OPTIMAL DESIGN FOR THE INTERIOR SHAPE OF A
ANNULAR DIFFUSER WITH DIVERGENT CAREENING
CONSIDERING THE MINIMUM WHOLE LOSS PRESSURE

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In this paper is presented a new mathematical model and a numerical
programme in Maple 11 programming language, for optimum design of the
interior shape of a hydraulic annular diffuser with interior divergent careening.

The mathematical model was determined to respect the minimum values of
the whole loss pressure. The proposed programme solves numerically the nonlinear
mathematical model and prints the results.

Keywords: hydraulic diffuser, divergent careening, loss pressure, numerical
simulation

1. Introduction

There are many engineering systems involving subsystems in which flexible
structures are subjected to fluid flow. Examples are certain types of jet into the
pumps, pistons and valves; oil and gas production systems etc.

This explains the interest in the study of fluid flow modelling on structures
in annular geometries [1, 2, 3].

Since then, the research effort on this topic, specially applicable to hydraulic
diffusers has been intensified and a number of interesting papers on the subject
may be found in the proceedings of various symposia in this area.

Moreover, a great variety of experimental works is reported in the literature
about optimization of hydraulic diffusers shape [4, 5, 6, 7, 8, 9, 10, 11].

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Due to the complexity of the model and the limited knowledge of the physics of the device, design of hydraulic diffusers with divergent careening is a difficult as well as a challenging task.

Fluid flow processes can be analyzed both numerically and experimentally. Experimental methods can help in understanding the flow behavior and also can help in assessing available theoretical predictions.

The purpose of this work is to obtain a new mathematical model and a numerical programme in Mapple 11 programming language, for optimal design of the interior shape of a hydraulic annular diffuser with divergent careening.

2. The mathematical model

The hydraulic diffusers are circuit elements which made connections and permit the fluid flow between two sections with distinct areas.

If a symmetrical body is placed into the diffuser and its symmetry axis is coincidental with the symmetry axis of the diffuser an annular diffuser with an interior divergent careening, is obtained through the increasing of its cross section [12, 13]. The sketch of this hydraulic element is given in Fig. 1 and the three dimensional model is shown in Fig. 2.

The performance characteristics of an annular diffuser with an interior divergent careening depends on their geometry and the inlet conditions. The whole loss pressure into the diffuser \( \Delta p \) [Pa] is calculated with the relation [13, 14]:

\[
\Delta p = \zeta \cdot \rho \cdot \frac{v_0^2}{2} = 8 \cdot \zeta \cdot \rho \cdot \left[ \frac{Q}{\pi(D_0^2 - d_0^2)} \right],
\]

(1)
where: \( \rho \) [kg/m\(^3\)] - the density of fluid; \( \zeta \) [dimensionless] - the Weissbach coefficient; \( D_0 \) [m] - the diameter of input pipe in diffuser; \( d_0 \) [m] - the diameter of spherical surface placed in front of the carrening body; \( Q \) [m\(^3\)/s] - the flow volume of fluid; \( v_0 \) [m/s] - the velocity of fluid to the input section.

The objective function \( \Delta p \rightarrow \min \), with \( \rho \) and \( v_0 \) constant, is equivalent with:
\[
\zeta \rightarrow \min
\]  
(2)

In practice, for this type of diffuser, the recommended constructive dimensions are given in [12]:
\[
L = K_1 \cdot D_0, \quad K_1 = 0.5, ..., 1.0;
\]
(3)
\[
d_0 = K_2 \cdot D_0, \quad K_2 = 0.6, ..., 0.8;
\]
(4)
\[
n_i = \frac{A_i}{A_0}, \quad n_4 = 2, ..., 4;
\]
(5)
\[
\alpha_i = 8^\circ, ..., 16^\circ.
\]
(6)

where: with \( n_1 \) [dimensionless] is denoted the divergence degree of diffuser.

