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NUMERICAL SIMULATION OF CHAOTIC MULTIMODE DYNAMICS OF A SEMICONDUCTOR LASER OPTICAL COUPLED WITH TWO EXTERNAL CAVITIES

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The dynamics of a double-reflector external cavity semiconductor laser (D-ECSL) system operating multimode with moderate feedback is given for every laser active mode by Lang-Kobayashi set of equations describing the rates of variation of the internal field and carrier density.
The developed multimode model simulates a system with 3 active modes (3 coupled single-mode systems). All modes are coupled having the same "feed" source, but evolving under different feedback conditions (feedback coefficient and round-trip time of the external cavity). The time series of the emission power represent the summing of the signals generated by the three modes.
The numerical results show that the developed numerical multimode model simulates with good approximation the chaotic dynamic of D-ECSL system for a specific set of parameter values.

Keywords: external cavity semiconductor laser, double external feedback, multimode, simulation model

1. Introduction

The external cavity semiconductor laser (ECSL) systems have received a special attention from scientific point of view on both experimentally and numerical analysis [1–3], but in conjunction with the potential applications (laser dynamic control, optical communications and data encryption) [4]. Recently, the study of these systems has contributed to the development of technical and information technology domains, i.e. the tenable high power laser diodes [5] or novel

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computational techniques [6], respectively. In these studies an important place is occupied by the numerical and experimental study of chaotic dynamics of the ECSL emission [1,7,8].

A variation of the ECSL system is the double-reflector external cavity semiconductor laser (D-ECSL) system and has theoretical and application importance in various domains, such as: laser systems development [9–15] and its dynamics stabilization [16–18], interferometer development [19], suppression of low-frequency fluctuations chaotic regime [20,21], information technology [6], data encoding transmission [22–24], spectroscopy [25] etc.

From theoretical point of view (numerical modelling and simulations) there are only a few studies that have approached the chaotic dynamics of multimode visible emission of the D-ECSL systems [26] compared to studies dedicated to the analysis of chaotic dynamics of single-mode or multimode emission of ECSL systems. There are approaches of the multimode [27] and visible [28] emission, but only for ECSL systems (with single external feedback). Also, the dynamics of D-ECSL systems was studied numerically, but only for single mode emission in infra-red [29].

In this work we aimed to develop a numerical model for simulating chaotic multimode dynamics of a semiconductor laser with compound external cavity, constituted by two optical reflectors (double external feedback) previously developed D-ECSL system [11,29,30], which operates in the so-called "low frequency fluctuation" (LFF) regime. This system consists of two external cavities, of C1 and C2 configurations, having as common branch the C1 configuration. In the experimental conditions, D-ECSL emission optical spectrum shows 3 active modes, centred at 662 nm wavelength. The numerical model, according to experimental observations simulates the 3 modes dynamics by coupling 3 single-mode systems that have the same feedback source and working under different feedback conditions values. For model development was used a multimode extension [28] of the Lang – Kobayashi [2] coupled rate equations model. The obtained numerical results show that the developed numerical model simulates with good approximation the chaotic dynamic of the D-ECSL system emission.

2. Experimental system and simulation model

2.1. Experimental system

The experimental set-up of D-ECSL system was described in details in Ref. [11]. The D-ECSL consists in a Fabry-Perot semiconductor laser (Mitsubishi, ML101J8) optically coupled with a double reflector external-cavity formed by combining two configurations of linear external cavities, C1 and C2, through a diffraction grating. These are delimited, the short one, C1 (L1= 0.445 m), by the diffraction grating, and the long one, C2 (L2= 0.652 m), by a mirror.

In this work, contrariwise the set-up given in Ref.[11,30], the diffraction grating assures feedback on 0-diffraction order, and the mirror on 1-diffraction order of the grating. Also, besides the photodiode and oscilloscope used to analyse the intensity time series, a spectrograph was used to monitor the D-ECSL emitted optical spectra. The used parameters were injection current I= 58 mA, semiconductor case temperature T= 24.9 ^oC, and coupling coefficients, c_1= 1.0 and c_2=0.72, corresponding to the neutral density filter transmissions in C1 and C2 cavities, respectively. The C1 and C2 cavities alignment procedures and the other used parameters are as they were described in the previous study.

