ASPECTS REGARDING THEORETICAL BASIC CONCEPTS OF THE PROBABILISTIC DESCRIPTION OF IMPACT DAMAGE SUSCEPTIBILITY FOR A FRUIT POPULATION (APPLES)

T. CĂŞANDROIU, GH. VOICU, Magdalena – Laura TOMA

In this paper it is theoretically grounded and experimentally validated that impact damage susceptibility of the apples, statistically expressed by damage probability which depends on impact level, can be best described by a logistic function as

$$p = \frac{1 - \exp(-\mu X)}{1 + \exp(C - \mu X)}$$

where $p$ – damage probability; $X$ – physical quantity characterising the impact level; $\mu$, $C$ – logistic coefficients resulted from experimental data. In our experiments, performed on a 36 apple lot from Jonathan variety, there was identified that the most appropriate characterisation of the impact level is $X = E_{ci}$ (initial kinetic energy of impact), for which there was found $\mu = 8,392$; $C = 1,698$ and a correlation coefficient $R^2 = 0.965$, from experimental data.

**Keywords:** impact level, damage, susceptibility, probability, logistic function.

**Introduction**

During mechanical manipulation of fresh fruit transport, sorting – packing, storage operations, after harvesting, they are subjected to different mechanical stresses causing damages. As a rule, stress application velocity, which defines impact stress, exceeds 25 – 30 cm/s [1 – 3]. That is why fruit impact one against the other or against protected or unprotected working surfaces of the technical
systems, represents the major cause of its bruises [1, 2, 4 – 8]. Bruises are produced when stresses exceed fruit’s pulp elastic limit [2, 4, 9 – 17] and they are highlighted by tissue bruises (crushes) and they manifest by changing in brown of the pulp colour or by pulp cracking (with or without cracking of the epidermis) [2, 3, 18 – 22]. These bruises develop in time reducing the marketing quality of the fresh fruits and also, the fruit storage period causing losses [2, 4, 23 – 27].

Damage degree and fruit damage percent, in various mechanical handling operations, depends on: fruits’ variety and its maturity degree (ripeness degree); growing and developing conditions in orchard; impact point on the fruit surface; its physical, mechanic and rheological characteristics; tissues heterogeneity and anisotropy; complexity and constructional and functional parameters of the technical system; working type and regime; specific time when operation is done (before or after a storage period) [2, 6, 12, 14, 20 – 22, 25, 29].

Both on international and national level there are developed intense researches regarding evaluation of the different aspects related to the impact behaviour of different fruit varieties (especially apples) [1 – 6, 8 – 38].

An important theoretical and experimental research direction aims to study the fruits impact behaviour for the direct evaluation of the bruising degree and for developing mathematical models which realise the correlation between bruising degree and impact level, allowing the impact bruise prognosis [3 – 8, 10, 12 – 15, 18 – 20, 25, 26, 29 – 32, 34 – 36, 38].

There is a large diversity of ways to appreciate the fruit tissue pulp bruising due to impact, namely: a) by the bruised tissues volume [2, 5, 7, 8, 25, 32]; b) by the bruise indent diameter visible on fruit surface [3, 5, 8, 13, 25]; c) by the absorbed or available specific kinetic energy to which the first crushed cells appear, invisible by the naked eye and which develop in time (they can be found at 1.5 – 2.5 mm under fruit epicarp) [3, 19]; d) by impact damage susceptibility coefficient (defined by available impact kinetic energy at which the fruits bruising probability with a damage degree under an admissible limit (ex.: bruised surface ≤ 1 cm² [9, 24]) does not exceed 0.05 or maximum 0.10 in accordance with E.U. standards [21 – 24, 27] or available impact kinetic energy at which first bruised tissues appear and the bruising probability does not exceeds 0.05 or 0.10 [33, 37]; e) by the bruising degree estimated by the probability that fruits are not containing over a certain number of bruising points or a certain level of bruised surface [39, 40].

From all these damage evaluations, lately, it is developed and perfected the model d) in which it is defined and grounded the impact damage susceptibility concept using the impact kinetic energy value for an accepted level of the bruised surface ≤ 1 cm². Impact damage probability does not exceed established level of 0.05 or maximum 0.10 [9, 23, 24]. Natural variability of the factors influencing fruit bruising and establishing a strong variability of the fruit impact damage appearance has required the introduction of the damage susceptibility concept.
In [33], we have proposed a general expression for the impact damage susceptibility coefficient using “impact level” as a replacement for “impact kinetic energy value”.