The input annular section in the diffuser is given by the following relation:
\[
A_0 = \frac{\pi \cdot (D_0^2 - d_0^2)}{4} = \frac{\pi \cdot D_0^2 (1 - K_2^2)}{4},
\]
(7)

and the interior and exterior rayons of output section from the diffuser are:
\[
R_{ii} = \frac{d_0}{2} + L \cdot \sin \alpha_1 = \frac{D_o}{2} \left[ K_2 + 2 \cdot K_1 \cdot \sin \alpha_1 \right];
\]
(8)
\[
R_{ie} = \frac{D_0}{2} + L \cdot \sin \alpha_2 = \frac{D_0}{2} \left[ 1 + 2 \cdot K_1 \cdot \sin \alpha_2 \right],
\]
(9)

where: \( \alpha_1 \) [°] - the angle of divergent careening; \( \alpha_2 \) [°] - the interior divergent angle of diffuser, fig.1.

The interior rayon of the output section from the diffuser could be determined starting from the relation (5):
\[
n_i = \frac{A_i}{A_0} = \frac{\pi \cdot (R_{ie}^2 - R_{ii}^2)}{A_0},
\]
(10)
\[
R_{ie} = \sqrt{R_{ii}^2 + \frac{n_i \cdot A_0}{\pi}},
\]
(11)

From the equality of relations (9) and (11), and then by substitution of \( R_{ii} \) [m] and \( A_0 \) [m\(^2\)], the divergent angle \( \alpha_2 \) is obtained:
\[
\alpha_2 = \arcsin \left\{ \sqrt{[K_2 + 2 \cdot K_1 \cdot \sin \alpha_1]^2 + n_4 (1 - K_2^2) - 1} \right\} / 2 \cdot K_1
\]
(12)

The Weissbach coefficient is determined by substituting the previous relations in relation (1):
The mathematical model given by the nonlinear objective function (13) must be minimized: \( \zeta = \zeta \left( K_1, K_2, n_1, \alpha_1 \right) \rightarrow \text{min} \), with restrictions given by the relations (3), (4), (5) and (6).

3. The numerical programme

With the availability of fast computers programming, it is now possible to solve the equations of flows using modelling principles to give accurate results at much shorter time. This will enable the design optimization by which the number of costly experiments can be reduced.

To solve the mathematical model concerning the nonlinear optimization [15, 16] an original programme for numerical calculus in Maple 11 programming language [17, 18] was designed. This programme determines through numerical calculation the minimum value of the Weissbach coefficient \( \zeta \) and for the following parameters: \( K_1, K_2, n_1 \) and \( \alpha_1 \). After that the programme determines the optimal sizes of the interior shape of the hydraulic diffuser.

4. The numerical study

The input data for numerical study of simulation are given in Table 1.

<table>
<thead>
<tr>
<th>Q</th>
<th>( K_1 )</th>
<th>( K_2 )</th>
<th>( N_1 )</th>
<th>( D_0 )</th>
<th>( \rho )</th>
<th>( \alpha_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m³/s</td>
<td>[dimensionless]</td>
<td>[m]</td>
<td>[kg/m³]</td>
<td>[°]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0001</td>
<td>0.5±1</td>
<td>0.6±0.8</td>
<td>2±4</td>
<td>0.01</td>
<td>10⁷</td>
<td>8±16</td>
</tr>
</tbody>
</table>

The optimum results of all parameters involved into the mathematical model, calculated with the numerical programme, are presented in Table 2.