At the used parameter, the D-ECSL system emits at 662 nm main wavelength a total power P_0= 1.79 mW, for a calculated feedback power P_{FB}= 0.313 mW, which corresponds to a moderate total feedback intensity (17%). This was calculated according to Ref. [11], $P_{FB}= P_0 \cdot (c_1)^2 \cdot C_{fb} \cdot C$, were c_1= 1.0, C_{fb}=0.65 is the calculated fraction from P_0 returned through the 0 and 1-diffraction orders, and C= 0.27 represents the sum of the power losses on the other optical components from the external cavity. In these conditions, the optical spectrum shows a multimode structure with 3 active modes (Fig. 1a). The intensity time series of the signal and the associated power spectrum are presented in Figs. 1b and 1c, respectively.

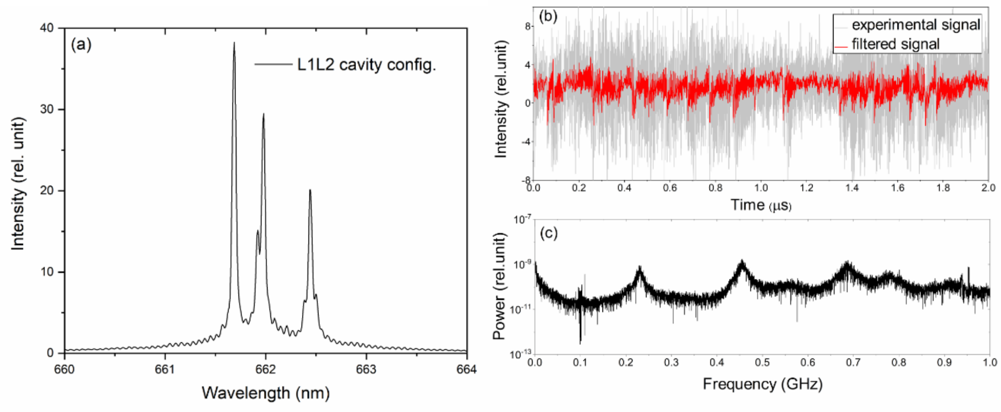


Fig. 1. D-ECSL system emission characteristics; (a) optical spectrum, (b) power intensity time series and (c) associated power spectrum for, *I*= 58 mA, *T*= 24.9 ^oC, and c_1= 1.0 and c_2=0.72.

It should be mentioned that, the signal acquisition was made under the conditions of Ref. [11] at high values of the coupling coefficients, c_1 and c_2. In the C1 configuration case it is returned the same power ratio for each mode, while in the C2 case the power is returned differentiated for each mode separately, depending on the position of the grating and the mirror. For the used experimental conditions, the components of the power spectra (frequencies) can correspond to low frequency fluctuations (LFF), with maxima up to 100 MHz, to external cavities oscillation frequencies, 337 MHz (C1 case) and 230 MHz (C2 case), and to their harmonics or to a mixture of them [11,29].

However, the effect of selective and non-selective feedback and the origin of all power spectrum components will be investigated in detail in a future study. The present study aimed to create a numerical model that reproduces the chaotic dynamics of the emission of the experimental D-ECSL system obtained under certain experimental conditions.

2.2. Differential equation system for modeling D-LSCE emission

The numerical simulation of the chaotic emission of the D-LCSE system is based on the Lang-Kobayashi [2] type rate equations for complex field, written for a compound cavity obtained by adding an mode-selective external feedback term to the standard laser equations. The dynamics of a semiconductor laser system, operating multimode with moderate feedback is given, for every laser active mode, by the set of equations describing the rates of variation of the internal field E_m (in complex form) and of the carrier density, N, [28,31]:

$$\frac{dE_m(t)}{dt} = (1 + i\alpha)(G_m(t) - \gamma_m)\frac{E_m(t)}{2} + \frac{k}{\tau_L}E_m(t-\tau)e^{-i\omega_{0m}\tau} + \sqrt{2\beta N(t)}\xi(t) \qquad (2.1)$$

$$\frac{dN(t)}{dt} = \frac{I}{e} - \frac{1}{\tau_s}N(t) - \sum_{m=-M}^{M} G_m(N)|E_m(t)|^2 \qquad (2.2)$$

$$G_m(t) = G_c(N - N_0)\left[1 - \left(\frac{m\Delta\omega_L + \frac{d\Phi_m}{dt} - \omega_N(N - N_{th})}{\Delta\omega_g}\right)^2\right], \qquad (2.3)$$

were the spontaneous emission is modelled by a complex, not correlated, Gaussian white noise term ξ of zero mean, and with a spontaneous emission rate β [31].

