OPERATIONAL PARAMETERS EVALUATION FOR OPTIMAL WIND ENERGY SYSTEMS DEVELOPMENT

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A method based on maximization of normalised average power is proposed to establish the operational parameters of wind turbine. The method is a compromise between those operational parameters that provide the highest values of the capacity factor and the rated power. The method can be used in developing stage of wind energy system, being a useful tool to make an adequate choice of the wind turbine with optimal operational parameters, which yield the highest amount of energy.

Keywords: Wind energy, capacity factor, normalized average power.

1. Introduction

In the last decades, a growing interest in renewable energy resources has been observed. Unlike other renewable energy sources, wind energy has become competitive with conventional power generation sources and therefore the application of wind turbine generators has the highest growth among other sources. Wind is one of fastest growing energy source and it is considered as an important alternative to conventional power generating sources.

The energy production from a wind energy system depends on many factors. These factors include the characteristics of the wind profile, and most importantly, the operational parameters of the wind turbine generator itself. The suitability of a wind turbine to a site involves designing of wind turbine’s operational parameters exactly according to the wind characteristics of the site. One more applicable method is to match a specified site with a suitable wind turbine among existing ones.

Numerous criteria have been proposed for the suitability procedure. Some authors determined the wind turbine generator’s parameters at maximum capacity factor or at maximum density rated output power [1,2], between these two methods there is a complementarity. Thus, if a higher capacity factor is desired, then turbine’s rated speed cannot be chosen too high. In this case, even if the wind turbine operates at rated power for most of the time, not too much of energy in higher speed wind will be captured. On the other hand, if the capacity factor is

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chosen too low, then although the wind turbine can generate a higher amount of output power at rated speed, the turbine will seldom operate at its rated power and will lose much of energy of lower speed wind.

A wind turbine should always extract the highest possible power from the wind. Therefore, in this paper is presented a method to identify optimal speed parameters of the wind turbines in order to yield maximum energy production. This means that the rated speed has to be selected such that the turbine delivers higher average power.

2. Wind speed modelling

Wind is a turbulent movement mass of air resulted from the differential pressure at different locations on the earth surface. The wind speed is continuously changing, making it desirable to be described by the statistical models. It is widely agreed that the randomly behaviour of wind speeds can be described by two parameters Weibull distribution. For Weibull distribution, the probability density function and the cumulative distribution function are given by:

\[ f_W(v) = \frac{\beta}{\alpha} \left( \frac{v}{\alpha} \right)^{\beta-1} \exp \left( -\left( \frac{v}{\alpha} \right)^\beta \right) \]

\[ F_W(v) = 1 - \exp \left[ -\left( \frac{v}{\alpha} \right)^\beta \right] \]  

(1)

where \( \alpha \) (m/s) is the scale parameter and \( \beta \) (dimensionless) is the shape parameter of the Weibull distribution. The shape and the scale parameters are related to the average wind speed by \( M(v) = \alpha \Gamma(1+1/\beta) \) and to the variance of wind speed by \( \text{Var}(v) = \alpha^2 \left[ \Gamma(1+2/\beta) - \left( \Gamma(1+1/\beta) \right)^2 \right] \). In many studies, the Rayleigh distribution is often used, the shape parameter being chosen to 2. The estimated parameters of the Weibull distribution can be found using different estimation methods [3,4].

3. Wind turbine’s output power modelling

The output power of a wind turbine is a function of the wind speed, the power curve of turbine gives the relation between the wind speed and the electrical output power. These curves come available from the wind turbine manufacturers or there are plotted using recorded wind speed and corresponding output power data.

The power curve of a pitch-regulated wind turbine is characterized by the following three speeds: the cut-in, the rated and the cut-off speed. The wind turbine starts generating power when the speed exceeds the cut-in speed \( (v_{ci}) \). The output power increases with the wind speed between the cut-in speed and the rated one \( (v_r) \), after that the output power remains constant at the rated power level \( (P_r) \). The cut-off wind speed \( (v_{co}) \) is the maximum wind speed at which the turbine
allows power generation, being usually limited by safety constraints. Thus, the power curve of a wind turbine can be modelled by four specific parameters, namely the cut-in speed, the rated speed, the cut-off speed and the rated power.

Different wind generators have different output power performances, thus the model used to describe the performances is also different. A literature survey indicates that the most used models for output power are the linear, the quadratic and the cubic models. The electric output power of a wind turbine generator can be accurately modelled using quadratic model, originally proposed in [5] and used in many other papers [1,6]:

$$P(v) = \begin{cases} \frac{P_r \cdot \left(A_2 v^2 + A_1 v + A_0\right)}{v_{ci}} & \text{for } v_{ci} < v < v_f, \\ P_r & \text{for } v_f < v < v_{co}, \\ 0 & \text{otherwise} \end{cases}$$

(2)

where the coefficients $A_2$, $A_1$ and $A_0$ were first time calculated in [7], having the following expressions:

$$A_2 = \frac{1}{(v_{ci} - v_f)^2} \left[2 - 4\left(\frac{v_{ci} + v_f}{2v_f}\right)^3\right],$$

$$A_1 = \frac{1}{(v_{ci} - v_f)^2} \left[4(v_{ci} + v_f)\left(\frac{v_{ci} + v_f}{2v_f}\right)^3 - 3(v_{ci} + v_f)\right],$$

$$A_0 = \frac{1}{(v_{ci} - v_f)^2} \left[v_{ci}(v_{ci} + v_f) - 4v_{ci} v_f \left(\frac{v_{ci} + v_f}{2v_f}\right)^3\right].$$

Equation (2) expresses the instantaneous value of the electrical output power as a function of the instantaneous wind speed.

