RESEARCH ON THE APPLICATION OF STABLE ATTITUDE ALGORITHM BASED ON DATA FUSION OF MULTI-DIMENSIONAL MEMS INERTIAL SENSORS

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In order to obtain the stable and accurate carrier attitude angle information in both static and dynamic conditions, we need to fuse the multi-dimensional MEMS inertial sensor data, to solve the problems of various errors of a single dimensional sensor. In this paper, we design a solution for attitude algorithm which based on Kalman filter. The attitude angle is calculated by using quaternion method. The accumulated error is corrected by the measured data, and dynamic tracking compensation of random drift error model is established. So, the stable attitude data of arbitrary state is obtained. In this paper, we take the example of the application of the balance vehicle, and get a good experimental result.

Keywords: MEMS inertial sensor; Kalman filter; Quaternion; Balance vehicle

1. Introduction

MEMS (micro electro mechanical system) sensors are widely used in many civil applications because of its small size, light weight, easy to wear and low cost. Using MEMS inertial sensors to obtain the carrier attitude angle data is the core technology of many strap down inertial navigation systems, such as balance vehicle, aircraft, game handle controller. But the performance of MEMS inertial sensors which in civil applications is much less than MEMS inertial sensors which in aerospace and military applications. It has disadvantages such as low precision, high noise and large error. At the same time, it is very difficult to obtain stable attitude information in both dynamic and static conditions by relying only on one kind MEMS inertial sensor. So, we should not only study the algorithm of each kind sensor, but also to study the data fusion solutions for multi-dimensional inertial sensors.

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2. Transformation relation of different coordinate system

When we describe the carrier attitude angle, we actually refer to the transformation rotation angle of the carrier coordinate system and the northeast sky coordinate system in the three-dimensional space. The northeast sky coordinate system, also known as the geographic coordinate system, is the most commonly used coordinate system in the inertial navigation system. The three axis direction of the northeast sky coordinate system is the direction of the vertical ground plane, the direction of the magnetic north pole and the direction of the East. The carrier coordinate system takes the center of carrier gravity as the origin of coordinate. Its x axis is parallel to the cross section of the carrier and is directed to the right of the carrier. Its y axis is parallel to the longitudinal section of carrier and points to the front of the carrier. Its z axis is perpendicular to the cross section of the carrier and points to the top of the carrier. The attitude angle and heading angle of the carrier are obtained by using the coordinate transformation matrix between the carrier coordinate system and the northeast sky coordinate system.

As shown above, O- $X_0Y_0Z_0$ is the northeast sky coordinate system. After rotating in the three-dimensional space; the carrier reaches the final position O - $X_2Y_2Z_2$. The attitude angle of the carrier is the space angle needed to rotate from the northeast sky coordinate system to the carrier coordinate system. The rotation angle of the space can be seen as a composite result of rotating around the three axis in turn. First, the coordinate systems
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O - X0Y0Z0 rotates around the Z0 axis, the rotation angles is \( \psi \), and get the coordinate system O - X1Y1Z0. Then, the new coordinates O - X1Y1Z0 rotates around the Y1 axis, the rotation angle is \( \theta \), and gets the coordinate system O - X2Y1Z1. Finally, after the two-rotation process, the new coordinate system O - X2Y1Z1 rotates around the X2 axis, the rotation angle is \( \gamma \), and get the final coordinate system O - X2Y2Z2. \( \psi \) is called the yaw angle, \( \theta \) is called pitch angle, and \( \gamma \) is called roll angle. They are called Euler angles. The coordinate transformation formula is as follows [1-3]:

\[
\begin{bmatrix}
    x_2 \\
    y_2 \\
    z_2
\end{bmatrix} = 
\begin{bmatrix}
    \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\
    \sin \gamma \sin \theta \cos \psi - \cos \gamma \sin \psi & \sin \gamma \sin \theta \sin \psi + \cos \gamma \cos \psi & \sin \gamma \cos \theta \\
    \cos \gamma \sin \theta \cos \psi + \sin \gamma \sin \psi & \cos \gamma \sin \theta \sin \psi - \sin \gamma \cos \psi & \cos \gamma \cos \theta
\end{bmatrix} \begin{bmatrix}
    x_0 \\
    y_0 \\
    z_0
\end{bmatrix}
\]

(1)

3. Attitude angle algorithm for MEMS inertial sensors

3.1 Attitude estimation of accelerometer

Accelerometer is the use of the inertia of the object motion to get the acceleration of the current moment of the object. It can accurately measure the gravity acceleration information under the condition of static or uniform motion. According to the three axis accelerometer in static or uniform motion state of the gravity component of the axis, the calculation of the roll angle and pitch angle of the carrier.