The coupled differential equations show a delay feedback of the cavity represented by the term $E_m(t-\tau)e^{i\omega\tau}$. The first equation models the low frequency envelope of the electromagnetic field in the cavity, the second and third equations represent the time variation of the carriers in the cavity, and the mode dependent gain, respectively. The equations (2.1) – (2.3) are written in the reference frame where the frequency at transparency of the laser is zero and neglecting the effects of lateral diffusion and spatial hole burning, and not taking into account the multiple reflections on the external resonators.

The used parameters where m= 0, ±1 (3 active modes; i.e. M= 1), and m = 0 corresponds to the mode located at the maximum of the gain curve of the solitary laser. The electric field amplitudes $E_m(t)$ are normalized so that $P_m(t) = |E_m(t)|^2$ measures the photon number in the m_{th} mode. The intrinsic laser parameters are the linewidth enhancement factor α, the mode-dependent cavity loss γ_m and the internal round-trip time τ_L, all of which are assumed equal for all modes. In the carrier density equation, τ_s is the lifetime of the electron–hole pairs, I is the injection current and e is the magnitude of the electron charge. The feedback parameters, namely the feedback level κ, and the round-trip time of the external cavity τ, can be set differently for each mode. In the first case, in the simulation model it is introduced a feedback coefficient B_m representing a subunit fraction of $B= \kappa/\tau_L$ value, and, in the second case, it is introduced τ_m representing the round-trip time depending on the external cavities length set for each mode, separately. In the case of κ, this assumption corresponds to a non-selective feedback, but with a different distribution of the intensity between modes. The phase shift $\omega_{0m}\tau$ appearing in the feedback term is due to the external cavity roundtrip, with ω_{0m} representing the nominal frequency of the m_{th} mode, i.e. $\omega_{0m} = \omega_c + m\Delta\omega_L$, where ω_c is the frequency of the gain peak of the solitary laser and $\Delta\omega_L = {}^{2\pi}/_{\tau_L}$ is the longitudinal mode spacing.

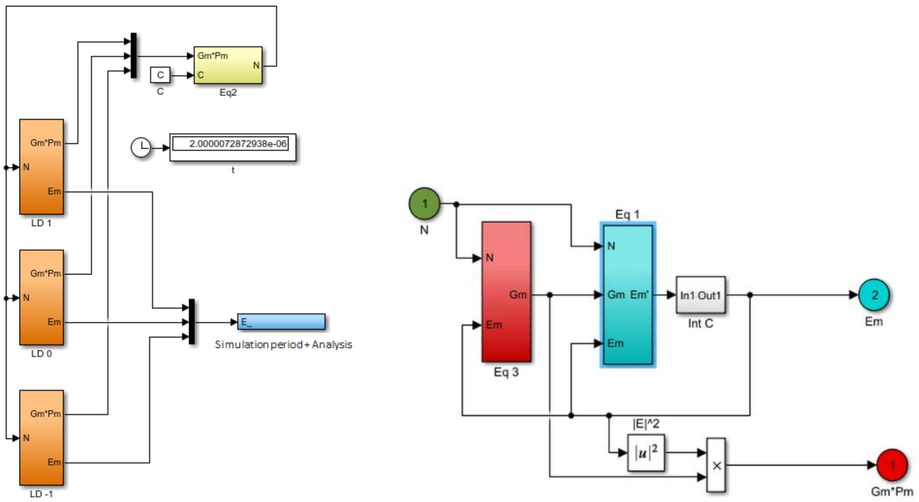
In the case of mode-dependent gain coefficient formula, G_c is the differential gain coefficient at the peak gain of the solitary laser, N_0 is the carrier number at transparency, $\Delta\omega_g$ is the gain width of the laser material, where ω_N is a constant and N_{th} is the carrier number at the laser threshold.

2.3. Development of the numerical simulation model

Numerical analysis of chaotic systems such as the D-ECSL systems that have the possibility to evolve on a chaotic deterministic trajectory, is usually the only possibility of complex and complete analysis [32]. Because the Matlab / Simulink [33] scientific programming environment is designed for easy user interface, having a very large number of general routines, with applicability in the numerical solution of various theoretical problems, and because it has an acceptable computational speed we chose this program to perform numerical simulations. The simulation and analysis of the chaotic phenomena of the D-LCSE system was done by sequential correlation between the physical experiment and the results of the numerical simulations.

