CAD PROCEDURE FOR BLADE DESIGN OF CENTRIFUGAL PUMP IMPELLER USING CONFORMAL MAPPING METHOD

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Proiectarea computerizată a pompelor a devenit o practică standard, fiind folosită pe scară largă atât în proiecte noi cât și pentru retehnologizarea pompelor vechi. Un asemenea cod de proiectare a fost creat de către autor. Totuși orice metodă de proiectare are anumite ipoteze prin care se neglijează în prima iteratie efectele tridimensionale datorate încărcărilor palei, precum și efectele vâscozității. Un proiect bun poate fi obținut doar prin analiză 3D a curgerii prin rotor, urmată de corecții asupra geometriei palelor. În lucrarea de față este prezentată o procedură automată de generare a geometriei 3D a canalului dintre pale pentru rotoare de pompe centrifugate pornind de la datele geometrice furnizate de codul quasi-3D. De aceea s-a dezvoltat o procedură originală ce ține cont de particolaritățile geometrice ale pompelor centrifugate. Lucrarea descrie modul de implementare a acestei proceduri.

Computerized pump design has become a standard practice in industry, and it is widely used for both new designs as well as for old pumps retrofit. Such a complex design code has been developed over the past decade by the author. However, any design method has to accept a set of hypotheses that neglect in the first design iteration the three-dimensional effects induced by the blade loading, as well as the viscous effects. As a result, an improved design can be achieved only by performing a full 3D flow analysis in the pump impeller, followed by a suitable correction of the blade geometry and/or the meridian geometry. The present paper presents a fully automated procedure for generating the inter-blade channel 3D geometry for centrifugal pump impeller, starting with the geometrical data provided by the quasi-3D code. This is why we developed an original procedure that successfully addresses all geometrical particularities of centrifugal pump. The paper describes the implementation of our procedure.

Keywords: CAD procedure, blade design, pump impeller, conformal mapping.

1. Introduction

The intermediary passage between entrance and discharge can be obtained by interpolating one of the significant kinematic physical quantities: the blade charge distribution over the impeller channels. The interpolation is made along the streamlines controlled through the curvilinear coordinate x and not by the current

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radius $r$ because the charge (charge distribution) is better quantified in the mixed zone. The hydraulic momentum is the physical quantity, which reflects the charge of the blades between entrance and discharge. For a current point heaving the curvilinear coordinate $x$ it is like [1]:

$$M_{hx} = \rho Q f (r, v_{ux} - r_1 v_{u1})$$  \hspace{1cm} (1)

From (1) explicating $(rv_u)_x$ result:

$$(rv_u)_x = r_1 v_{u1} + \frac{M_{hx}}{\rho Q} = f(x)$$  \hspace{1cm} (2)

The physical quantity directly linked to the hydraulic momentum is $\Delta (rv_u)$, therefore by controlling the variation over the impeller channel of the product $rv_u$, results the variation of the hydraulic momentum with the radius, repartition of partial head, and distribution of the pressure differences on the blade faces. The variation of the $rv_u$ product implies the variation of the $\beta$ angle required by the blade construction in the hypothesis that the relative speed is tangent at the surface of the blade. The height of the velocity triangles is given by the meridian velocity, $v_u$, corrected with the obstruction degree given by the thickness of the blades. If we consider a current point on the stream line marked with “i”, then the current angle $\beta$ results from equation:

$$\beta_i = \arctg \left( \frac{v_{ui}}{\rho_1 u_i - v_{ui}} \right)$$  \hspace{1cm} (3)

We observe that in (3) appear the factors $r_i$ ($u_i = r_1 \omega$) and $v_{uis}$, therefore indifferent of the two variants interpolate we obtain the same type of information. The best choice is that $\beta$ angle to be directly linked of the blade orientation in the rotor channel. The variation of the $\beta$ angle between inlet and outlet must be chosen in such a way that the charge of the blade is relatively uniform and the variation is rising strictly on the entire domain. For a better engagement of the stream at entrance and discharge it is recommended that in the vicinity of the limit points the blade to be made as an inactive blade. By analyzing many rotors are established that this condition is realized if the variation curves of $\beta$ has a zero derivate at entry and discharge.

