SHORT-CIRCUIT CURRENTS CALCULATION IN DISTRIBUTION ELECTRICAL NETWORKS IN THE PRESENCE OF DISTRIBUTED GENERATION

Tudor-Ion ZĂBAVĂ¹, Ion TRIŞTIU², Alexandru MANDIŞ³, Constantin BULAC⁴

This paper presents a methodology for calculating short-circuit currents in the distribution of electrical networks. The methodology uses the method of equivalent voltage source at the fault location. The originality, consists in applying the principle of backward-forward sweep method used for steady state calculation in distribution networks for calculation of short-circuit currents in electrical distribution networks in the presence of distributed generation. The calculation procedure was tested on the IEEE 33-bus distribution network, considering four distributed generators.

Keywords: Distribution electrical networks, Distributed Generation (DG), Short-circuit currents

1. Introduction

The primary objective of the distributed generation consists in injecting active power in the electrical networks, accompanied by the generation or consumption of reactive power. These sources are based on the exploitation of renewable forms of energy (wind, solar, hydro, biomass, etc.) as well as of non-renewable (natural gas, petroleum derivatives). The powers generated by distributed sources are relatively small as compared to the large power plants (thermal, hydro and nuclear) which recommends the connection of these generators to low, medium or high voltage networks.

In general, urban electrical distribution networks have a meshed or strongly meshed configuration, but under normal conditions, for technical and economical reasons, they operate radially or arborescent. Rural distribution electrical networks usually have an arborescent structure, consisting of one main line and several derivations.

Arborescent configurations have some particularities that allow the use of specific analytical methods, in which certain calculations can be easily performed.

¹ PhD student, Power Engineering Faculty, University POLITEHNICA of Bucharest, Romania
² Assoc. prof., Power Engineering Faculty, University POLITEHNICA of Bucharest, Romania
³ Assistant, Power Engineering Faculty, University POLITEHNICA of Bucharest, Romania
⁴ Professor, Power Engineering Faculty, University POLITEHNICA of Bucharest, Romania
An arborescent electrical network consists of one source node and one or more load nodes. In such networks there are no closed loops, so each load is fed by the source node on a single path.

Based on previous observations, various methods for steady state calculations have been developed, one of these methods is known in the literature as “backward/forward sweep”. The principle of this method is iterative and consists of performing two phases: the backward sweep for branch currents calculation and the forward sweep for node voltages calculation [1]-[3].

The presence of distributed sources in the electrical networks influences both steady state and disturbed operating conditions. The influences are determined by several factors such as: type of the generator, installed power, the network connection point of DG, electrical network characteristics, etc. When a short-circuit occurs in the electrical network, distributed generators cause an increase in the current at the fault location and change its repartition through the network branches.

The calculation of short-circuit currents represents an important step in analyzing the integration of distributed generation in electrical networks. Given the values of short-circuit currents it is necessary to verify the compliance of technical requirements as regards the connection of distributed sources to the electrical networks.

It is important to know to what extent each source increases the values of short-circuit currents and which is their repartition through the network branches. The contribution of distributed generation on the short-circuit currents values depends on the generator type and their connection interface to the network: synchronous or asynchronous generators, directly connected or through static power converters.

Depending on the rated voltage and the considered assumptions, various methods for short-circuit currents calculation have been developed. An important method is the “impedance” method, used to calculate fault currents at any point in an installation with a high degree of accuracy [4]. Standard IEC 60909 applies to all networks, radial or meshed, up to 550 kV. This method, based on the Thévenin theorem, calculates an equivalent voltage source at the short-circuit location and then determines the corresponding short-circuit current [5]. The methods mentioned above do not take in consideration the presence of distributed generators in electrical networks. The North American standard [6] treats various aspects extensively related to short-circuit currents calculation in electrical network without considering the presence of distributed generators. Recent papers suggest adapting classical methods for different types of distributed generators. Paper [7] presents the available analytical equations to calculate the short-circuit current, and makes a comparison between the IEC 60909 and the results obtained by simulations on a test network that incorporates such generation units. The
methodology of IEC Standard 60909 is also applied to distribution networks with DG resources, to determine the maximum fault level [8]-[10].