Next we will present data regarding the development of the numerical simulation model. A test program was developed for a simple single-mode system with visible emission, starting from equations (2.1-2.3), to verify the typical behaviors of the system, and to be later able to easily modified it in order to adapt the behavior of the model to the different types of modeling we want to analyze. After this stage was developed the model that simulates a system with 3 coupled simple single-mode systems (3 modes). All the modes (chaotic oscillators) are coupled having the same “feed” source (N), but being under diffrent feedback conditions (B_m, and τ_m). The image of the general modules is shown in Figure 2.

The typical values for the parameters used in simulation are: a) for equation (2.1): wavelength λ= 662.0e-9 m, τ_L= 5.4e-12 s which corresponds to a laser resonator length of 400 µm, α= 4, γ_m = 5e11 s^{-1}, κ= 7.5e-2, $\tau = 2 \cdot L/c$, were L is the set length of the external cavity, and c the speed of light in vacuum, $\omega_c = 2 \cdot \pi \cdot (c/\lambda)$, and β= 1.1e3 s^{-1}; b) for equation (2.2): τ_s =2e-9s, $I_{th} \approx$ 18.590 mA; c) for equation (2.3): G_c= 4e3 s^{-1}, N_0= 1.1e8, $\Delta\omega_g$= 2·π·2.82e12 Hz, ω_N = 2.4e12 Hz, N_{th}= 1.8e8.



(a) (b)

Fig. 2. Images of (a) general modules for the numerical model with 3 active modes made in Similink-Matlab, and (b) LD1 module details.

To numerically solve the differential delay equations we used a Runge-Kutta pair of Bogacki and Shampine integrations [26,34]. Neglecting the optical feedback (κ =0), a stable state solution at a current higher than the threshold current (in this case $I_{th} \approx$ 18.590 mA) can be obtained. Fig. 3 shows the time series of the numerical solutions obtained for the power of the electric field emitted by a one ECSL system (I= 18.630 mA, , B_m= B_0=1·B) with diffrent cavity lengths, 0.652 m (Figure 3a) and 0.445 m (Figure 3b), in red being shown the filtered signal down to a frequency close to that of a usual optical detector (corresponding to Fig. 1b). The used external cavity lengths were those determinated experimentally.

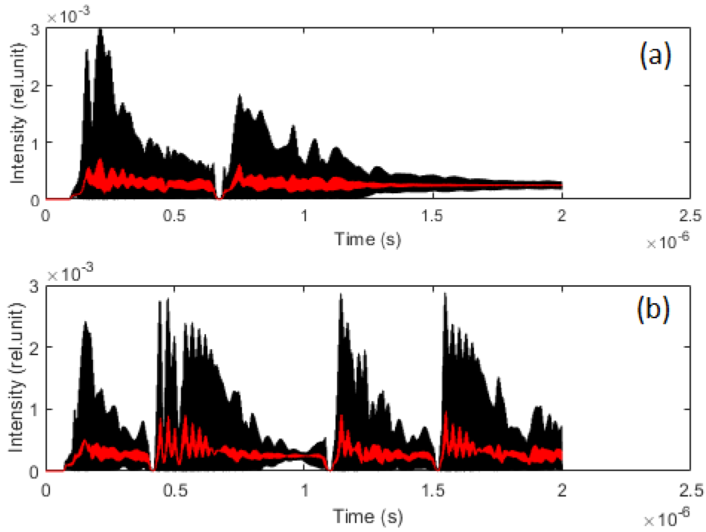


Fig. 3. Single mode numerical simulation. Time series of the ECSL system emission power (black) and the optically filtered signal (red), having the external cavity length (a) 0.652 m, and (b) 0.445 m, and I= 18.630 mA, B_0=1·B.

The obtained oscillations are characteristic of the low frequency fluctuations regime, dynamic states of an ECSL system obtained for injection current near the laser emission threshold and moderate feedback. These oscillations are characteristic of a high dimensional chaotic system.