In this new situation it can be used for “impact level” characterisation the best suited physical quantity from the proposed ones \((E_{ci} – \text{impact kinetic energy}; v_i – \text{initial impact energy}; a – \text{maximum indent radius of fruit deformation during impact})\) on the base of the best concordance with experimental data of the logistic mathematical model having two parameters. It associates to the damage fruit percentage the impact damage probability in accordance to the impact level [33], namely:

\[
p = \frac{\exp(C + m X)}{1 + \exp(C + m X)},
\]

where: \(p\) is damage probability; \(X\) is physical quantity characterising the impact level; \(m, C\) are logistic coefficients.

Developing a method (which might become a standard) for the determination of a fruit impact bruising coefficient, we can obtain very important and useful data in genetic improvement, in variety selection, in crop growing and development techniques, in environmental and agro technical evaluation, in choosing the post harvesting treatment, in design improvement of the handling technical systems, in choosing the best working regime, in engineering activity et. al. [4, 5, 8, 9, 20, 24 – 27, 33, 37].

The subject of this paper consists in finding an impact damage susceptibility measurement which can be statistically described by damage probability in correspondence with an impact level limit.

The objectives of the paper are: a) to establish the theoretical basic concepts of the mathematical model for the statistic description of the damage probability in accordance with the impact level; b) to test the theoretical model using experimental data from Jonathan variety apples; c) to identify the most adequate physical quantity to characterise the impact level and the evaluation of the maximum threshold of the admissible impact level for a specified damage probability tolerance.

1. **Theoretical considerations. Statistic model**

The experimental data that we posses [22, 24, 27, 33, 37] regarding probabilistic interpretation of impact damaged fruit percentage have shown that the graphic representation is a sigmoid curves, well described by logistic models, for which we intend to give a theoretical explanation.

For the impacted fruits at a known impact level, some of them could present a bruise that exceeds the specified accepted bruise, others could present a bruise under the specified one, and others can be undamaged. All fruits for which the accepted damage was exceeded can be considered “rejected” and the rest “accepted”. This fact is statistically represented as a binomial probability. Once the impact level grows, the rejected fruits volume will also grow. This could be
considered a relation between impact level and damage probability, representing a useful description of the damage susceptibility.

The damage susceptibility notion represents the best natural variability of the influence factors determining the strong variability of damage appearance and, therefore, of the susceptibility requiring a statistic – probabilistic tackling of the phenomenon.

We will use, in our analysis, the analogy to the limited grow probabilistic model of a micro organisms population (logistic model) [41, 42].

One considers that at an impact level $X$, the numerical dimension of the 

$\text{damaged fruit population is } \mathcal{N}_v(X)$. 

Infinitesimal variation of damaged fruits due to an impact level variation $X$ can be considered, at a first approximation, proportional to the numerical dimension of the undamaged fruit population $N$ which is a part from the number of the fruit population $\mathcal{N}_o$ subjected to impact ($\mathcal{N}_v + N = \mathcal{N}_o$). If there are no constrains ($\mathcal{N}_o \rightarrow \infty$), the growth of the number of the damaged fruits can be expressed by the equation:

$$\frac{d\mathcal{N}_v}{dX} = \mu N$$  \hspace{1cm} (1)

where $\mu$ represents the speed of growth parameter or relative speed of growth (specific speed of growth). In limited growth conditions, ($\mathcal{N}_v \rightarrow \mathcal{N}_o, \mathcal{N}_o = \text{finite}$), the relative speed of growth is no longer constant. It decreases with respect to the increase of the number of damaged fruit population.

In logistic model, specific speed of growth is often supposed to decrease linearly with the number of damaged fruit population. Thus the development equation of the damaged fruits population is:

$$\frac{d\mathcal{N}_v}{dX} = \mu N \left(1 - \frac{N}{K}\right)$$  \hspace{1cm} (2)

with initial conditions $X = X_o$, $N = \mathcal{N}_o$ ($\mathcal{N}_v = 0$), in which $\mu$ is the velocity coefficient for potential exponential growth of the population (in accordance with relation (1)), while $K$ has the meaning of a bearing ability of the damaged fruit population having values beyond the numerical value of the micro organisms population ($K > \mathcal{N}_o$ for determining the $\mathcal{N}_v$ growth).