<table>
<thead>
<tr>
<th>( \zeta_{opt} )</th>
<th>( K_{1, opt} )</th>
<th>( K_{2, opt} )</th>
<th>( n_{1, opt} )</th>
<th>( \alpha_{1, opt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[dimensionless]</td>
<td>[°]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.455</td>
<td>1</td>
<td>0.8</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

The optimal sizes of the diffuser are presented in Table 3.
Table 3

<table>
<thead>
<tr>
<th>d₀</th>
<th>Rᵢ₀</th>
<th>Rₑ₀</th>
<th>L</th>
<th>A₀</th>
<th>A₁</th>
<th>α₂</th>
<th>K₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>[m]</td>
<td>[m²]</td>
<td>[°]</td>
<td>[°]</td>
<td>[dimensionless]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>0.0054</td>
<td>0.0069</td>
<td>0.01</td>
<td>2.82·10⁻⁵</td>
<td>5.65·10⁻⁵</td>
<td>10°45'</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Based on the optimal values calculated through numerical simulation, the minimum loss pressure is \( \Delta p = 2850 \) [Pa].

The input data are the optimum values, previous determined:

\[
\alpha_{\text{opt}} = 8°; \quad n_{\text{opt}} = 2; \quad K_1 = 0.5,...1; \quad K_2 = 0.6,...0.8, \quad (14)
\]

Based on the input data, the numerical programme calculates the analytical relations for: \( d_0, L, Rᵢ₀, Rₑ₀, A₀, K₃, α₂, A₁, ζ, v₀, Δp \), then, the corresponding graphical representations are presented in Fig. 3 to Fig.10.

\[
L = 0.01 \cdot K_1, \quad (15)
\]

\[
d_0 = 0.01 \cdot K_2, \quad (16)
\]

\[
A₀ = 7.854 \cdot 10^{-5}(1 - K_2^2), \quad (17)
\]

\[
A₁ = \pi \left[ 5 \cdot 10^{-5} - 2.5 \cdot 10^{-5} \cdot K_2^2 + 1.392 \cdot 10^{-5} \cdot K_1 K_2 + 0.19 \cdot 10^{-5} \cdot K_1^2 - (5 \cdot 10^{-3} \cdot K_2 + 1.392 \cdot 10^{-3} \cdot K_1)^2 \right], \quad (18)
\]

\[
Rᵢ₀ = 1.391 \cdot 10^{-3} K_1 + 5 \cdot 10^{-3} K_2, \quad (19)
\]

\[
Rₑ₀ = 0.2 \cdot 10^{-2} \sqrt{12.5 - 6.25 \cdot K_2^2 + 3.478 \cdot K_1 K_2 + 0.484 \cdot K_1^2}, \quad (20)
\]

\[
α₂ = \arcsin \left( \frac{0.2 \sqrt{12.5 - 6.25 K_2^2 + 3.478 K_1 K_2 + 0.484 K_1^2}}{250000} \right), \quad (21)
\]

\[
v₀ = \frac{10^{-4}}{7.854 \cdot 10^{-5}(1 - K_2^2)} \quad (22)
\]
Fig. 5. The 3D variations of $K_3 = f(K_1, K_2)$

Fig. 6. The 3D variations of $\alpha_2 = f(K_1, K_2)$

Fig. 7. The 3D variation of inner area areas $A_1 = f(K_1, K_2)$

Fig. 8. The 3D variations of $\zeta = f(K_1, K_2)$

Fig. 9. The variation of velocity $v_0 = f(K_2)$

Fig. 10. The 3D variation of $\Delta p = f(K_1, K_2)$
The numerical programme also permits to find the analytical relations for all the sizes involved into the mathematical model, the function of variations and the corresponding plots.

5. Conclusions

In this paper a new mathematical model and a numerical programme in Mapple 11 programming language, for optimum design of the interior shape of a hydraulic annular diffuser with divergent careening is presented.

The predictions were made using the standard model and the predicted results have been validated with experiments. The proposed model was found to be a satisfactory physical model to obtain good predictions for different flow characteristics of a hydraulic annular diffuser with divergent careening.

The mathematical model was determined to respect the minimum values of the whole loss pressure owing to friction, which is the most important parameter of design from the exploitation point of view. The proposed numerical programme solves the nonlinear mathematical model and prints the graphical representations.

The present study indicates that the proposed procedure used in the paper is a powerful tool in the optimum design of the interior shape of a hydraulic annular diffuser with divergent careening.

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REFERENCES