3. Results and discussions

In D-ECSL system, if we are in the case of facilitating the occurrence of LFF (injection current near the threshold, I= 18.630 mA) and high coupling coefficients are used, several modes, in our case 3 modes are present in the system emission spectrum and a chaotic behavior similar to the cases in which only one mode is active (Figure 3) is observed in the emission dynamics (Figure 4). The time series of the power was obtained by summing the signals generated by the three modes and the duration of the power drops is similar to the single-mode case, ≈0.5μs.

It should be mentioned that the simulation was performed for non-selective extended cavities, grating and mirror that return the same power ratio for each mode. It should also be noted that each mode assumed to interacts only with its delayed field, with no influences of the neighboring modes. Also, for each m mode, the cavity length L_m (implicitly, the round-trip time in the external cavity) and the feedback coefficient B_m were established by performing tests so that, the number, position and intensity distribution of the components (frequencies) in the power spectrum are as close as possible to experimental data. Thus, a combination of cavity lengths and feedback coefficients values set on the three modes, m= 1, 0, -1, and which lead to a power spectrum similar to the one obtained experimentally is L_m: L_1= 0. 652/ L_0= 0.445/ L_{-1}= 652 m, and B_m: B_1 = 0.9·B / B_0 = 0.85·B / B_{-1} = 0.9·B (or noted such as B_m= 0.9/ 0.85/ 0.9) (Figure 4c). The power spectrum components of the numerical results (Fig. 4c, orange /middle line) correspond in good approximation with those of the experimental spectrum from Figure 4c, marked by dashed black lines. Any other combination of cavity lengths or other feedback generates power spectra that do not correspond to the experimental one (i.e. Fig. 4c, B_m= 1/ 1/ 1 case, or Figure 5).

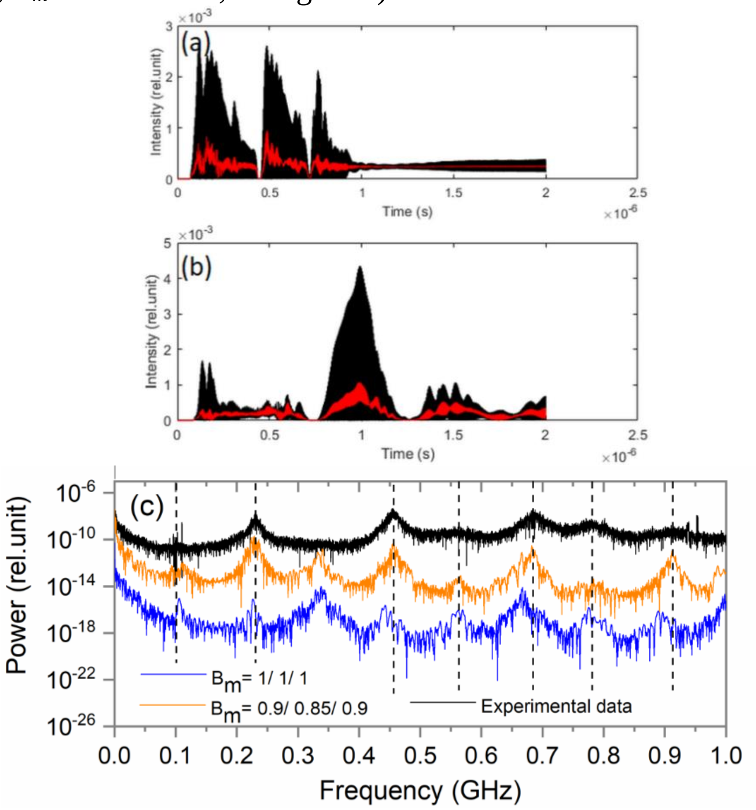


Fig. 4. Multimode numerical simulations. Time series of the D-ECSL system emission power (black) and optical filtered signal (red) for (a) B_m = 1/ 1/ 1, and (b) B_m = 0.9/ 0.85/ 0.9, and (c) the associated power spectra for experimental, and simulation at I= 18.630 mA, Lm= 0. 652 / 0.445 / 652 m; dashed black lines mark the components of experimental spectrum.

In Fig. 4c is observed that for the same combination of cavity lengths on the all modes, and equal feedback coefficients, B_m = 1/1/1, the power spectrum shows components that differ from the experimental ones, both by the value of frequencies and their power.