In the formula (1), the vector \([x_2 \ y_2 \ z_2]\) is the projection of the three axis of gravity in the carrier coordinate system. Use the vector \([0 \ 0 \ 1]\) to replace the vector \([x_0 \ y_0 \ z_0]\). Then we use the inverse trigonometric function to obtain the roll angle \( \gamma \) and pitch angle \( \theta \). When the pitch angle reaches 90 or -90 degrees, we usually set the roll angle gamma is 0 degrees to avoid singular value [2-5].

\[
\theta = \sin^{-1}(-x_1)
\]

(2)

\[
\gamma = \sin^{-1}(y_1)/\cos \theta
\]

(3)

The premise of using the accelerometer to obtain the accurate attitude of the carrier is that the carrier is in a static state or uniform motion. That is, there is no outside force except gravity, only the distribution of the gravity component in the Z axis, so the default value of the northeast sky coordinate system in the formula (1) is determined. When the carrier is subjected to shock motion, accelerometer can’t distinguish between gravity and external
force, it will produce a lot of error. The estimates of the attitude angle results will appear distortion. A simple example is when the accelerometer does low speed movement, X axis and Y axis angle basically does not change, both at about 0 degrees. Once it moves quickly, X, Y axis will appear large angle changes, and in fact the accelerometer has been in a horizontal state, no change.

### 3.2 Attitude estimation of magnetometer

The magnetometer is a kind of sensor which is used to measure the earth’s magnetic field strength. In the algorithm of solving attitude angle, it is used to calculate the heading angle. At any moment, the magnetic strength of the earth’s magnetic field measured by the magnetometer is:

\[
M = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}
\]  \hspace{1cm} (4)

Calculate the heading angle using the measured values in the X axis and the Y axis:

\[
\psi = \tan^{-1} \left( \frac{m_y}{m_x} \right)
\]  \hspace{1cm} (5)

In actual use, due to the earth's magnetic field is not stable, easily disturbed, magnetometer is rarely used alone. In general; we will combine the magnetometer and other inertial sensors to measure attitude angle.

### 3.3 Attitude estimation of gyroscope

The gyroscope sensor is a sensing device which is made of the mechanical principle of gyroscope. It can sense the angular rate of rotation of the carrier. In the strap down inertial navigation system, it uses gyro data to do integral operation, and then gets the attitude angle of the carrier. However, this method can only be used to solve small rotation angle due to the coupling problem of the three axis. Large errors will occur when the rotation angle is large. To avoid this problem, use the angular velocity to solve the differential equation of the Euler angle, the direction cosine matrix or quaternion, and update the attitude angle in real time. The use of Euler angle in the computation of trigonometric functions, not easy to be calculated. The calculation of direction cosine matrix is larger; by comparison, quaternion method has the advantage of a small amount of calculation and high precision, so it is the most commonly used. The quaternion consists of one real unit and three imaginary unit I, j, k. Its form
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is \( \mathbf{q} = \lambda + p_1 \hat{i} + p_2 \hat{j} + p_3 \hat{k} \). The quaternion can represent the rotation of three-dimensional space. Its scalar represents the size of the rotation angle, and the vector section indicates the direction of the rotating shaft.

