity and direction of variation coincide, the output signal increases very quickly.

In order to define an instability indicator, the simultaneous satisfaction of the above conditions is evaluated by logical “and” operations, modelled with multiplier circuits. The active states correspond to positive values of signals, selected by limiter circuits. A non-null value for stability indicator corresponds with the dominance of positive feedback. Thus, after a certain transient regime, the increasing average value of the indicator shows an unstable evolution and a decreasing one corresponds to a damping stabilization process.

To verify the relevance of the defined indicator, the compensator behaviour was studied by means of numerical simulations, with a critical value of the C1 capacitor, determined through numerical experiments with the basic scheme (Fig.4) to be at 0.4mF. Fig.9 shows the supply current superimposed with the stability indicator evolution at the stability limit. In this case, after 1.2ms when the starting transient regime disappears, the inductor current becomes very low and the stability indicator value reaches a low and stable value. For C1 values higher than 0.5mF the overall indicator value will become negligible. The domain of increasing indicator values may be interpreted as a reserve of stability.
7. Conclusions

The presented analysis is based on analogical models. This approach enables us to identify main problems and characterize the technical solution. In addition, this representation corresponds directly to associated functional blocks. Although a compensation process based on a minimisation principle has advantages, having no need of reference signals and linear signal processing, a quantitative analysis is difficult to be done. The presented combined techniques try to make the task of the designer easier, eliminating empirical approaches. Thus, different optimisation criteria may be formulated. The content of the paper offers several tools concerning phase error estimation and the dynamical and stability characterisation of the compensator.
The analogical models presented above have many advantages like simplicity and no numerical computations for phase difference and reference current. A real implementation based on this model is very simple, containing few components. Using a minimisation principle, no precision devices are necessary. On the other hand, a digital solution for this model is expensive. Powerful DSP circuits are necessary, due to the needs for numerical resolution and floating point computations.

An important aspect must be highlighted. Sinusoidal compensators have a restrained functionality, compared with active filters, when harmonics are present. However, their power factor accuracy and high frequency noise characteristics are always better, because no chopping processes are used.

REFERENCES