Fig. 5 shows comparative results when to each mode, the C1 cavity length and the associated feedback coefficient (0.85) is assigned successively, while to the other two modes are assigned the C2 cavity length and associated feedback coefficient (0.9). It is observed that the best correspondence with the experimental data (position and power of the maxima) is obtained for the cavities configuration and associated feedback values, B_m, previously specified in the Fig. 4 case.

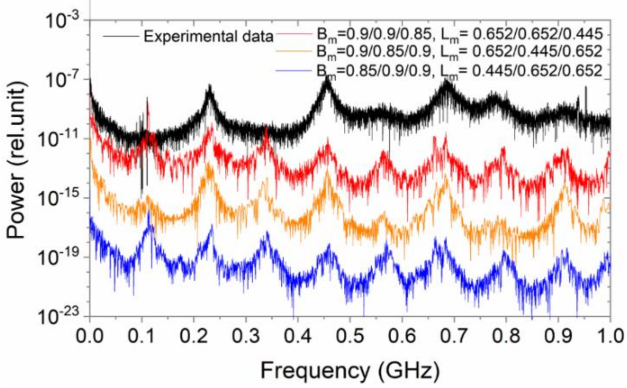


Fig. 5. Power spectra for experimental and simulation data, obtained for different combinations of the external cavities L_m and feedback coefficients B_m at the same injection currents I=18.630 mA.

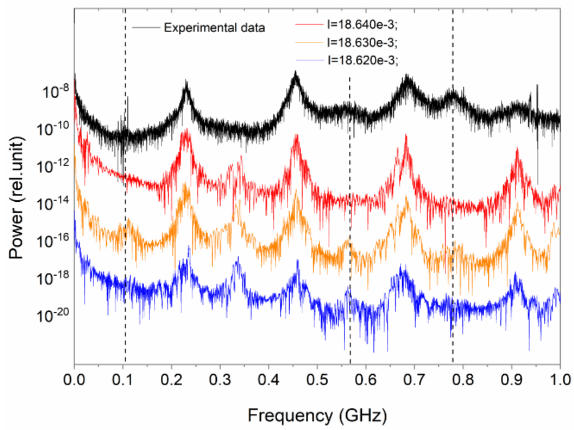


Fig. 6. Power spectra for experimental and simulation data, obtained at different injection currents I for the same combination of the external cavity lengths, L_m= 0. 652 / 0.445/ 652 m, and feedback coefficients, B_m = 0.9/ 0.85/ 0.9; dashed black lines mark the secondary maxima detailed in text.

If we vary only the injection current, I, keeping fixed the cavities configuration on modes and the associated feedback coefficients (L_m= 0. 652/ 0.445/ 652 m, and B_m = 0.9/ 0.85/ 0.9) (Fig. 6), power spectra are obtained that have the same components as the experimental one. However, the intensity of these maxima change depending on the set current. Thus, for the analyzed values, the spectrum obtained at I=18.630 mA remains the closest to the experimental one, mainly, in terms of the relative intensity of the secondary maxima, namely those marked with a dashed black line in Fig. 6.

The obtained numerical results show that the developed model simulates with good approximation the chaotic dynamic of the experimental D-ECSL system only for a specific set of parameter values.

Thus, both, the qualitative aspect and the characteristics of frequency and emission power of the obtained chaotic oscillations, compared with those in the literature [28,35], allow us to sustain that the numerical multimode model is a valid approximation of possible real behaviors of equivalent systems.

4. Conclusions

In conclusion, we have developed a numerical model for simulating the chaotic dynamics of multimode visible emission of a system with compound external cavity, with two optical reflectors, experimentally implemented.

This system consists of two linear external cavities, C1 and C2 configurations, made using a diffraction grating and having as common branch the C1 configuration. In the experimental conditions, D-ECSL emission optical spectrum shown 3 active modes. The components of the power spectrum (frequencies) associated to intensity time series correspond to low frequency fluctuations, external cavities oscillation frequencies, their harmonics, and to a mixture of them.

Next was developed the model that simulates a system with 3 coupled single-mode systems (3 modes). All the modes are coupled having the same "feed" source (N) and working under different feedback conditions. Thus, the time series of the power was obtained by summing the signals generated by the three modes.

The obtained numerical results show that the developed multimode model simulates with good approximation the chaotic dynamic (power spectrum) of experimental D-ECSL system for a specific set of parameter values. Thus, both, the qualitative aspect and the characteristics of frequency and emission power of the obtained chaotic oscillations allow us to sustain that the numerical multimode model is a valid approximation of possible real behaviors of equivalent systems.