The first step, the first moment of the attitude angle is converted to quaternion. The formula is as follows [6-8]:

\[
\begin{bmatrix}
\lambda \\
q_1 \\
q_2 \\
q_3
\end{bmatrix}
= 
\begin{bmatrix}
\cos \frac{\omega}{2} \cos \frac{\theta}{2} + \sin \frac{\omega}{2} \sin \frac{\theta}{2} \\
\sin \frac{\omega}{2} \cos \frac{\theta}{2} - \cos \frac{\omega}{2} \sin \frac{\theta}{2} \\
\cos \frac{\omega}{2} \sin \frac{\theta}{2} + \sin \frac{\omega}{2} \cos \frac{\theta}{2} \\
\cos \frac{\omega}{2} \cos \frac{\theta}{2} - \sin \frac{\omega}{2} \sin \frac{\theta}{2}
\end{bmatrix}
\]  

(6)

The second step, differential equation for the quaternion formula:

\[ \dot{q} = \frac{1}{2} q \omega \]  

(7)

By using four order Runge Kutta method to solve [4, 9-12]:

\[ K_1 = \frac{1}{2} q(t) \omega(t) \]  

(8)

\[ K_2 = \frac{1}{2} \left( q(t) + \frac{K_1}{2} T \right) \omega \left( t + \frac{T}{2} \right) \]  

(9)

\[ K_3 = \frac{1}{2} \left( q(t) + \frac{K_2}{2} T \right) \omega \left( t + \frac{T}{2} \right) \]  

(10)

\[ K_4 = \frac{1}{2} \left( q(t) + K_3 T \right) \omega \left( t + \frac{T}{2} \right) \]  

(11)

Finally obtain the formula:

\[ q(t + T) = q(t) + \frac{T}{6} (K_1 + 2K_2 + 2K_3 + K_4) \]  

(12)

T is the update cycle, \( q (T) \) is the previous quaternion, \( q(t+T) \) is the updated quaternion.

The third step, the updated quaternion is brought into the attitude matrix, and the attitude angle is obtained.
The attitude angle is obtained from the attitude matrix.

\[
\theta = -\sin^{-1}(T_{13})
\]

(14)

Roll angle \( \gamma \) : \[
\gamma = \tan^{-1}\left(\frac{T_{23}}{T_{33}}\right)
\]

(15)

Gyroscope is susceptible to temperature, unstable moment and other factors, resulting in random drift error. Moreover, the angular velocity integral can cause the accumulated error. Cumulative error is not obvious in short time. As long as the correct handling of zero drifts, the drift error within 1 minute is very small. But after a long time, it will produce a lot of cumulative errors, may be 5 minutes later, the result has been seriously distorted.

4. Data fusion algorithm

How to get a high precision and no drift attitude angle? Gyro precision is high, but it is easy to be affected by the environment resulting from drift error, a long time there will be cumulative error. The accelerometer does not have the problem of long term drift, but the dynamic accuracy is poor. Can we integrated three kinds of sensors advantages, and get the attitude angle accurately? The answer is yes, the method is to use the Kalman filter to do data fusion.

4.1 Random error model of gyroscope

The gyroscope is easy to be affected by the factors such as temperature, unstable moment, and so on. So, it is necessary to analyze the random error model. The measurement model of the gyroscope is:

\[
\omega_{\text{gyro}}(t) = \omega(t) + \epsilon(k) + \nu_{\text{gyro}}(t)
\]

(16)

The \( \omega_{\text{gyro}}(t) \) is the output of the gyro. The \( \omega(t) \) is the true value of angular velocity. The \( \nu_{\text{gyro}}(t) \) is measured white noise. The \( \epsilon(k) \) is random drift error, can be expressed as [13-16]:

\[
T = \begin{bmatrix}
(\lambda^2 + p_1^2 - p_2^2 - p_3^2) & 2(p_1 p_2 + \lambda p_3) & 2(p_1 p_3 - \lambda p_2) \\
2(p_1 p_2 - \lambda p_3) & (\lambda^2 + p_2^2 - p_1^2 - p_3^2) & 2(p_2 p_3 + \lambda p_1) \\
2(p_1 p_3 + \lambda p_2) & 2(p_2 p_3 - \lambda p_1) & (\lambda^2 + p_3^2 - p_1^2 - p_2^2)
\end{bmatrix}
\]
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\[ \varepsilon(t) = C_1 \left( 1 - e^{\frac{t}{T}} \right) + C_2 \]  \hfill (17)

After differentiation, the difference equations are obtained by discretization.

\[ \varepsilon(k+1) = \frac{T}{T+T_3} \varepsilon(k) + \frac{T_3}{T+T_3} \left( C_4 + C_2 \right) \]  \hfill (18)

The \( T \) is the sampling time, the parameters \( C_1 \) and \( T \) need to be data fitted by the actual measured values of the gyroscope. Because the gyroscope random drift error is nonlinear, Levenberg-Marquardt nonlinear least square method can be used for data fitting. In the process of calculating the attitude angle by the quaternion, gyro output data is corrected, and the influence of random error is reduced.