In this case has been chosen interpolation with two connected parabola arcs.

2. Interpolation with two connected parabola arcs

In case first case the connection of two parabola arcs is made after common tangent at the point $x_3$. The function that defines the two parabola arcs with vertically focus axes is noted with $f_1$ and $f_2$ heaving the general equations:
We observe that for a correct definition is necessary to know the coefficients \( a_1, b_1, c_1, a_2, b_2, c_2 \). Therefore we need six equations with six variables resulting from the equations (4). Putting analytical conditions of position and connection we will have in accordance with fig. 1. Analytical translating the six conditions will result a six equations system with six variables (5) that is exactly solved by Gauss eliminating algorithm.

\[
\begin{align*}
&f_1(x_5) = f_2(x_3) \quad (I) \\
&f_1'(x_1) = 0 \quad (II) \\
&f_2'(x_2) = 0 \quad (III) \\
&f_1(x_5) = f_2(x_3) \quad (IV) \\
&f_1(x_1) = \beta_1 \quad (V) \\
&f_2(x_2) = \beta_2 \quad (VI)
\end{align*}
\]

Fig. 1 \( \beta \) interpolation with two parabola arcs direct connected

3. Blade calculus in projection on a plane perpendicular on the rotation axis

In both cases is possible to modify the points position \( x_3 \) or \( x_4 \) on the interval \((x_1, x_2)\) until is found an optimum variation for the blade shape. The image in projection in a perpendicular plane on the rotation axes is calculated and represented in polar coordinates system \((r, \varphi)\), where \( r \) is the radius point from the streamline and \( \varphi \) is the wrapping angle of the blade, measured between entry and exit. The \( \varphi \) angle is calculated with the equation:

\[
\varphi = \int_{\varphi_{in}}^{\varphi_{out}} \frac{dx}{r \cdot tg \beta}
\]
The integration is done numerically by summing the partial surfaces using the trapeze method. From hydrodynamic field calculation can result 3...11 or more streamlines. Applying formula (8) for each streamline will result a $\varphi_{\text{max}}$ angle. Using relation (6) for the wrapping angle of the blade calculus, $\varphi_{ij}$ (in 3D is identified with $\theta$ variable of cylindrical reference coordinate system $(r, \theta, z)$) where $i$ – controls current point along a streamline, $j$ – controls the number of streamlines, leads to some independent results, so that at the end when they are assembled in a blade frame surface, it can result with curls on width, which from a hydraulic point of view is unacceptable. From that is necessary a strategy for problem solving, which consist of imposing some supplementary conditions. These conditions are function of impeller type characterized by $n_q$. For radial impeller with $n_q < 40$ (I-case of the paper) the solution is possible by a specific modality, and for $n_q > 40$ (II) the solution is completed by supplementary aspects.

4. Transposition of the geometric data of the pump blade in the conformal transformation mapping plan

After the computer aided design of the centrifugal impeller blade using the method presented above, the 3D frame surface of the blade and the band and crown surface for the leading of the fluid to the blade zone, the blade zone and after the blade zone is obtained. For the numerical computation using FLUENT it is needed that the frame surface of the blade to be completed with a thickness function which will generate the suction side and pressure side afterwards. For this operation the most efficient and precise method is the conformal transformation mapping method. First the camber line of the blade is transpose to the conformal mapping plan. If $x$ is the curvilinear coordinate measured along the
streamline between inlet and outlet, then after previous calculus for \( N \) discrete points, indexed with \( i \) (\( i=0…N \)) the following data are known: \( r_i, z_i, x_i, \beta_i, \varphi_i \). For the construction of the camber line in the conformal mapping plan, \( \Delta A_i \) is determined using the equation:

\[
\Delta A_i = \frac{r_i + r_{i+1}}{2} (\varphi_i - \varphi_{i+1}) \quad \Rightarrow \quad A_i = \sum_{i=0}^{i} \Delta A_i \tag{7}
\]