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R E F E R E N C E S

[1] *J. Ohtsubo*, Semiconductor Lasers: Stability, Instability and Chaos, 4th Edition, Springer International Publishing, Cham, 2017.

[2] *R. Lang, K. Kobayashi*, External optical feedback effects on semiconductor injection laser properties, IEEE Journal of Quantum Electronics, 16 (1980) 347–355.
[3] *Z. Fang, H. Cai, G. Chen, R. Qu*, External Cavity Semiconductor Lasers, in: Single Frequency Semiconductor Lasers, Springer Singapore, Singapore, 2017: pp. 117–166.
[4] *B. Mroziewicz*, External cavity wavelength tunable semiconductor lasers - a review, Opto-Electronics Review, 16 (2008).
[5] *A.A. Podoskin, V.S. Golovin, P.S. Gavrina, D.A. Veselov, V.V. Zolotarev, V.V. Shamakhov, D.N. Nikolaev, I.S. Shashkin, S.O. Slipchenko, N.A. Pikhtin, P.S. Kopèv,* Properties of external-cavity high-power semiconductor lasers based on a single InGaAs quantum well at high pulsed current pump, Journal of the Optical Society of America B, 37 (2020) 784.
[6] *Y. Hou, G. Xia, W. Yang, D. Wang, E. Jayaprasath, Z. Jiang, C. Hu, Z. Wu,* Prediction performance of reservoir computing system based on a semiconductor laser subject to double optical feedback and optical injection, Optics Express, 26 (2018) 10211.
[7] *D.-A. Iordache, P. Sterian, A.R. Sterian, F. Pop*, Complex Computer Simulations, Numerical Artifacts, and Numerical Phenomena, International Journal of Computers Communications & Control, 5 (2010) 744–754.
[8] *H. A. Abdullah*, Secure speech transmission using chaotic system, U.P.B. Sci. Bull., Series C, 82 (2020) 115–126.
[9] *L. Jaurigue, B. Lingnau, K. Ludge*, Advanced control schemes for passively mode-locked lasers: Coupled lasers and dual-feedback approaches, in: 2015 International Conference on Numerical Simulation of Optoelectronic Devices (NUSOD), IEEE, Taipei, 2015: pp. 123–124.
[10] *A. Laurent, P. Chanclou, M. Thual, J. Lostec, M. Gadonna*, Double external cavity laser diode for DWDM applications, Journal of Optics A: Pure and Applied Optics, 2 (2000) L6–L8.
[11] *C. Onea, P.E. Sterian, I.R. Andrei, M.L. Pascu,* High frequency chaotic dynamics in a semiconductor laser with double-reflector selective cavity, U.P.B. Sci. Bull., Series A, 81 (2019) 261–270.
[12] *H. Lin, Y. Hong, K.A. Shore,* Experimental Study of Time-Delay Signatures in Vertical-Cavity Surface-Emitting Lasers Subject to Double-Cavity Polarization-Rotated Optical Feedback, Journal of Lightwave Technology, 32 (2014) 1829–1836.
[13] *H. Lin, A. Khurram, Y. Hong,* Time-delay signatures in multi-transverse mode VCSELs subject to double-cavity polarization-rotated optical feedback, Optics Communications, 377 (2016) 128–138.
[14] *J.-W. Wu, H.-B. Bao,* Nonlinear Dynamics in Semiconductor Quantum Dot Laser Subject to Double Delayed Feedback: Numerical Analysis, Brazilian Journal of Physics, 50 (2020) 594–601.
[15] *M.W. Lee, P. Rees, K.A. Shore, S. Ortin, L. Pesquera, A. Valle*, Dynamical characterisation of laser diode subject to double optical feedback for chaotic optical communications, IEE Proceedings - Optoelectronics, 152 (2005) 97.
[16] *K. Doi, Y. Minabe, T. Sato, T. Maruyama, M. Ohkawa, T. Tsubokawa*, Compact double optical feedback external-cavity diode laser system and its frequency stabilization, in: M. Osinski, F. Henneberger, Y. Arakawa (Eds.), San Jose, CA, 2007: p. 64680E.
[17] *S.S. Rusu, T. Oloinic, V.Z. Tronciu*, Quantum dots lasers dynamics under the influence of double cavity external feedback, Optics Communications, 381 (2016) 140–145.
[18] *F.R. Ruiz-Oliveras, A.N. Pisarchik*, Phase-locking phenomenon in a semiconductor laser with external cavities, Optics Express, 14 (2006) 12859.
[19] *M. Wang, G. Lai,* A self-mixing interferometer using an external dual cavity, Meas. Sci. Technol., 14 (2003) 1025–1031.