4.2 Kalman filter

In addition to the problem of random error, the gyroscope angular velocity integral operation will cause cumulative error. At this time, we need the Kalman filter for data fusion, use the measured values obtained from the accelerometer and magnetometer to correct the accumulated results of the gyroscope. Kalman filter is an optimal recursive data processing algorithm. It uses a feedback control approach to estimate a process [6-8]: The filter first estimates the state of the process at a certain time, and then adjusted by a number of measurement values that carry noise. Kalman filter equation contains two parts: time update equation and measurement update equation. The time update equation is responsible for predicting the current state and computing a priori estimates of covariance. The measurement update equations are responsible for feedback control, i.e., the combined new measurement results and a priori estimates are obtained by an improved posterior estimator. The time update equation can be regarded as a correction procedure. As shown in Fig. 2:
The state equation is the quaternion equation of the 3.3 section, i.e., the angular velocity of the gyro output is brought into the differential equation of the quaternion at last moment, and calculates the current prediction attitude angle. The measurement equation is the attitude angle equation of the 3.1 and 3.2 section, i.e., according to the output data of accelerometer and magnetometer to calculate attitude angle, which for correction prediction value. The data $P_k^{-}$ is a priori estimate of the error covariance of the state equation. The previous moment $P_{k-1}^{-}$ and the state noise covariance $Q$ is added in each recursive operation. The value $K_k$ is Kalman gain, based on the value $P_k^{-}$, combined with the noise covariance $R$ generated by the measurement equation. Both $Q$ and $R$ can be calculated by the recently measured data series [17-21].

In the initial static environment, initial attitude angle is calculated by the measured value of magnetometer and accelerometer. Then the initial quaternion is obtained from the initial attitude angle. According to the angular velocity data of the gyro output, the attitude angle which as the estimated value of the next moment is obtained by solving quaternion differential equation. The output of the accelerometer and magnetometer as measured value to correct the estimation attitude angle. Then the state noise covariance $Q$ and the measurement noise covariance $R$ are calculated. Using Kalman filtering algorithm to realize the fusion of estimation angle and measuring angle. The optimal estimation angle is obtained. The obtained optimal estimation angle is regarded as the starting point of the next

<table>
<thead>
<tr>
<th>Time update equation (Predict)</th>
<th>measurement update equation (correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Predict current status $x_k^{-} = A_k x_{k-1} + B_k u_{k-1}$</td>
<td>1. Kalman gain $K_k = P_k^{-} H_k^T (H_k P_k^{-} H_k^T + R)^{-1}$</td>
</tr>
<tr>
<td>2. Predict error covariance $P_k^{-} = A_k P_{k-1} A_k^T + Q$</td>
<td>2. Using measured value to update estimates value $x_k = x_k^{-} + K_k (z_k - H_k x_k^{-})$</td>
</tr>
<tr>
<td></td>
<td>3. Update error covariance $P_k = (1 - K_k H_k) P_k^{-}$</td>
</tr>
</tbody>
</table>

Fig. 2 the complete process of Kalman filter
calculation. Repeated data fusion algorithm to get a series of stable optimal estimation. Its flow chart is shown in Fig. 3.