On the Y axis \( Lm_i \) is plotted:

\[
Lm_i = \sum_{i=0}^{i} \Delta Lm_i = \sum_{i=0}^{i} \Delta x_i = \sum_{i=0}^{i} (x_i - x_{i-1}) \tag{8}
\]

The thickness function is introduced towards the camber line. For the centrifugal pump impeller this function is of constant thickness and only for the leading edge is profiled. If \( s \) is the thickness of the blade, then the offset curve that bounds the camber line is determined for the pressure side and suction side with the equations:

\[
\begin{align*}
A_{\text{ps}i} &= A_i + \frac{s}{2} \sin \beta_i \\
L_{\text{ps}i} &= Lm_i - \frac{s}{2} \cos \beta_i \\
A_{\text{si}i} &= A_i - \frac{s}{2} \cos \beta_i \\
L_{\text{si}i} &= Lm_i + \frac{s}{2} \sin \beta_i \tag{9}
\end{align*}
\]

On the leading edge the profile is made with ellipse arcs connected with the offset lines of the transpose thickness. If \( a \) and \( b \) are the semi-axes of the ellipse then the coefficient \( k_e = \frac{a}{b} \) is introduced and becomes a control parameter of the leading edge profile, and \( b \) will be \( b = \frac{s}{2} \). Depending on the curved coordinate \( x_{cl} \) (\( x \) along the camber line) in the local coordinate system (figure 3), the coordinates are determined with the equations:

\[
\begin{align*}
x_M &= k_e \frac{s}{2} - x_{cl} \\
y_M &= \frac{1}{k_e \sqrt{(k_e \frac{s}{2})^2 - x_M}} \tag{11}
\end{align*}
\]
This method of profiling the leading edge is valid only for impeller blade of constant thickness. On the trailing edge the blade is cut off. This model is tested in the first phase of the study and in the future the trailing edge will be profiled or cut off after a circle with the radius equal with the outlet radius. In figure 4 the result of this phase of the study is presented:

Using the same method for all the calculus sections all the profiles are obtained as presented in the figure 5.
It can be observed that the shape of the camber line is very much alike with the shape generated directly in the conformal mapping plane, and has no inflexions and it is monotone increasing between inlet and outlet. This aspect suggests the possibility to couple the optimisation of the blade shape through the classic method with the method of the conformal transpose.

5. Transposition of the geometric data of the impeller blade from the conformal mapping plan on the stream surfaces

The images of the blade profile from the conformal mapping plane have an identical correspondent on the stream surfaces from where the camber lines come. The transposition is made on small consecutive intervals, which were obtained from the meshing of the surface between inlet and outlet. The coordinates \((r, \varphi, z)\) of the points are obtained through linear interpolation.

The examination of the results is made with AutoCAD, using a script file which generates mesh surfaces. Through rotation, zooming and rendering the precision of the images is studied as shown in figure 6.

![Fig. 6. 3D image of the impeller blades (the band surface was removed)](image)

After everything is checked and validated these files of the coordinates of the boundary surfaces from the crown and band, the surface of the blade is imported in GAMBIT and FLUENT. These are the initial data for the procedures of the discretization of the impeller channel needed for the numerical simulation of the flow [6].

6. Conclusions

Using these interpolation methods of \(\beta\) angle between inlet and outlet, optimized forms of the centrifugal pump blades can be obtained. What before, when the computer was not used, was approximately and supposed a great
calculus effort, now it is possible to obtain in a few minutes. Starting with this modality of solving the problem it is possible to imagine other interpolation functions with other restrictions.

The shape of the camber line resulted from this mode of interpolation of the angle $\beta$ between inlet and outlet is very much alike with the one generated directly in the conformal transformation mapping plan, meaning that there are no inflexions and it is uniform increasing between inlet and outlet. This aspect suggests the possibility to couple the optimization of the blade shape through the classic method with the conformal transformation method.

The transposition of the thickness to the camber surface of the blade with the conformal transformation method gives this a very good precision compared to the classic method, and results that the geometry of the blade is much more improved.

REFERENCES