Generally the calculation of short-circuit currents is based on numerical matrix computation, which involves a significant amount of computations. In this method are calculated the nodal admittances matrix and is inverted to obtain the impedances matrix, the short-circuit current at the fault location, residual node voltages and short-circuit branch currents. Starting from the observation that in arborescent networks there are no closed loops, the calculation of short-circuit currents can be simplified, not being necessary to use the matrix computation.

In this paper a methodology for calculating short-circuit currents in arborescent networks in the presence of distributed generation is presented. Considering the equivalent voltage source method, are calculated short-circuit currents in the upstream and downstream area of the fault location. The calculation principle is similar to that used for steady state calculation based on backward-forward sweep method.

2. Overview of short-circuit calculation

The short-circuit represents the accidental contact through a low value impedance between two or more points of an electrical circuit which normally operate at different potentials. In electrical networks short-circuits can occur between phases or between phases and earth.

In a three-phase electrical network the following types of short-circuits can occur (Fig. 1):
- symmetrical three-phase short-circuit ($k3$);
- phase-to-phase short-circuit clear of earth ($k2$);
- phase-to-phase-to-earth short-circuit ($k2E$);
- phase-to-earth short circuit ($k1$).

Although three-phase short-circuit is characterized by a low frequency rate of occurrence (5-10% of cases), represents a basic element for design and operating studies of electrical networks.

2.1. Basic principle of short-circuit calculation

Between the four types of short-circuits that may occur in electrical networks, the three-phase short-circuit is symmetrical, and the others are non-symmetrical. The calculation of short-circuit currents (both symmetrical and non-symmetrical) is performed by using the symmetrical components method. This method requires the use of three independent components (positive, negative and zero sequence), the relations between them being imposed by the type of short-circuit.
Fig. 1. Types of short-circuits: \( a \). symmetrical three-phase short-circuit; \( b \). phase-to-phase short-circuit clear of earth; \( c \). phase-to-phase-to-earth short-circuit; \( d \). phase-to-earth short circuit

Depending on the electrical distance between the fault location and the generators from the network, are defined two types of short-circuits: near to generator and far from generator. During the occurrence of a short-circuit the phase currents have variable values over time in a transient regime, the transient regime being followed by a steady state in which the currents have constant amplitude. One of the quantities important for short-circuit regimes is the initial symmetrical short-circuit current \( I_{Ik} \), which is the effective value of the a.c. symmetrical component and serves as a basis for the calculation of the peak asymmetrical short-circuit current, \( i_p \), as well as the breaking current and capacity \[11\].

For a faster calculation of short-circuit currents in acceptable precision conditions some simplifying assumptions are required. The most important assumptions are \[5\]:

- the short-circuit current, during a three-phase short-circuit, is assumed to occur simultaneously on all three phases;
- for the entire duration of the short-circuit, the voltages responsible for the flow of the current and the short-circuit impedance do not change significantly;
- arc resistances are not taken into account;
- line capacitances and the parallel admittances of non-rotating loads, except those of the zero-sequence system, are neglected;
- transformer regulators or tap-changers are assumed to be set to a medium position;
- load currents are neglected.

In addition to previous assumptions, it is considered that all electromotive voltages of the generators are in phase, thus leading to a calculated value of the short-circuit current higher than the real one.
For the calculation of short-circuit currents the method of the equivalent voltage source at the fault location is used. The only source in the network is considered as an ideal voltage source \( E = \frac{cU_n}{\sqrt{3}} \) connected at the fault location, present only in the positive sequence diagram. All network feeders are replaced by their internal impedance (positive, negative and zero sequence). The value of voltage factor \( c \) depends on the rated voltage at the fault location and by the considered fault regime (maximum or minimum). Fig. 2 shows an example of application of this method.

For a three-phase fault at point F of the network from Fig. 2, the initial symmetrical short-circuit current value is given by:

\[
I_s^* = E \frac{cU_n}{\sqrt{3}Z_k} = \frac{cU_n}{\sqrt{3}(Z_s + Z_T + Z_L)}
\]

where \( Z_k \) is the equivalent short-circuit impedance of the electric network at the short-circuit location F. The impedances of the network feeder \( (Z_s) \) and the transformer \( (Z_T) \) are calculated relative to the rated voltage of the fault location. For the impedance \( Z_T \) a correction factor \( K_T \) must be applied.