Taking into account that $\mathcal{N}_v + N = \mathcal{N}_o$, equation (2) becomes:

$$\frac{dN}{dX} = -\mu N \left(1 - \frac{N}{K}\right)$$  \hspace{1cm} (3)

It represents the differential logistic model for the dimensional variation of the undamaged fruit population $N$ which is related to the impact level $X$ to which the population dimension $\mathcal{N}_o$ is subjected.

Separating the variables, equation (2) can be written as:
Aspects regarding theoretical basic concepts of the probabilistic description of impact damage

\[
\frac{K}{N(K - N)} \, dN = -\mu dX
\]  

(4)

Imposing \(N(X) = N_o\) for \(X = X_o\), after integrating equation (4), it results:

\[
\ln\left(\frac{N}{K - N} \cdot \frac{K - N_o}{N_o}\right) = -\mu(X - X_o)
\]  

(5)

Exponentially expressed, after calculations, it is found:

\[
N(X) = \frac{K}{1 + \frac{K - N_o}{N_o} e^{\mu(X - X_o)}}
\]  

(6)

Relation (6) represents the logistic equation for undamaged fruit population dimension \(N(X)\) from a fruit population \(N_o\) subjected to impact level \(X\).

Taking into account relation (6), percentage \(p\) of damaged population \(N_v\) from population \(N_o\) subjected to impact level \(X\), defined by \(p = \frac{N_v}{N_o} = 1 - \frac{N}{N_o}\), the percentage assimilated with damage probability, after calculations is:

\[
p = \frac{1 - e^{-\mu(x - x_o)}}{1 + e^{c - \mu(x - x_o)}}
\]  

(7)

where:

\[
\frac{N_o}{K - N_o} = e^c.
\]  

(8)

For the particular case \(X_o = 0\), from (7) there results:

\[
p = \frac{1 - e^{-\mu x}}{1 + e^{c - \mu x}}
\]  

(9)

One notices that for an impact level \(X = X_o\) in relation (7), respectively \(X_o = 0\) in (9), it results \(p = 0\) and for an impact level indefinite large \((X \to \infty)\), it is found \(p = 1\). This fits better to the real situation, compared to logistic relations proposed in [22, 24, 27, 33, 37].

From the relation (2) it comes out that \(N_v(X)\) increases constantly with the impact level \(X\), if \(0 < N_v < N_o\) because \(\frac{dN_v}{dX} > 0\).

Second derivative of \(N_v(X)\) is:

\[
\frac{d^2N_v}{dX^2} = -\mu^2 \left(1 - \frac{2}{K} \right) N \left(1 - \frac{N}{K}\right)
\]  

(10)

Using relation (10) and \(N_v + N = N_o\) one can show that \(\frac{dN_v}{dX}\) is increasing if \(N_v(X) < N_o - \frac{K}{2}\) and decreasing if \(N_v(X) > N_o - \frac{K}{2}\), while for \(N_v(X) = N_o - \frac{K}{2}\),
\[ \frac{d^2 N}{dX^2} = 0. \] This means that there is an inflexion point in \( N_v(X) \) depending on \( X \) graphic and the sigmoid curve has a S shape. This is experimentally proved in \([24, 33]\).

If we denote \( K = \alpha N_o \) \((\alpha > 1)\), at the curve inflexion point \( N_{o_1}(X) = N_o - \frac{K}{2} \) and

\[ N_i = N_o - N_{vi} = \frac{\alpha}{2} N_o. \]

The curve slope \( N_v(X) \) results from relation (2) and it is

\[ \left( \frac{dN_i}{dX} \right) = \frac{\mu \alpha}{4} N_o > 0. \quad (11) \]

From experimental data it results that \( \mu \ll \alpha \). Therefore, curve gradient \( N_v(X) \) in inflexion point for a given \( N_o \) (constant) is practically determined by \( \mu \) value and can be considered as a measure for curve steepness. The values of \( \mu \) increase at the same time with fruit ripeness degree and, therefore, steepness degree of sigmoid curve appreciated by \( \mu \) can be taken as a measure for the ripeness degree. Higher values of \( \mu = \mu_1 \) and of steepness for logistic curve of impact damage probability correspond to advanced fruit ripeness degrees. For lower ripeness degrees, lower values of \( \mu = \mu_2 \) and of steepness correspond because for \( \mu_1 > \mu_2 \) there results \( \left( \frac{dN_i}{dX} \right)_1 > \left( \frac{dN_i}{dX} \right)_2 \), from relation (11).