4. Wind turbine’s performance indicators

A measure of the wind turbine performance is given by the average power value, being a very important parameter of wind energy conversion, determining the total energy production. The average power is related to rated output power by capacity factor. The capacity factor is defined as the ratio of the energy generated in real operation, over a time period, to the energy that could have been generated at constant rated power operation during the same period.

Considering the capacity factor definition, the average of output power may be expressed as product between rated power ($P_r$) and capacity factor (CF) value:

$$P_{avg} = P_r \cdot CF$$

(3)
The rated power output at rated wind speed $v_r$ is given by:

$$P_r = \frac{1}{2} \cdot \eta_r \cdot \rho \cdot A \cdot v_r^3$$  \hspace{1cm} (4)

where $\eta_r$ is the rated overall efficiency at rated power, $\rho$ (kg/m$^3$) is the air density, and $A$ (m$^2$) is the turbine swept area. The rated overall efficiency, $\eta_r = C_{pr} \cdot \eta_{mr} \cdot \eta_{gr}$, includes the coefficient of performance, the mechanical transmission efficiency and the generator efficiency, all evaluated at rated power.

The capacity factor can be calculated by integrating the normalised power curve multiplied by the wind speed distribution, over all wind speed values. Thus, the capacity factor can be derived based on the power curve of wind turbine generator and the parameters of wind speed distribution. Considering the quadratic model of power curve and Weibull distribution of wind speed, the authors have been developed an analytical model for capacity factor evaluation, expressed by the following equation:

$$\text{CF} = \sum_{k=0}^{\beta} A_k \cdot \alpha^k \cdot \Gamma\left(\frac{k}{\beta} + 1\right) \left[ \frac{v_r}{\alpha} \right]^{\beta} \left(\frac{v_r}{\alpha} \right)^{\frac{k}{\beta} + 1} - \exp\left(-\frac{v_r}{\alpha}\right) - \exp\left(-\frac{v_r}{\alpha}\right)^{\beta}$$

(5)

where $\Gamma(\cdot)$ and $P(\cdot)$ are the gamma and the lower incomplete gamma functions. The details of the capacity factor evaluation can be founded in [8].

Therefore, the average electrical output power can be expressed considering the above relationship:

$$P_{avg} = \frac{1}{2} \cdot \eta_r \cdot \rho \cdot A \cdot v_r^3 \cdot \text{CF}$$

(6)

In equation (6), the choice of rated wind speed, which maximize the average power, will not depend on the rated overall efficiency, the air density, or the cross sectional area of wind swept by turbine blades. Thus, these quantities can be normalized out. Also, since the capacity factor is expressed in normalized wind speeds, it is convenient to do likewise for average power by dividing to $\alpha$ in order to get the term $(v_r/\alpha)^3$. Therefore, a new indicator is obtained. This indicator, namely the normalised average power ($P_{N}$), takes both capacity factor and rated power into account, being utilized in this study to estimate the optimal rated speed parameter [9,10]. This indicator is defined as the normalised average power:
The equation (7) expresses the effect of the cut-in, the rated and the cut-off speeds, respectively the wind distribution parameters, on the normalised average power values. Thus, for a given wind profile, with \(\alpha\) and \(\beta\) parameters known, the values of the wind turbines’ operational parameters which lead to the maximum normalised average power, can be established.

From the three speed operational parameters, the rated speed is the most important for the wind turbine design. The cut-in speed is that fraction of the rated speed, which contains enough power to cover all the wind turbine’s strengths. On the other hand, the choice of the cut-off speed depends on the capacity of the turbine control system to maintain a constant power output for that wind speed values over the rated speed [11,12]. Thus, it has been shown that the rated wind speed has a significant effect on the other speed parameters of the wind turbine, and also, in the normalised average power value.

If the rated wind speed is chosen too low, too much energy generated under higher wind speeds will be lost. Instead, if the rated speed is too high, the turbine seldom operates at its rated power and will lose too much energy at lower speed winds. Thence, the average power output of wind turbine can reach a maximum value at a specific rated wind speed, as is depicted in figure 1.

**5. Matching wind turbine generators with a wind profile. A study case**

In this section, a study for optimal operational parameters’ evaluation is developed, based on maximization of the normalized average power. In this study, the rated speed \(v_r\) is chosen to maximize \(P_N\) with the prespecified \(v_{ci}\) and \(v_{co}\).
values. The typical commercial wind turbines have the values of $v_{ci}$ between 3 and 4 m/s and $v_{co}$ between 20 and 25 m/s [13].