\begin{center}
\begin{tikzpicture}

\node[draw, rounded corners] (start) {Start};
\node[below=of start, draw, rounded corners] (acquisition) {Acquisition of gyroscope, accelerometer and magnetometer data};
\node[below=of acquisition, draw, rounded corners] (first) {Is it the first time to run?};
\node[below=of first, draw, rounded corners] (initial_attitude) {Calculate the initial attitude angle};
\node[below=of initial_attitude, draw, rounded corners] (initial_quaternion) {Calculate the initial quaternion};
\node[below=of initial_quaternion, draw, rounded corners] (gyro_update) {Using gyro angular velocity update quaternion};
\node[below=of gyro_update, draw, rounded corners] (process_noise) {Calculation process noise covariance Q};
\node[below=of process_noise, draw, rounded corners] (noise_covariance) {Calculation of noise covariance R};
\node[below=of noise_covariance, draw, rounded corners] (kalman_filter) {Kalman filter for data fusion obtain the optimal attitude angle};
\node[below=of kalman_filter, draw, rounded corners] (attitude_angle) {Attitude angle is converted to the quaternion at the next moment};
\node[below=of attitude_angle, draw, rounded corners] (end) {End};

\draw[->] (start) -- (acquisition);
\draw[->] (acquisition) -- (first);
\draw[->] (first) -- node[anchor=south] {yes} (initial_attitude);
\draw[->] (initial_attitude) -- (initial_quaternion);
\draw[->] (initial_quaternion) -- (gyro_update);
\draw[->] (gyro_update) -- (process_noise);
\draw[->] (process_noise) -- (noise_covariance);
\draw[->] (noise_covariance) -- (kalman_filter);
\draw[->] (kalman_filter) -- (attitude_angle);
\draw[->] (attitude_angle) -- (end);
\end{tikzpicture}
\end{center}

5. Experimental result analysis

We apply the data fusion algorithm in this paper to the balanced vehicle. The balanced vehicle is a typical strap down inertial navigation system. The balance vehicle can perceive the human posture change by the inertial sensor technology, and simulate the human upright walking mode, so as to control the self-balancing behavior. We want to test is that, whether the balance vehicle in the stationary state or in motion state, can the data fusion algorithm is able to capture the human body accurate attitude information?

Fig. 4 is the balance vehicle in a stationary state, without the data fusion algorithm, just use the gyroscope output data to get the pitch angle and roll angle. It can be seen that, although the gyroscope has been in a state of the level, but the attitude angle has been changing over time, and it is distorted in a short time. This is the result of the random drift error of the gyroscope accumulate with time.
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Fig. 4 The attitude angle obtained from gyroscope data in the static state

Fig. 5 is the pitch angle and roll angle after Kalman filter. After being corrected by measuring value, we can see the pitch angle and roll angle near 0°, the error control in 0.2°. Contrast Fig. 4, error accumulation of several tens of degrees in 1 minutes; it can be proved that the random drift error of the gyroscope is corrected effectively by Kalman filter, and the problem of error accumulation is solved.

Fig. 5 The attitude angle obtained by the data fusion in the static state

Fig. 6 is the results of the dynamic attitude estimation for the balanced vehicle. It can be seen that the angle of the accelerometer is measured has the obvious burr waveform and the mutation component, may cause the shock of the body, affecting the safety of the driver. In this paper, the data fusion algorithm is used to effectively eliminate the interference noise and the mutation component, and achieve the correct estimation of the vehicle body posture.
6. Conclusion

Simply relying on one kind inertial sensor to obtain the attitude of the carrier cannot obtain satisfactory results. The dynamic accuracy of the gyroscope is very high, and the accurate rotation angle can be obtained in a short time, but it will generate accumulate errors in a slightly longer time. The accelerometer can obtain accurate attitude angle in the condition of static and uniform speed. However, once it is subjected to other external forces in addition to gravity, it also cannot guarantee accuracy.

This paper studies the algorithm of the gyroscope, accelerometer and magnetometer sensor. On the basis of this, this paper designs a suitable data fusion algorithm based on Kalman filter. The attitude angle is estimated and corrected by the combined measurement results of multi-dimensional sensors. Proved by experiment, the design of the data fusion algorithm base on Kalman filter and quaternion, not only effectively corrects the gyro random drift error, solves the problem of error accumulation, but also avoids the noise of accelerometer in the dynamic case, achieved good experimental results. Therefore, the accurate vehicle body posture can be obtained both in the dynamic and static conditions, to ensure the safety of the driver.

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