The ripeness degree at which fruits tissues soften and \( \mu \) values increase indicating higher steepness of logistic curves can be related.

For impact level characterisation \( X \), the most adequate of the following physical quantities is used: \( E_{ci} \) – initial kinetic energy; \( v_i \) – initial impact energy; \( a \) – maximum indent radius on the fruit surface during impact. This is experimentally determined on the base of logistic law testing (7) or (9) using experimental data, and choosing the physical quantity for which the best concordance between logistic law and experimental data is realised. It is proved that the best physical quantity for impact level characterisation \( X \) is the initial impact energy \( E_{ci} \) \([24, 33]\).

Mathematical models (7) and (9) will be tested by experimental data obtained for Jonathan apples.

2. Apparatus, materials and procedures

A 36 Jonathan apple population were used, after a 6 months storage period, at a temperature of 2 – 4ºC in frigorific cells. Experiments were performed in the laboratory of “Physical proprieties of agricultural materials”, Agricultural Machinery Department from Biotechnical Systems Engineering Faculty of U.P.B. Four impact velocity levels \((0.79; 1.0; 1.22; 1.53 \text{ m/s}) \) were used in experiments. Apples impacts with a rigid plane surface were performed in our laboratory, using
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A pendulum apparatus [3, 19, 36]. Before impact fruits were weighted with a technical laboratory balance having a 0.1 g precision, while their geometrical dimensions were measured with a 0.1 mm precision calliper rule. Supposed to be spherical in the proximity of the impact points, their surfaces radii were measured with a 0.01 mm precision spherometer [2, 3]. After impact, fruits were kept at environmental temperature (18 – 20ºC) in compartmented cardboard boxes and they were examined on subgroups of 3 apples after 3, 5 and 10 days after impact. The impact area (marked) was examined by the naked eye or by a magnifier (5X), at the surface and in section, for the identification of the bruised tissues highlighted by pulp colour change (in brown).

To determine the maximum impact area, the apple is covered in the presumed impact area by a coloured substance (using an indigo paper moist in mineral oil). This colour will let an indentation on a piece of white paper. Thus, the geometrical shape and its dimensions can be determined [3, 19, 36].

Either by direct measurements, or by calculus, the following quantities required for impact damage analysis were determinate: fruit mass \( m \), geometrical dimensions \( D – \) equatorial diameter, \( H – \) fruit height), velocity at the beginning of impact \( v_i \), radius of the maximum contact surface \( a \), kinetic energy at the beginning of impact \( E_{ci} \).

3. Experimental data. Interpretation. Comments

Primary data obtained by experiments are presented in Table 1. The mean values \( \bar{X} \) partially processed and the standard deviation \( \sigma \) of fruit masses, the maximum contact surfaces radii during impact and initial impact kinetic energies for all four impact levels, are presented in Table 2.

It comes out that for mean values of apples masses in range from 126.7±6.0 to 156.1±13.1 g, having initial impact velocities level from 0.79 to 1.53 m/s, the mean values level of the initial kinetic energies grow from 0.101±0.005 to 0.352±0.006 J, while the mean values of the maximum contact surfaces radii during impact grow from 8.1±0.3 to 10.3±0.6 mm, as well as the bruises percentage from 22.2 to 77.8%.

Using mean values data from Table 2 the correspondence of the theoretical logistic models expressed by relations (7) and (9) with experimental data was tested. Using the specialized software MicroCal Origin 6.0 on a Pentium IV PC the coefficients values from the two equations, together with the correlation coefficients values \( R^2 \) and the concordance test \( \chi^2 \) corresponding for each of the physical quantities \( E_{ci}, \ v_i, \ a \), used for impact level \( X \) characterisation, were determinated. The obtained values are given in Table 3.

Examining the correlation coefficients values \( R^2 \) and the test \( \chi^2 \) there results a good concordance of the data obtained in experimentations by the help of
logistic function (7) or (9). A best fitted physical quantity, for impact level characterisation, of $E_{ci}$ ($R^2 \geq 0.965$) was used. This physical quantity is more complex because it includes the fruit mass and its impact velocity ($E_{ci} = \frac{mv_i^2}{2}$).