[20] *F. Rogister, P. Mégret, O. Deparis, M. Blondel, T. Erneux,* Suppression of low-frequency fluctuations and stabilization of a semiconductor laser subjected to optical feedback from a double cavity: theoretical results, Optics Letters, 24 (1999) 1218.
[21] *F. Rogister, D.W. Sukow, A. Gavrielides, P. Mégret, O. Deparis, M. Blondel*, Experimental demonstration of suppression of low-frequency fluctuations and stabilization of an external-cavity laser diode, Optics Letters, 25 (2000) 808.
[22] *V.Z. Tronciu, I.V. Ermakov, P. Colet, C.R. Mirasso,* Chaotic dynamics of a semiconductor laser with double cavity feedback: Applications to phase shift keying modulation, Optics Communications, 281 (2008) 4747–4752.
[23] *T. Heil, J. Mulet, I. Fischer, C.R. Mirasso, M. Peil, P. Colet,* W. Elsasser, ON/OFF phase shift keying for chaos-encrypted communication using external-cavity semiconductor lasers, IEEE Journal of Quantum Electronics, 38 (2002) 1162–1170.
[24] *J.-G. Wu, G.-Q. Xia, Z.-M. Wu,* Suppression of time delay signatures of chaotic output in a semiconductor laser with double optical feedback, Optics Express, 17 (2009) 20124.
[25] *A.K. Chernyshov, P.A. Mikheyev, N.N. Lunev,* Diode Laser with External Double Reflector for Gas Analysis, Bulletin of the Lebedev Physics Institute, 45 (2018) 83–86.
[26] *W. Chun-Lin, W. Jian, L. Jin-Tong*, Low frequency fluctuation with two external cavity reflectors, Chinese Physics, 12 (2003) 1120–1123.
[27] *F. Rogister, P. Mégret, O. Deparis, M. Blondel,* Coexistence of in-phase and out-of-phase dynamics in a multimode external-cavity laser diode operating in the low-frequency fluctuations regime, Physical Review A, 62 (2000).
[28] *J.M. Buldu, F. Rogister, J. Trull, C. Serrat, M.C. Torrent, J. Garc a-Ojalvo, C.R. Mirasso*, Asymmetric and delayed activation of side modes in multimode semiconductor lasers with optical feedback, Journal of Optics B: Quantum and Semiclassical Optics, 4 (2002) 415–420.
[29] *A. Bakry, S. Abdulrhmann, M. Ahmed,* Theoretical modeling of the dynamics of a semiconductor laser subject to double-reflector optical feedback, Journal of Experimental and Theoretical Physics, 122 (2016) 960–969.
[30] *C. Onea, P. E. Sterian, I. R. Andrei, A. Baleanu, M. L. Pascu,* Chaotic low-frequency fluctuations of laser diode emission at injection currents above threshold, U.P.B. Sci. Bull. Series C, 81 (2019).
[31] *A. Sanchez-Diaz, C.R. Mirasso, P. Colet, P. Garcia-Fernandez*, Encoded Gbit/s digital communications with synchronized chaotic semiconductor lasers, IEEE Journal of Quantum Electronics, 35 (1999) 292–297.
[32] *V. Stefanescu, D. Stoichescu, M. Frunzete, B. Florea*, Influence of Computer Computation Precision in Chaos Analysis, U.P.B. Sci. Bull., Series A, 75 (2013) 151–162.
[33] Simulink - Simulation and Model-Based Design, (n.d.).
[34] *J. Mork, B. Tromborg, P.L. Christiansen*, Bistability and low-frequency fluctuations in semiconductor lasers with optical feedback: a theoretical analysis, IEEE Journal of Quantum Electronics, 24 (1988) 123–133.
[35] *W. Ray, W.-S. Lam, P.N. Guzdar, R. Roy,* Observation of chaotic itinerancy in the light and carrier dynamics of a semiconductor laser with optical feedback, Physical Review E, 73 (2006).