### 2.2. Short-circuit calculation in network with DG

Distributed energy sources currently connected to the electrical distribution networks are: gas turbines, wind turbines, photovoltaic panels and small hydropower plants. Depending on their energy conversion system, these sources can be connected into electrical networks in two ways:

- directly, suitable for synchronous and asynchronous generators operating at fixed speed;
- indirectly using static power converters, connection suitable for static generators (photovoltaic, fuel cells) and rotating generators (synchronous or asynchronous) operating at variable speed.

In order to determine the contribution of DG on short-circuit currents the method of equivalent voltage source at the fault location is used. Fig. 3 shows an example of application of this method.
For a three-phase fault at point F of the electrical network in Fig. 3, the initial symmetrical short-circuit current is given by:

$$I_k = \frac{E}{Z_k} = \frac{cU_n}{\sqrt{3}Z_k} = \frac{cU_n}{\sqrt{3}(Z_{DG} + Z_T + Z_L)}$$

where $Z_{DG}$ is the equivalent impedance of DG calculated relative to the rated voltage of the fault location. For the impedance $Z_I$ a correction factor $K_T$ must be applied.

The influence of DGs upon the short-circuit currents is different and depends on the type of the generator and how these generators are connected to the network. Thus, in case of rotating generators directly connected to the grid, the impedance $Z_{DG} = R_{DG} + jX_{DG}$ may be determined as follows [9], [10]:

- for synchronous generators the resistance is $R_{DG} = 0.15X_{DG}$, and the reactance $X_{DG}$ is determined based on the time elapsed from the occurrence of the fault by considering the subtransient reactance (the first 10 to 20 milliseconds of the fault), transient reactance (up to 500 milliseconds) and steady-state reactance (or synchronous reactance);
- for the impedance $Z_{DG}$ a correction factor $K_G$ must be applied;

- for asynchronous generators the resistance is $R_{DG} = 0.1X_{DG}$, and the reactance $X_{DG}$ is determined as ratio of locked-rotor current to rated current;

The short-circuit contribution of Converter Interfaced Distributed Generators is low and mainly determined by the thermal limit of the semiconductors, therefore it is usually limited up to 2 p.u. of the nominal current.

A particular case is that of doubly-fed induction generators (DFIG) used for wind turbines. Although the rotor is connected to the electrical network via an electronic power converter, DFIG behaves as an asynchronous generator. The duration of their contribution, however, should be limited to 3-5 cycles [9].
3. Calculation methodology of short-circuit currents in distribution electrical networks with DG

In arborescent electrical networks there are no closed loops and the current flow is unidirectional in normal and abnormal states. Thus, in case of a short-circuit at the point F of the network form Fig. 4, two areas can be defined:

- **upstream area**, located between the supply node S and the point F. For this area, the contribution of the short-circuit current is given by the supply node and eventually by the distributed generators;
- **downstream area**, located between point F and end nodes of the network. For this area the contribution of short-circuit current depends exclusively of distributed generators.

![Diagram](image)

**Fig. 4. Arborescent distribution network**

The short-circuit current at the fault location F is given by:

\[
I_k = I_{k, up} + I_{k, down}
\]

(3)

where \( I_{k, up} \) and \( I_{k, down} \) are calculated based on equivalent voltage source method depending on upstream and downstream equivalent impedance related to point F:

\[
I_{k, up} = \frac{E}{Z_{k, up}} ; \quad I_{k, down} = \frac{E}{Z_{k, down}}
\]

(4)

Based on previous observations a methodology for short-circuit currents calculation has been developed, the principle of this method being similar to that used to calculate the steady state by backward-forward sweep method. For each of the two areas of the network the calculations are carried out in two steps:

- in the first step (backward sweep), starting from the DG nodes, respectively the supply node toward the fault point, the equivalent impedances between these nodes and the neutral point are calculated;
- in the second step (forward sweep), starting from the fault point, for which the short-circuit current was calculated, toward DG nodes, respectively supply node, the current through the branches are calculated.
In order to exemplify the application of the methodology, the network from Fig. 5 is considered, where a three-phase short-circuit occurs in node $k$.

For the calculation of short-circuit currents using the method of equivalent voltage source at the fault location, load currents are neglected, distributed sources and feeding source are replaced by their equivalent impedances, and the equivalent voltage source $E$ is inserted at the fault location (fig. 6).