**Table 1**

Primary data obtained in experiments regarding Jonathan apple impact, after a 6 months storage period; $m$ – mass (g); $v_i$ – initial impact velocity (m/s); $a$ – maximum contact surfaces radius (mm); $E_{ci}$ – kinetic energy at the beginning of impact (J).

<table>
<thead>
<tr>
<th>$v_i$ (m/s)</th>
<th>$m$ (g)</th>
<th>$a$ (mm)</th>
<th>$E_{ci}$ (J)</th>
<th>Observations regarding damage</th>
<th>Damage probability – damaged fruits percentage $p$</th>
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<td></td>
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<td>Hard damaged tissue</td>
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</table>
Mean values ($\bar{X}$) and standard deviations ($\sigma$) of the maximum contact surfaces radii ($a$), masses ($m$), initial impact kinetic energies ($E_{ci}$) and damaged fruits percentages assimilated to the impact damage probability ($p$) for each of the 4 impact velocity levels ($v_i$) for Jonathan variety apples

$$\frac{\sum x_i}{n} = \left(\frac{\sum (x_i - \bar{x})^2}{n-1}\right)^{1/2}, \quad x_i - \text{actual value; } n - \text{number of experiments.}$$

<table>
<thead>
<tr>
<th>$v_i$ (m/s)</th>
<th>$m$ (g)</th>
<th>$a$ (mm)</th>
<th>$E_{ci}$ (J)</th>
<th>$p$ (%)</th>
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Table 3

Coefficients values $\mu$, $C$, $X_0$ from equations (7) and (9), and corresponding values $R^2$ and $\chi^2$, for each of the physical quantities $E_{ci}$, $v_i$, $a$ used for impact level $X$ characterisation

<table>
<thead>
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<th>$X$</th>
<th>Logistic equation (7)</th>
<th>Logistic equation (9)</th>
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<td>$\mu$</td>
<td>9.907</td>
<td>3.324</td>
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<td>$C$</td>
<td>8.808</td>
<td>8.962</td>
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<td>$X_0$</td>
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<td>-1.477</td>
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<td>$\chi^2$</td>
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</tbody>
</table>

Fig. 1 (a, b, c). Logistic curves concordance, represented by logistic functions (7) and (9), with experimental data, for Jonathan apples, for each of the 3 physical quantities which characterise the impact level: ■ – experimental points, —— logistic curve, function (7), ——— logistic curve, function (9).

Apple’s impact damage probability described by logistic functions (7) and (9) is represented in Fig. 1 (a, b, c) where the validity limits in comparison to the experimental domains was extrapolated. The graphic from Fig. 1(a), related to the other ones, proves the best concordance of the experimental data described by the help of the logistic function (7) or (9).
In this case we obtained the logistic function for the prediction of the impact damage probability described by the equation:

\[
p = \left[1 - \exp(-8.392 \cdot E_{ci})\right]\left[1 + \exp(1.698 - 8.392 \cdot E_{ci})\right]
\]

(12) where \(E_{ci}\) is initial impact kinetic energy (available).

Impact damage probability \((p)\) can be assimilated to the percentage of the damaged fruits from the impacted fruits population.

We present two practical examples to prove the utility of the relation (12), that is:

a) imposing a tolerance of a 10% admissible damage percentage \((p = 0.1)\) [9, 22, 24], from relation (12) it is obtained \(E_{ci\,\text{max}} = 0.064\) J; meaning that for \(E_{ci} \leq 0.064\) J there results a damaged fruits probability \(p \leq 10\%\). The value \(E_{ci} = 0.064\) J can be used either in designing or in a correct choosing of the working kinematical parameters for an apple sorting – packing set;

b) for an admissible initial impact kinetic energy \(E_{ci}\) (ex.: \(E_{ci} = 0.05\) J [3, 19, 24]) at which the first bruises appear on the pulp tissue, from relation (12), it is obtained \(p = 0.075\), meaning a potential percentage of damaged fruits of a maximum 7.5%. If the validity of relation (12) is admitted outside the \(E_{ci}\) domain from experimentations, at a kinetic energy level of \(E_{ci} = 0.06\) J, it is obtained \(p = 0.092\), respectively 9.2%.