For a numerical analysis, a real wind speed database is used. The wind speed database was collected from the north-east of Romania, for the year 2008, the hourly average values being recorded [14]. The parameters of the Weibull distribution, used for fitting the database, have been evaluated with Maximum Likelihood Method and they were founded as the scale parameter $\alpha=4.9399$ m/s and the shape parameter $\beta=1.8656$. With specified $v_{ci}=3.5$ m/s and $v_{co}=20$ m/s, the capacity factor and rated power curves are calculated and shown in figure 2. It can be seen that the capacity factor and the rated power curves reach their peaks at different rated speeds, involving that a compromise should be made to achieve the best result. Since normalised average power curve takes capacity factor and rated power into account, its peak can be used to specify the optimal rated speed of the wind turbine.

To determine the optimum wind turbine that best matches the analysed wind profile, the capacity factor and the normalised power are calculated by varying normalised power rated ($v_r/\alpha$) and the interest points from curves are summarized in Table 1.

<table>
<thead>
<tr>
<th>$v_r/\alpha$</th>
<th>$v_r$</th>
<th>$v_{ci}$</th>
<th>$v_{co}$</th>
<th>$PN$</th>
<th>$Prat$</th>
<th>$CF$</th>
</tr>
</thead>
<tbody>
<tr>
<td>At CF max</td>
<td>0.7</td>
<td>3.5</td>
<td>3.5</td>
<td>20</td>
<td>0.2031</td>
<td>0.343</td>
</tr>
<tr>
<td>At PN max</td>
<td>2.22</td>
<td>10.966</td>
<td>3.5</td>
<td>20</td>
<td>1.219</td>
<td>10.941</td>
</tr>
</tbody>
</table>

Fig. 2. The capacity factor, rated power and normalised average power curves
From Table 1, it can be seen that the normalized power is the greatest at \( v_r/\alpha = 2.22 \), therefore, the optimum rated wind speed is then 10.97 m/s.

To evaluate the accuracy of the optimum rated wind speed, in the following, it will be evaluated the average power for two wind turbines with different rated speeds, for which an optimization for matching to the analysed wind profile is required. It is considered that both wind turbines have the 1 MW rated power, 3.5 m/s cut-in speed, 20 m/s cut-out speed, 100 m hub’s height, but the first one has the 9 m/s rated speed and the second one has the 13 m/s rated speed.

The average power values for unmodified wind turbines for analysed wind profile depend on their rated power and the capacity factor. As can be seen in figure 2, for the first wind turbine, the capacity factor is \( CF_1 = 0.1901 \) and for the second one is \( CF_2 = 0.0646 \). Considering these values for capacity factor, the averages power for unmodified wind turbines are \( P_{avgWT1} = 1000 \times 0.1901 = 190.1 \) W and \( P_{avgWT2} = 1000 \times 0.0646 = 64.6 \) kW, respectively.

Considering the optimization of wind turbines, the rated speed and the rated power will be adjusted to optimal values. Therefore, for first wind turbine, the rated power, assuming all efficiencies remains the same, will just be in the ratio of the cube of the wind speeds, namely \( P_{rat1O} = 1000 \times (10.97/9)^3 = 1810.9 \) kW. For optimum rated speed wind speed, the capacity factor is \( CF_{1O} = 0.1114 \). These values for rated power and capacity factor are used to calculate the turbine average power output, \( P_{avgWT1O} = 1805.9 \times 0.1114 = 201.73 \) kW.

The same computations have been done for the second wind turbine. Thus, the rated power, assuming all efficiencies remains the same, it is \( P_{rat2O} = 1000 \times (10.97/13)^3 = 600.9 \) kW, but it will function with a capacity factor equal with 0.1114. Under these conditions the turbine average power output is \( P_{avgWT2O} = 66.93 \) kW. Thus, as can be seen, for both wind turbines under consideration, the highest average power is determined for optimal rated speed.

### 6. Conclusions

A method of site matching of wind turbine generator problem based on normalized average power maximization is presented in this paper. The method is proposed to determine the operational parameters of the wind turbine, considering the maximization of energy production. The method is a useful tool to make an adequate choice of a wind turbine generator with optimum speed parameters to give higher energy.

A real wind data base has been evaluated to estimate the optimal wind turbine generator power curve and optimal operational parameters in order to yield the highest energy production.
It is found that the optimal rated speed of a wind turbine, that will be placed in site under consideration must, have about 11 m/s. thus, a wind turbine with the 3.5 m/s cut-in speed, 11 m/s rated speed and 20 m/s cut-off speed will allow yielding highest energy production.

Acknowledgement

This paper was supported by the project PERFORM-ERA "Postdoctoral Performance for Integration in the European Research Area" (ID-57649), financed by the European Social Fund and the Romanian Government.

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