For each upstream and downstream area, the process of the short-circuit current calculation at the fault point and the contribution of distributed generators to this current is the same, based on completion of both steps backward and forward. Further, the upstream area, between nodes $S$ and $k$, is considered. In order to reduce the equivalent diagram, the branches with distributed generators are replaced by their equivalent impedances (fig. 7):

$$Z_{ki}^{'} = Z_{ki} + Z_{DGi} \quad i = 1, 2, ..., j$$

In the backward sweep, starting from the supply node $S$ and going towards node $k$, the equivalent impedance of each node with respect to the neutral point is calculated with the following expressions:
The impedance $Z_{kl}$ represents the equivalent impedance of the upstream area $Z_{k_{up}}$. Using relation (4), the short-circuit current $I_{k_{up}}$ is obtained.

In the forward sweep, starting from the node $k$ and going towards node $S$, using the current $I_{k_{up}}$ calculated before, the short-circuit currents for each distributed generator are calculated by applying the expressions:

$$
I_{k_{DGi}} = I_{k_{(i+1)}} \frac{Z_{ki}}{Z_{ki} + Z_{ki'}} \quad i = j, j - 1, \ldots, 1
$$

The current $I_{k_{i}}$ represents the short-circuit current of the supply node $I_{k_{S}}$.

4. Case study

To test the methodology presented above the IEEE 33-bus distribution network, operating at a nominal voltage of 20 kV, is considered (Fig. 7). Synchronous generators are connected at nodes 11, 20, 23 and 29, having the following parameters: $S_{nG} = 950$ kVA, $U_{nG} = 0.69$ kV, $\cos \phi_{nG} = 0.9$, $x_d' = 18\%$ and $R_{G}/X_{G} = 0.15$. The transformers have the following nominal parameters: $S_{nT} = 1000$ kVA, $U_{nMV} = 20$ kV, $U_{nLV} = 0.69$ kV, $\Delta P_0 = 10$ kW and $u_k = 6\%$.

The short-circuit power at the supply node is $S_{kS} = 500$ MVA.

Considering a three-phase short-circuit occurring at node 3, the reduced equivalent diagram is presented in Fig. 9. Table 1 presents the results obtained using the proposed methodology for short-circuit currents calculation.
Fig. 9. The reduced equivalent diagram of IEEE 33-bus distribution network system with 4 DGs

### Table 1

**Network feeder**

\[
Z_{s} = \frac{c \cdot U_{s}^2}{S_{s}} = \frac{1.1 \cdot 20^2}{500} = 0.88 \Omega \\
R_{s} = \frac{1}{X_{s}} = 0.1
\]

\[
\Rightarrow Z_{s} = (0.8756 + j0.0876)\Omega
\]

**DGs**

**Generator**

\[
X_{g} = \frac{U_{g}^2}{100} = \frac{18 \cdot 0.69^2}{950} = 0.0902 \Omega \\
R_{g} = \frac{1}{X_{g}} = 0.15
\]

\[
K_{g} = \frac{c_{max}}{1 + x_{g} \sin \varphi_{g}} = \frac{1.1}{1 + 0.18 \cdot \sin(acos(0.9))} = 1.019973
\]

**Transformer**

\[
R_{t} = \frac{\Delta P_{t} \cdot U_{m}^2}{S_{t}} = \frac{10 \cdot 20^2}{10000} = 4 \Omega \\
Z_{t} = \frac{U_{t}}{100} = \frac{20^2}{100} = 24 \Omega
\]

\[
\Rightarrow Z_{t} = (4 + j23.6643)\Omega
\]

\[
K_{t} = \frac{0.95 \cdot c_{max}}{1 + 0.06 \cdot x_{t}} = \frac{0.95 \cdot 1.1}{1 + 0.06 \cdot 0.05916} = 1.009178
\]

**Equivalent impedance**

\[
Z_{DG} = K_{G} \cdot Z_{G} \cdot N_{r}^2 + K_{T} \cdot Z_{T}
\]

\[
N_{r} = \frac{U_{m}^2}{U_{m}^2} = \frac{20}{0.69} = 28.99
\]

\[
\Rightarrow Z_{DG} = (15.632 + j101.185)\Omega
\]

**Distribution network**

**Network impedances**

\[
Z_{0,1} = (0.0922 + j0.047)\Omega \\
Z_{0,2} = (0.493 + j0.2511)\Omega \\
Z_{1,2} = (0.366 + j0.1864)\Omega \\
Z_{2,3} = (1.2001 + j0.9011)\Omega \\
Z_{3,4} = (3.5436 + j2.5227)\Omega \\
Z_{4,0} = (2.0777 + j1.9903)\Omega \\
Z_{4,23} = (1.3492 + j1.0174)\Omega \\
Z_{2,0} = (2.8579 + j2.1409)\Omega
\]