The \(\mu, C, X_0\) coefficients values from the logistic functions (7) and (9) determine the position and the steepness of the logistic curve in the variation limits of the energy level used in experimentations. A fruit population with a medium ripeness degree will be characterised by a coefficient \(\mu_1 = 8.411\), the others having a ripeness degree more advanced will have, for example, \(\mu_2 = 12.350\), while the last one, in an early stage of ripeness, will have \(\mu_3 = 6.125\). The values of \(C\) are insignificantly different \(C_1 \approx C_2 \approx C_3 = 1.705\). For \(C = 1.705\), from relation (8), it is obtained \(K = 1.182 N_o\), meaning \(\alpha = 1.182\). In the curve inflection point \(p_i = 0.409\), the adequate slopes are:

\[
\left(\frac{dp}{dE_{ci}}\right)_{1i} = 2.485; \left(\frac{dp}{dE_{ci}}\right)_{2i} = 3.649; \left(\frac{dp}{dE_{ci}}\right)_{3i} = 1.81
\]

while the initial kinetic energies are: \((E_{ci})_{1i} = 0.203\) J; \((E_{ci})_{2i} = 0.138\) J; \((E_{ci})_{3i} = 0.278\) J.

The logistic curves for all three situations are graphically represented in Fig. 2. It shows that logistic curves steepness is directly related to \(\mu\) values which can be associated with the fruits ripeness degree.

The impact damage probability related to the initial kinetic energy, for a fruit population (apples) with a high ripeness degree is represented by the logistic curve having a high steepness \((\mu_2)\) because the damage probability will rapidly change after the kinetic energy grows beyond critical threshold. A fruit population having a low ripeness degree will have a logistic curve more bended (low steepness \((\mu_3)\)) because the damage probability will change slower at kinetic energies growing over the critical threshold.
Aspects regarding theoretical basic concepts of the probabilistic description of impact damage

Fig. 2. Comparative presentation of the logistic curve as position and steepness related to the $\mu$ values in accordance with fruits ripeness degree ($\mu_1 = 8.411$ medium ripeness; $\mu_2 = 12.350$ advanced ripeness; $\mu_3 = 6.125$ incipient ripeness).

**Observations** [3, 19]. At the impacted area examination, for the lowest initial kinetic energy level ($E_{ci} = 0.085$ J), to some fruits, the existence in section, of a bruised pulp tissue at about 1.5 – 2.5 mm deep under epidermis was observed. During impact, the normal compressive stresses are accompanied by shearing stresses whose maximum value (in case of elastic impact) are manifested under epidermis at a depth of about 0.48a [2, 4]. This fact suggests a possible explanation for the apparition of the cellular tissue bruising under epicarp, mainly, because of the shearing stress causing slides between cell layers and producing cell shearings (ruptures) (although tangential stresses represents about 0.27 of the maximum normal compression stresses [2, 4]). A probable explanation consists in fact that cellular tissues bear much better the normal compression stresses compared to the sheering ones. This ruptures free some enzymes which develop in time and provoke the tainting of the adjacent tissues leading to quality depreciation of the fruit. For $E_{ci} = 0.085$ J [3], from relation (12), it results $p = 0.139$, respectively 13.9%, representing a potential value of the damage probability.

**Conclusions**

Our theoretical and experimental researches have highlighted that the impact damage probability of the apple, related to the impact level, for Jonathan variety, can be well described by a logistic function as relation (7) or (9). It was identified that the impact level can be adequately characterised by the impact kinetic energy $X = E_{ci}$. For this situation the constant coefficients of the two functions were determinated, for a
correlation coefficient $R^2 = 0.965$. The relation (12) can be used to predict the potential impact damage probability for an imposed impact level $E_{ci}$.

Relation (12) can be useful in designing and in the rational utilisation of the technical systems of mechanical handling, transport, sorting – packing of the apples.

Following the methodology presented in this paper, for different fruit varieties, similar relations, evaluating the coefficients from the relations (7) or (9), can be determined.

Also, the methodology from this paper can stand at the base for elaboration of some evaluation criteria of the fruit physical characteristics, useful in genetic selection and variety improvement.

In the end, the paper has theoretically grounded the concept of impact damage susceptibility coefficient, a quantity that expresses more adequate the impact damage, by including in evaluations the natural variability of the fruit physical and mechanical characteristics, by the probabilistic interpretation of the impact damage phenomenon.

For Jonathan apple variety, the impact damage susceptibility value corresponds to the initial kinetic energy threshold $E_{ci} = 0.064 \text{ J}$ for which it is accepted a tolerance of the potential impact damage probability $p = 10\%$ (value admitted by E.U. standards [6, 21, 22, 24]).

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