**DGs branch impedances**

\[
Z_{1,1} = Z_{DG} + Z_{1,20} = (17.7099 + j103.175)\Omega \\
Z_{1,2} = Z_{DG} + Z_{1,23} = (16.9814 + j102.2021)\Omega \\
Z_{1,25} = Z_{DG} + Z_{5,20} = (18.4901 + j103.3256)\Omega \\
Z_{1,11} = Z_{DG} = (15.6322 + j101.1847)\Omega
\]

**Backward sweep** (impedance calculation)
### Upstream area

\[
Z_{41} = Z_{45} + Z_{0-1} = (0.1798 + j0.9226) \Omega
\]
\[
Z_{42} = Z_{1-2} + \frac{Z_{41} \cdot Z_{4V}}{Z_{41} + Z_{4V}} = (0.671 + j1.1656) \Omega
\]
\[
Z_{43} = Z_{2-3} + \frac{Z_{42} \cdot Z_{1V}}{Z_{42} + Z_{1V}} = (1.037 + j1.352) \Omega
\]

### Downstream area

\[
Z_{45} = Z_{DG} + Z_{0-1} = (19.1758 + j103.7074) \Omega
\]
\[
Z_{43} = Z_{1-5} + \frac{Z_{45} \cdot Z_{4V}}{Z_{45} + Z_{4V}} = (10.6159 + j52.6596) \Omega
\]

### Forward sweep (short-circuit currents calculation)

#### Upstream area

\[
\dot{I}_{4\text{up}} = \frac{C \cdot U_{s}}{\sqrt{3} Z_{4\text{up}}} = (4.569 - j5.984) \text{kA}
\]
\[
\dot{I}_{41} = \dot{I}_{4\text{up}} \frac{Z_{2V}}{Z_{12} + Z_{2V}} = (4.54 - j5.892) \text{kA}
\]
\[
\dot{I}_{DG25} = \dot{I}_{4\text{up}} \frac{Z_{42}}{Z_{42} + Z_{1V}} = (0.028 - j0.092) \text{kA}
\]
\[
\dot{I}_{41} = \dot{I}_{42} \frac{Z_{4V}}{Z_{41} + Z_{4V}} = (4.501 - j5.839) \text{kA}
\]
\[
\dot{I}_{DG26} = \dot{I}_{42} \frac{Z_{41}}{Z_{41} + Z_{4V}} = (0.039 - j0.053) \text{kA}
\]

#### Downstream area

\[
\dot{I}_{4\text{down}} = \frac{C \cdot U_{s}}{\sqrt{3} Z_{4\text{down}}} = (0.047 - j0.232) \text{kA}
\]
\[
\dot{I}_{DG1} = \dot{I}_{4\text{up}} \frac{Z_{4V}}{Z_{45} + Z_{4V}} = (0.024 - j0.116) \text{kA}
\]
\[
\dot{I}_{DG29} = \dot{I}_{4\text{up}} \frac{Z_{41}}{Z_{45} + Z_{4V}} = (0.023 - j0.116) \text{kA}
\]

### Short-circuit current at the fault location

\[
\dot{I}_{4} = \dot{I}_{4\text{up}} + \dot{I}_{4\text{down}} = (4.616 - j6.216) \text{kA}
\]
\[
I_{4} = 7.742 \text{kA}
\]

### 5. Conclusions

The paper presents a simple methodology for calculating the short-circuit currents in arborescent distribution networks whit distributed generators. This methodology does not involve a great computational effort because is not necessary to calculate the nodal admittance matrix and its inverse. The originality of this paper, consists in applying the principle of backward-forward sweep method used for steady state calculation in distribution networks for calculation of short-circuit currents in electrical distribution networks in the presence of DGs. Based on the methodology proposed in this paper, a simple and efficient calculation algorithm for short-circuit currents can be developed. This algorithm...
can be implemented in a program used for the study of integration of distributed sources in electrical networks. The proposed methodology was applied for calculation of a three-phase short-circuit on IEEE 33-bus distribution network, in which 4 DGs were included. This methodology can be used for any type of short-circuit, by considering the corresponding symmetrical sequence schemes.

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